Low-Frequency $1/f$ Fluctuations of Resistivity in Disordered Metals

August Ludviksson and Reiner Kree
Institut für Theorie der Kondensierten Materie, Universität Karlsruhe, D-7500 Karlsruhe, Federal Republic of Germany

and

Albert Schmid[(a)]
Physics Laboratory I, H. C. Ørsted Institute, University of Copenhagen, Copenhagen, Denmark
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The concept of tunneling systems has been very successful in explaining phenomena which occur in disordered solids at small energies and with long relaxation times. It is shown in this Letter that fluctuations in the tunneling systems will also produce fluctuations in the electric resistivity and that the power spectrum of these resistivity fluctuations is inversely proportional to the frequency.

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There is a rather large variety of metals which show fluctuations in resistivity on a very long time scale.[1] It has been found, in particular, that the power spectrum of these fluctuations is approximately inversely proportional to the frequency; for this reason, this phenomenon is commonly referred to as $1/f$ noise.[2]

Though there may be various mechanisms which produce this type of noise, it has been recognized[1] that activated random processes may easily lead to long-time fluctuations. It is the purpose of this note to recall that, besides thermal activation, quantum mechanical tunneling may play an important role, particularly at low temperatures. It is known that there are tunneling systems in disordered solids[3] which exhibit fluctuations on very large time scales; and we will show how resistivity fluctuations arise by coupling of conduction electrons to the tunneling systems.

There is no truly microscopic theory for tunneling systems available. However, one assumes[3] that one system consists of a generalized coordinate of ion configurations with two minima in the potential energy. These two states with bare energy $\pm\frac{1}{2}\Delta$ are connected by a tunneling matrix element $\Delta_0 = \hbar\omega_0 \exp(-\lambda)$, where ω_0 is of order of the Debye frequency. Consequently, this system has a Hamiltonian of the form

$$\hat{H}'_{\mathrm{TS}} = \frac{1}{2}\begin{pmatrix} \Delta & -\Delta_0 \\ -\Delta_0 & -\Delta \end{pmatrix} = \tfrac{1}{2}\Delta\hat{\sigma}'_3 - \tfrac{1}{2}\Delta_0\hat{\sigma}'_1. \tag{1a}$$

It is a most important experimental fact that the distribution $P(\Delta,\gamma)$ of a collection of tunneling systems is almost independent of Δ and λ; hence

$$P'(\Delta,\lambda) = \bar{P}. \tag{2a}$$

We take the interaction of one tunneling system with the conduction electrons to be of the form

$$\hat{H}'_{\mathrm{int}} = \frac{1}{V}\sum_{\vec{k}\vec{q}s} \tfrac{1}{2}[u(\vec{q})\hat{1}' + v(\vec{q})\hat{\sigma}'_3] c^{\dagger}_{\vec{k}s} c_{\vec{k}+\vec{q},s}, \tag{3a}$$

where $c^{\dagger}_{\vec{k}s}$, $c_{\vec{k}s}$ are the usual creation and annihilation operators of conduction electrons. This type of interaction is of standard form[3b] except for a new term proportional to the unit matrix. Though being irrelevant for relaxation processes, this term is most important here since it leads to a difference in the scattering probabilities $\frac{1}{4}|u(\vec{q}) \pm v(\vec{q})|^2$ of the electrons in the two states of the tunneling system.

The Hamiltonian (1a) can be diagonalized by a unitary transformation,

$$\hat{H}_{\mathrm{TS}} = \tfrac{1}{2}E\hat{\sigma}_3, \quad E = (\Delta^2 + \Delta_0^2)^{1/2}. \tag{1b}$$

Similarly, we obtain from Eq. (3a)

$$\hat{H}_{\mathrm{int}} = \frac{1}{V}\sum_{\vec{k}\vec{q}s} \tfrac{1}{2}[u(\vec{q})\hat{1} + (\Delta/E)v(\vec{q})\hat{\sigma}_3 + (\Delta_0/E)v(\vec{q})\hat{\sigma}_1] c^{\dagger}_{\vec{k}s} c_{\vec{k}+\vec{q},s}. \tag{3b}$$

Transitions in the tunneling system will be induced by the term proportional to $\hat{\sigma}_1$. Using Fermi's "golden rule,'' we obtain the relaxation rate for the return of the pseudospin $\hat{\sigma}_3$ to its equilibrium value[3]

$$\Gamma = \frac{\pi}{\hbar} N(0)^2 \langle |v|^2 \rangle_{\rm FS} \frac{\Delta_0^2}{E} \coth\frac{E}{2kT}, \tag{4}$$

where $N(0)$ is the electronic density of states (per spin and unit volume), and where $\langle \cdots \rangle_{\rm FS}$ means an appropriate average over the Fermi surface.

