Nature of the Beam-Density Effect on Energy Loss by Nonrelativistic Charged-Particle Beams

D. W. Rule
Naval Surface Weapons Center, White Oak, Silver Spring, Maryland 20910

and

Oakley H. Crawford
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

(Received 28 October 1983)

The authors present a new formulation of the beam-density effect on energy loss by charged particles passing through matter, which exhibits an increased loss with a beam-shape dependence. This arises from a long-range dipolelike term contained in the two-particle vicinage function for cooperative energy loss by a pair of nonrelativistic particles. A new analytic expression for the vicinage function, which exhibits the long-range term, is also presented.

PACS numbers: 52.40.Mj, 29.70.Gn, 34.50.-s, 52.20.-j

Efficient coupling of beam energy to a target medium is of crucial importance to the viability of inertial-confinement fusion, as well as to other applications of charged-particle beams. For this reason much effort has been devoted to searches for energy-deposition–enhancement mechanisms.[1] The enhancements, observed or predicted, have been attributed to several phenomena; for example, the increase of effective path length in the target by applied or self-generated fields (see Ref. 1 for a review and original references), collective beam-target interactions,[2] modification of the single-particle deposition rate because of finite target temperatures,[3–5] and the beam-density effect, first considered by McCorkle and Iafrate.[6] The latter two effects are closely related in that the beam-density effect is *coherent,* cooperative energy deposition by beam particles, while finite–target-temperature effects can be viewed as the *incoherent* interaction of individual beam particles with free medium electrons which were produced by ionizing collisions of preceding beam particles.[5]

The origin of the beam-density effect is the two-particle vicinage, or proximity, contribution to energy loss. This phenomenon has been investigated extensively for nonrelativistic molecular ion clusters, both theoretically[7–11] and experimentally (see Gemmell[12] for an introduction to the experiments and further references). Derivations of the relativistic form of the proximity function have also been given, including the Fermi-density effect.[13,14] In this Letter we first present an analytic expression for the nonrelativistic vicinage function and show that it has a dipolelike behavior for large separations between beam particles. This long-range behavior, which has not previously been realized, causes the beam-density effect to depend on the shape of the beam. The implications of these results are that calculations of the beam-density effect based on the short-ranged expressions are not qualitatively nor quantitatively correct and that calculations based on angle averaging of the vicinage term before integration over a beam are also not generally correct. Finally, we present the beam-density effect obtained for a model beam shape.

A pair of particles separated by $\vec{R}_{ij}$ has a total energy loss per unit path length W_{ij} which can be written as the sum of the usual single-particle terms $W_S = Z^2 S$, plus the vicinage term $W(\vec{R}_{ij})$:

$$W_{ij} = Z_i^2 S + Z_j^2 S + W(\vec{R}_{ij}), \tag{1}$$

where Z is the nuclear charge and S is the stopping power of a proton. If the two particles coalesce, then $\vec{R}_{ij} \to 0$ and $W(\vec{R}_{ij}) \to 2Z_iZ_jS$; therefore,

$$\lim_{R_{ij}\to 0} W_{ij} = (Z_i + Z_j)^2 S. \tag{2}$$

If attention is focused on the ith particle in a beam consisting of N particles, then the total energy loss by that particle is $W = W_S + W_B$, where

$$W_B = \tfrac{1}{2}\sum_{j=1}^{N} W(\vec{R}_{ij}), \quad j \neq i, \tag{3}$$

is the contribution from cooperative energy loss, the beam-density[6] term. The factor of $\frac{1}{2}$ in (3) eliminates double counting of pairs.

The starting point for the derivation of W_B is the proximity, or vicinage, term for energy loss by a pair of relativistic particles with charges Z_1 and Z_2 as derived in Ref. 13, under the assumption of equal velocities,

$$W(z,b) \equiv (4/\pi) Z_1 Z_2 e^2 (\omega_p/v)^2 \mathrm{Re}\int_0^\infty i\,d\nu\,\nu \cos(\omega_p z\nu/v)\epsilon^{-1}(1-\beta^2\epsilon) \times K_0(b_> \omega_p \nu(1-\beta^2\epsilon)^{1/2}/v), \quad b_> = \max(a,b), \tag{4}$$

where a is the minimum impact parameter. The Fourier transform frequency ν is in units of ω_p with $\omega_p^2 = 4\pi Ne^2/m$, where N is the total electron density of the medium and e and m are the electronic charge and mass. The components of $\vec{R}_{ij}$ which are parallel and transverse to the velocity are z and b, respectively, and $\beta = v/c$. $K_0(Z)$ is a modified Bessel function.

