Comment on "Coupling of Spin Waves with Zero Sound in Normal ^{3}He"

Ketterson recently proposed a pulsed NMR and acoustic experiment to study the transverse spin-wave excitations of normal ^{3}He.[1] He suggests preparing ^{3}He in a uniform, nonequilibrium state in which the magnetization is tipped at an angle θ_H relative to the static field $\vec{H}_0$ and rotates with angular velocity $\vec{\Omega}_0 = -\gamma \vec{H}_0$. The angle θ_H is constant for times short compared with the spin-relaxation time, which is long enough for sound to be used as a probe of the spin system. Ketterson also proposes that particle-hole asymmetry (pha), a small effect from the energy dependence of the density of states, provides the coupling between zero sound and the spin-wave modes. With ^{3}He prepared as a bath of homogeneous spin waves, zero sound can be used to excite spin waves with finite wave vector in a process where the $\vec{q}=0$ spin waves stimulate the decay of zero sound into spin waves with frequency $\omega_{\rm sw}(\vec{q}) = \omega(\vec{q}) - \Omega_0$. Thus, tuning the field and sound frequency ω to observe the decay of zero sound should determine the spin wave spectrum $\omega_{\rm sw}(\vec{q})$. Although Ketterson's idea is appealing, spin waves are not excited by zero sound as suggested in Ref. 1. This fact follows from spin rotation invariance. The initial nonequilibrium state is described by precession of the tipped magnetization, $\vec{M}_0 = \vec{R}[\theta_H \hat{y}]\cdot(\chi \vec{H}_0)$, about the static field; $\vec{M}(t) = \vec{R}[\vec{\Omega}_0 t]\cdot \vec{M}_0$. The corresponding density matrix is $\hat{\sigma}(\vec{M}) = \hat{\rho}[\hat{H} - \gamma\chi^{-1}\vec{M}\cdot\hat{\vec{S}}]$, where $\hat{\sigma}(\chi\vec{H}_0)$ is the equilibrium density matrix, $\hat{H}$ is the Hamiltonian, and $\hat{\vec{S}}$ is the total spin. Dipolar interactions are neglected (as in Ref. 1) so that $\hat{H}$ is invariant under spin rotations, implying that the linear response of the spin density $\hat{\vec{s}}(\vec{x})$ to a scalar field that couples to the density $\hat{n}(\vec{x})$ can be written in terms of operators carrying the time dependence generated by $\hat{H}$;

$$\vec{\chi}_{sn}[\vec{M}_0] = \vec{R}[\vec{\Omega}_0 t]\cdot(-i\theta(t-t')\mathrm{tr}\{\hat{\sigma}(\vec{M}_0)[\hat{\vec{s}}_H(\vec{x}t),\ \hat{n}_H(\vec{x}'t')]\}).$$

So $\vec{R}^{-1}[\vec{\Omega}_0 t]\cdot\vec{\chi}_{sn}$ (i.e., the response function in the rotating frame) is the *equilibrium* response function calculated in the tipped field $\chi^{-1}\vec{M}_0$. Thus, $\vec{\chi}_{sn}$ is simply described in terms of orthonormal coordinate axes $\{\hat{u}_1, \hat{u}_2, \hat{u}_3\}$, with $\vec{M}(t)$ parallel to $\hat{u}_3(t)$. For any spin rotation $\vec{R}[\Lambda]$,

$$\vec{\chi}_{sn}[\vec{M}_0] = \vec{R}[\vec{\Omega}_0 t]\cdot\vec{R}[\vec{\Lambda}]\cdot\vec{R}^{-1}[\vec{\Omega}_0 t]\cdot\vec{\chi}_{sn}[\vec{R}^{-1}[\vec{\Lambda}]\cdot\vec{M}_0];$$

specifically $\vec{\Lambda} = (\pi/2)\hat{M}_0$ implies $\vec{\chi}_{sn} = \chi^3 \hat{u}_3(t)$. In the rotating frame, $\vec{\chi}_{sn}$ is a purely longitudinal magnetic response along the same tipped field $\vec{M}_0$ that appears in the density matrix $\hat{\sigma}$. Since there are no longitudinal spin-wave modes in ^{3}He, χ^3 does not contain contributions from any spin-wave mode.

However, there is an interesting magnetic response to zero sound at frequencies $\omega \pm \Omega_0$ due to pha in Ketterson's proposed experiment. The particle-hole transformation, represented by a unitary operator C, is an approximate symmetry of the low-energy (quasiparticle) part of the Hamiltonian; $C\hat{H}C^\dagger = \hat{H} + $pha terms.[2] If all pha terms are neglected, then $C\hat{\vec{s}}C^\dagger = \hat{\vec{s}}$, $C\hat{n}C^\dagger = -\hat{n}$, $C\hat{\sigma}C^\dagger = \hat{\sigma}$, and $\vec{\chi}_{sn} = -\vec{\chi}_{sn} = 0$. The pha terms imply that $\vec{\chi}_{sn}$ is nonzero and first-order in pha, $\chi^3 \sim \eta\chi_{nn}$, when $\eta \sim \hbar\Omega_0/(\text{Fermi energy}) \ll 1$, and χ_{nn} is the density response function. The magnetic response to a density wave, $\delta n = \delta n(\vec{q}\omega)e^{i(\vec{q}\cdot\vec{x}-\omega t)}$, is $\delta\vec{m} = (\gamma\hbar/2)(\chi^3/\chi_{nn}) \times \delta n(\vec{x},t)\hat{u}_3(t)$; there is a magnetization wave induced by zero sound which is transverse to the static field $\vec{H}_0$,

$$\delta m_x \pm i\delta m_y = (\tfrac{1}{2}\gamma\hbar)\eta\sin\theta_H \delta n(\vec{x},t)\exp(\pm i\Omega_0 t). \quad (1)$$

These oscillations are the driven response of the magnetization along the instantaneous direction of the rotating magnetization $\vec{M}(t)$. In the stationary frame these oscillations are modulated by the uniform rotation of $\vec{M}(t)$. This is a different physical picture from that proposed by Ketterson; for $\theta_H \neq 0$ and with pha, the spin variables that couple to zero sound have no natural oscillations. They are not propagating spin waves; consequently, there is no resonant coupling between zero sound and the spin-wave system as is implied by the quantum decay process suggested in Ref. 1.

The pha effect described above may be observable since the induced magnetic oscillations, which are small in magnitude, are separated in frequency from both the unperturbed rotating magnetization and the sound wave.

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Received 19 March 1984
PACS numbers: 67.50.Dg

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[2] J. W. Serene, in *Quantum Fluids and Solids—1983,* edited by E. D. Adams and G. G. Ihas, AIP Conference Proceedings No. 103 (American Institute of Physics, New York, 1983), p. 305.