Of importance will be the power spectrum of the pseudospin fluctuations,

$$S(\omega) \equiv \langle (\delta\hat{\sigma}_3)^2 \rangle_\omega \equiv \int dt\, e^{i\omega t} \{ \tfrac{1}{2} \langle [\hat{\sigma}_3(t), \hat{\sigma}_3]_+ \rangle - \langle \hat{\sigma}_3 \rangle^2 \} = \hbar \chi''(\omega) \coth\frac{\hbar\omega}{2kT}.$$

The right-hand side follows from the dissipation-fluctuation theorem. In the following, we are interested in the regime where $\hbar\omega, \hbar\Gamma << kT$. Then the pseudospin fluctuations are purely thermodynamic fluctuations.[4] In this case one may assume that the susceptibility is of relaxational form, $\chi \propto \Gamma(-i\omega + \Gamma)^{-1}$. It follows that

$$S(\omega; E, \Gamma) = \frac{1}{\cosh^2 E/2kT} \frac{2\Gamma}{\omega^2 + \Gamma^2}, \tag{5}$$

where the prefactor is the magnitude of the ''equal-time'' fluctuations.[5]

We take the average of S with respect to the distribution (2a) most conveniently by introducing[3]

$$P(E, \Gamma) = \tfrac{1}{2} \bar{P} \Gamma^{-1} (1 - \Gamma/\Gamma_M)^{1/2}. \tag{2b}$$

Note that the crucial dependence on the inverse of Γ appears whenever Γ depends on some power of Δ_0. Thus, Eq. (2b) is valid also when, at higher temperatures, phonon scattering contributes predominantly to Γ. The maximal relaxation rate $\Gamma_M(E) = \Gamma(E, \Delta_0 = E)$ results from the requirement $\Delta_0 \leq E$. Quite generally, Γ_M is very large compared to the frequencies we are interested in. Thus, any corrections depending on Γ_M^{-1} may be neglected and we obtain

$$\bar{S}(\omega) = \int dE\, d\Gamma\, P(E, \Gamma)\, S(\omega; E, \Gamma) = (\pi kT/|\omega|) \bar{P}. \tag{6}$$

For the sake of simplicity, let us assume that the electrical resistivity is determined predominantly by scattering of electrons at static impurities. Thus, we define $\rho_0 = m/e^2 n \tau_{\rm imp}$, where the scattering rate is given by[6]

$$1/\tau_{\rm imp} = (2\pi/\hbar) N(0) n_{\rm imp} \langle |v_{\rm imp}|^2 \rangle_{\rm FS}.$$

In this relation, $v_{\rm imp}$ is the impurity potential, and $n_{\rm imp}$ is the impurity density.

Consider now the scattering of electrons at the tunneling systems. We observe that the duration of a scattering event is very short on the time scale of pseudospin fluctuations. This allows us to consider the pseudospins as adiabatic variables which have definite values at any instant of time. Consequently, the contribution of a tunneling system (labeled by j) to the scattering rate can be calculated according to[7]

$$\left(\frac{1}{\tau} \right)_{\rm TS}^j = \frac{\pi}{2\hbar} N(0) \frac{1}{V} \left\langle \left| u^j + \frac{\Delta^j}{E^j} v^j \hat{\sigma}_3^j(t) \right|^2 \right\rangle_{\rm FS}.$$

Evidently, this expression contains a time-dependent part[8] which is linear in $\hat{\sigma}_3^j(t)$. Thus, fluctuations in the pseudospins of the tunneling systems produce a time-dependent addition to the resistivity which we present in the form

$$\delta\rho(t) = (\partial\rho_0/\partial n_{\rm imp}) \frac{1}{V} \sum\nolimits_j \alpha^j \delta\hat{\sigma}_3^j(t), \tag{7a}$$

where

$$\alpha^j = \tfrac{1}{2} \operatorname{Re} \langle u^{j*} v^j \rangle_{\rm FS} / \langle |v_{\rm imp}|^2 \rangle_{\rm FS}. \tag{7b}$$

Note that $|\Delta^j/E^j| = 1$ is an adequate approximation in the present problem.

As a result of their local nature, there are no correlations between different tunneling systems. For simplicity, we will neglect any interdependence between the interaction potentials and the energy parameters of a given system. Using Eq. (6), we obtain for the power spectrum of resistivity fluctuations

$$S_\rho(\omega) \equiv \langle (\delta\rho)^2 \rangle_\omega = \left(\frac{\partial\rho_0}{\partial n_{\rm imp}} \right)^2 \langle \alpha^2 \rangle_{\rm av} \frac{1}{V} \bar{P} \frac{\pi kT}{|\omega|}, \tag{8}$$

where the quantity $\bar{P}$ has now to be taken per unit volume.