The expression for the inverse of the dielectric function, $\epsilon^{-1}(\nu)$, is taken to be that developed by Sternheimer[15] in his calculations of the Fermi-density effect:

$$\epsilon^{-1}(\nu) \approx 1 + \sum_{j=1}^{n} \frac{f_j}{(\nu-\alpha_j)(\nu-\beta_j)}, \tag{5}$$

where

$$\alpha_j = -i\eta_j + (l_j^2 - \eta_j^2)^{1/2}, \quad \beta_j = -i\eta_j - (l_j^2 - \eta_j^2)^{1/2}, \tag{6}$$

with $l_i^2 = \rho^2 \hat{\nu}_i^2 + f_i$. The Sternheimer factor[16] ρ is adjusted to reproduce the experimentally determined values of the Bethe logarithm, $\ln I = \sum f_i \ln(\hbar\omega_p l_i)$. In Eqs. (5) and (6), $\hat{\nu}_i$ is the ionization potential for the ith subshell of an atom and $2\eta_i$ is the linewidth for bound electrons, while for free electrons $2\eta_i$ is a plasmon width in conductors, or the collision frequency in plasmas, all in units of ω_p. The oscillator strengths f_i are normalized so the $\sum f_i = 1$. If ϵ represents a material containing a fraction f_i of free electrons, then $\hat{\nu}_i = 0$ and their associated plasma frequency is $l_i\omega_p = f_i^{1/2}\omega_p$.

The beam loss for our model beam can now be written in terms of the proximity function $W(z,b)$ of Eq. (4) as

$$W_B = \tfrac{1}{2}\int_{-\infty}^{\infty} dz \int_0^{2\pi} d\theta \int_0^\infty db\, b\rho_B(z,b)\, W(z,b), \tag{7}$$

where the beam density is $\rho_B(z,b)$.

In order to investigate the beam-shape dependence of W_B in more detail for finite beams, we first evaluate $W(z,b)$ for the nonrelativistic case in which $\beta = 0$ in $W(z,b)$, Eq. (4). The frequency integration is accomplished by use of contours around the first and fourth quadrants of the complex ν plane.[14] Looking at Eqs. (5) and (6), one can see that the contour around the fourth quadrant encloses poles at $\nu = \alpha_j$. The contribution of the residues of these poles to $W(z,b)$ is

$$W_R = 4Z_1 Z_2 e^2 \left(\frac{\omega_p}{v}\right)^2 \mathrm{Re}\sum_{j=1}^{N} \frac{f_j \alpha_j}{\alpha_j - \beta_j} \exp\left(-\frac{i\omega_p}{v}|z|\alpha_j\right) K_0\left(\frac{b\omega_p}{v}\alpha_j\right). \tag{8}$$

The integrals along $\nu = \pm iy$ yield a nonresonant contribution to $W(z,b)$, W_N, which may be evaluated numerically. For large r, the nonresonant contribution has a dipolelike dependence $P_2(\cos\theta) r^{-3}$ to lowest order, which may be seen in the asymptotic expansion for W_N[17]:

$$W_N \approx -4Z_1 Z_2 e^2 \left(\frac{\omega_p}{v}\right)^2 \mathrm{Re}\sum_{j=1}^{N} f_j \frac{i\alpha_j}{\alpha_j - \beta_j} \sum_{k=1}^{\infty} (-1)^k (2k)! \left(\frac{\alpha_j \omega_p r}{v}\right)^{-(2k+1)} P_{2k}(\cos\theta), \tag{9}$$

where $r = (z^2 + b^2)^{1/2}$ and $\cos\theta = z/r$. With use of this, an asymptotic expression for $W(z,b) = W_R + W_N$ is obtained.

Returning to Eq. (7), the general expression for W_B, we now consider a Gaussian radial profile for the beam density,

$$\rho_B(z,b) = n_b \exp[-(b/b_0)^2]\theta(z+L_1)\theta(L_2 - z), \tag{10}$$

where θ is the Heaviside step function. Using this density function in Eq. (7) and performing the integrals over z and b, while neglecting the small correction for $b < a$, Eq. (4), one obtains

$$W_B = n_b b_0^2 e^2 \frac{\omega_p}{v} \mathrm{Re}\int_0^\infty i\,d\nu \left[\sin\left(\frac{\nu\omega_p}{v}L_1\right) + \sin\left(\frac{\nu\omega_p}{v}L_2\right)\right] \frac{1}{\epsilon(\nu)} \exp\left[\left(\frac{\nu\omega_p b_0}{2v}\right)^2\right] E_1\left(\left(\frac{\nu\omega_p b_0}{2v}\right)^2\right), \tag{11}$$

where $E_1(x)$ is the exponential integral function.