Concerning the magnitude of $\langle \alpha^2 \rangle_{\rm av}$, we consider the simplified model of a tunneling system presented in Sec. 8.3 of Ref. 3b. It consists of two ions with coordinates $\vec{R}_i = \vec{R}_i^0 \pm \vec{d}_i/2$ ($\hat{\sigma}_z = \pm 1$) and with scattering amplitudes $\mu(\vec{q}) \exp i\vec{q}\vec{R}_i$. Then $\frac{1}{2} \operatorname{Re} u^* v = 4|\mu|^2 \sin\Phi \sin\delta$, where $\Phi = \vec{q}(\vec{R}_1^0 - \vec{R}_2^0)$ and $\delta = \vec{q}(\vec{d}_1 - \vec{d}_2)/2$. One estimates that Φ and δ are of order 1 and 10^{-1}, respectively. Let us assume that $\langle \sin^2\Phi \rangle_{\rm av} = \frac{1}{2}$ and $\langle \sin^2\delta \rangle_{\rm av} = \frac{1}{2} \times 10^{-2}$. Furthermore, one expects $N(0)|\mu|$ as well as

$N(0)|v_{imp}|$ to be in the range 10^{-1}–1, which is typical for scattering centers of atomic dimensions. We take the smallest value. Therefore,

$$\langle\alpha^2\rangle_{av} = 10^{-2};\ (\partial\rho_0/\partial n_{imp}) = 10^{-26}\ \Omega\ \text{cm}^4;$$

$$\bar{P} = 10^{32}\ \text{erg}^{-1}\ \text{cm}^{-3},$$

where the last figure is considered to be typical for disordered solids.[3] Then, it follows that

$$S_\rho(\omega) = (0.7\times10^{-38}\ \Omega^2\ \text{cm}^5\ \text{K}^{-1})\,V^{-1}T(2\pi/|\omega|).$$

At $T=100$ K, the value of this expression is of the same order of magnitude as a typical experimental noise power.[9]

The $(1/f)$-frequency dependence in Eq. (8) is a consequence of the *Ansatz* (2), where the distribution function $P'(\Delta,\lambda)$ has been taken to be independent of λ. Clearly, there have to be restrictions on this form. The simplest one is in the form of an upper cutoff λ_{max} in the tunneling parameter which causes a low-frequency cutoff in the $(1/f)$ noise spectrum. From experimental evidence, one expects λ_{max} to be as large as 20. In addition, there is the possibility of a weak λ dependence in the distribution function. In this case, the noise power would be proportional to $f^{-\alpha}$, with α being close to 1.

The linear temperature dependence in Eq. (8) reflects the property of $P'(\Delta,\lambda)$ to be independent of Δ. One expects that this is true only for energies Δ smaller than, say,[10] 100 K. In case $P'=P'(\Delta)$ is independent of λ, one obtains a temperature dependence $S_\rho \propto \int d\Delta\, P'(\Delta)/\cosh^2(\Delta/2T)$.

The resistivity fluctuations as discussed here result from many independent and localized elementary processes. Thus, these fluctuations are Gaussian distributed and uncorrelated in space. However, there exist angular correlations of the type discussed by Weissman and Black *et al.*[11] In terms of the simplified model[3b] presented previously, such correlations depend on the relative orientation of the vectors $\vec{R}_1^0-\vec{R}_2^0$ and $\vec{d}_1-\vec{d}_2$.

We wish to recall the picture of a tunneling system where two minima in potential energy are separated by a barrier. As an estimate, we take the average height of the barrier equal to $2\hbar\omega_0\lambda$. Then, it is clear that for temperautres $kT \gg \hbar\omega_0$, thermal activation will dominate the transitions between the two minima (levels). Consequently, the relaxation rate of Eq. (4) should be replaced by

$$\Gamma = \omega_0\cosh(E/2T)\exp(2\hbar\omega_0\lambda/kT),$$

and one obtains a temperature dependence of S_ρ which is larger than the expression obtained previously by a factor of $kT/\hbar\omega_0$.

However, it is most likely that at higher temperatures, the concept of two-level systems loses its meaning. Rather, one expects multilevel systems which consist of many minima in the potential energy to become important. Also, the differences in energy may now be comparable or much smaller than the thermal energy. Collective aspects recede, and one will preferrably talk about reorientation and migration of defects. On this basis, a theory of thermally activated $1/f$ noise has been developed recently.[12]

We return to low temperatures and to tunneling systems where the present theory is also able to explain enhanced resistivity fluctuations of a metal at its superconducting transition. It is known that there is a weak dependence of the transition temperature on the disorder which is induced by a change in the electron-phonon coupling due to impurity scattering. According to a theory[13] there is a change in the transition temperature proportional to the scattering rate, $\delta T_c/T_c = \hbar b/\epsilon_F\tau_{imp}$, where[13] b may be of the order 1–10. That theory can also be applied to the present problem. In close analogy to $\delta\rho(t)$ of Eq. (7), we obtain now a time-dependent shift $\delta T_c(t)$ of the transition temperature. Furthermore, the power spectrum of these fluctuations is given by

$$S_{T_c}(\omega) \equiv \langle(\delta T)^2\rangle_\omega = \left(\frac{\partial T_c}{\partial n_{imp}}\right)^2 \langle\alpha^2\rangle_{av}\frac{kT\bar{P}}{V}\frac{2\pi}{|\omega|}. \qquad (9)$$

It is clear that at the resistive transition, fluctuations in T_c have the same effect as temperature fluctuations.[14] Therefore, $S_\rho(\omega) = \beta^2 S_{T_c}(\omega)$, where $\beta = \partial\rho/\partial T_c$ is large if the resistive transition is sharp. The observable effect may now be many orders of magnitude larger than far above T_c. As a disadvantage we note the difficulty of varying the transition temperature (e.g., by an applied magnetic field). Furthermore, no conclusion on spatial correlations is possbile since there is no simple theory of the resistive transition in a superconductor.

In conclusion, we have shown how to connect $1/f$ noise of resistivity fluctuations with tunneling systems. These systems are an inherent feature of disordered solids; but it is worthy of note that they have been found at rather large densities even in solids with almost perfect crystalline order.[15] Though there is no microscopic theory of tunneling systems available, their properties are already reasonably well known from experiments. Resistivity noise measurements could add further information, particularly for larger energies and for very small tunneling probabilities.

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[(a)]On leave from the Institut für Theorie der Kondensierten Materie, Universität Karlsruhe, Karlsruhe, Germany.

[1]For reviews, see P. Dutta and P. M. Horn, Rev. Mod. Phys. 53, 497 (1981); M. Nelkin, in *Chaos and Statistical Mechanics,* edited by Y. Kuramoto (Springer, Berlin, 1983).

[2]Occasionally, it is also called flicker noise.

They have been introduced by P. W. Anderson *et al.,* Philos. Mag. 25, 1 (1972), and by W. A. Phillips, J. Low Temp. Phys. 7, 351 (1972).

[3b]For a review, see J. L. Black, in *Glassy Metals,* edited by H. J. Güntherodt and H. Beck (Springer-Verlag, Berlin, 1981).

[4]L. D. Landau and I. M. Lifshitz, *Course of Theoretical Physics* (Pergamon, London, 1958), Vol. 5, Chap. 12.

[5]Note that the integral $\int d\omega/2\pi$ is taken from $-\infty$ to $+\infty$, whereas $f=\omega/2\pi$ is generally assumed to be positive.

[6]Strictly, the scattering rate appropriate to momentum relaxation should appear here. However, this leads only to changes in numerical coefficients. Therefore, we may ignore it here and in the following.

[7]The physics of this effect is very different from the one proposed by R. W. Cochrane *et al.,* Phys. Rev. Lett. 35, 676 (1975). There, the tunneling systems are coupled rather strongly to localized d electrons.

[8]The time-independent part should be added to $1/\tau_{\text{imp}}$.

[9]In the first paper of Ref. 1, the magnitude of voltage fluctuations at $f=20$ Hz is quoted to be $S_v=(3\times10^{-38}\ \Omega^2\ \text{sec cm}^5)J^2L/A$, where J is the current density and L and A are the length and the cross section of the sample.

[10]As a rule, properties of tunneling systems have been tested in the past only for low temperatures ($\lesssim 1$ K). Recently, however, it has been observed [U. Bartell and S. Hunklinger, J. Phys. (Paris), Colloq. 43, C9-489 (1982)] that an applied pressure did not change appreciably the low-temperature properties of a glassy substance. It has been estimated that the pressure applied contributes the energy $\gamma e\hat{\sigma}$ to the Hamiltonian (1a), where γe is about 150 K. Thus, one has to conclude that the distribution is rather flat in this range.

[11]M. B. Weissman, J. Appl. Phys. 51, 5872 (1980); R. D. Black *et al.,* Phys. Rev. Lett. 51, 1476 (1983).

[12]Sh. M. Kogan and K. E. Nagaev, Fiz. Tverd. Tela 24, 3381 (1982) [Sov. Phys. Solid State 24, 1921 (1982)].

[13]B. Keck and A. Schmid, J. Low Temp. Phys. 24, 611 (1976). Concerning the coefficient b, see Eq. (48) in this reference.

[14]In this respect, our theory provides an alternative to the thermal fluctuation model of R. F. Voss and J. Clarke [Phys. Rev. B 13, 556 (1976)].

[15]Cf. M. St. Paul *et al.,* J. Phys. C 15, 2375 (1982).