This integral has been treated by two methods: (a) A numerical contour integration was done along the same contours used for $W(z,b)$ and (b) the integrand was approximated by replacing $e^x E_1(x)$ with $\ln(1+1/x)$. The result was integrated analytically along $\nu = \pm iy$ to obtain the nonresonant contribution W_N to W_B. If the terms in Eq. (11) containing L_1 and L_2 are considered separately, then we can write $W_B = W_B(L_1,b_0) + W_B(L_2,b_0)$. Furthermore, the resonant and nonresonant parts of $W_B(L_1,b_0)$ can be denoted by $W_R(L_1,b_0)$ and $W_N(L_1,b_0)$.

The resonant terms from the poles of $\epsilon^{-1}(\nu)$ at $\nu = \alpha_j$ are

$$W_R(L_1,b_0) = \pi b_0^2 n_b e^2 \frac{\omega_p}{v} \operatorname{Re} \sum_{j=1}^{N} \frac{if_j}{\alpha j - \beta_j} \exp\left(\frac{-i\alpha_j\omega_p}{v} L_1\right) \ln\left[1 + \left(\frac{2v}{\alpha_j \omega_p b_0}\right)^2\right]. \tag{12}$$

The nonresonant terms are

$$W_N(L_1,b_0) = -\pi b_0^2 n_b e^2 \frac{\omega_p}{v}$$
$$\times \operatorname{Re} \sum_{j=1}^{N} \frac{if_j}{\alpha_j - \beta_j} \left\{ \exp\left(\frac{-i\alpha_j\omega_p}{v} L_1\right) \left[E_1\left(\frac{-i\alpha_j\omega_p}{v} L_1\right) - E_1\left(\frac{-i\alpha_j\omega_p}{v} L_1 + \frac{2L_1}{b_0}\right)\right] \right.$$
$$\left. + \exp\left(\frac{i\alpha_j\omega_p}{v} L_1\right) \left[E_1\left(\frac{i\alpha_j\omega_p}{v} L_1\right) - E_1\left(\frac{i\alpha_j\omega_p}{v} L_1 + \frac{2L_1}{b_0}\right)\right] \right\}. \tag{13}$$

$W_N(L_1,b_0)$ depends on the ratio L_1/b_0; therefore W_B exhibits a beam-shape dependence originating from W_N, Eq. (9). Thus, an angle averaging of $W(z,b)$ will not result in a physically meaningful calculation of W_B: Note that the angle average of $P_{2k}(\cos\theta)$ in Eq. (9) gives zero. Furthermore, simple arguments, based on field screening distances taken from single-particle stopping theory, are not adequate to determine the qualitative nature of the beam-density effect.

Equations (12) and (13) treat bound and free electrons in materials on the same basis and are valid for solids, gases, and plasmas, when appropriate values are used for the parameters in Eqs. (5) and (6). From Eqs. (12) and (13) we can show that W_B is proportional to n_b, inversely proportional to the square root of the medium density through ω_p, and inversely proportional to $|\alpha_j|^{-4}$. The scaling of the functional dependence of W_B on the beam shape is given by $v/|\alpha_j\omega_p|$. Thus the beam-density effect is important mainly when there are conduction electrons present, but, if the $\hat{\nu}_i$ represent bound-bound transitions of sufficiently low energy, they could also be important for current densities achievable in the laboratory. We have calculated W_B for solid-density aluminum and found that even for mega-ampere per square centimeter current densities, W_B was negligible. This led us to consider weakly ionized gases for which the smallest value of $|\alpha_j\omega_p|$, for the free electrons, can be orders of magnitude smaller than for metals.

Figure 1 illustrates the evaluation of W_B, using Eqs. (12) and (13), for a semi-infinite beam with a Gaussian radial profile and a step-function head. The ratio of the beam contribution to the single-particle energy loss W_S is plotted versus the distance L_2 of a beam particle from the beam front on the beam's axis. Since $L_1 \rightarrow \infty$ for a semi-infinite beam, we have $W_B = W_R(L_2,b_0) + W_N(L_2,b_0)$,

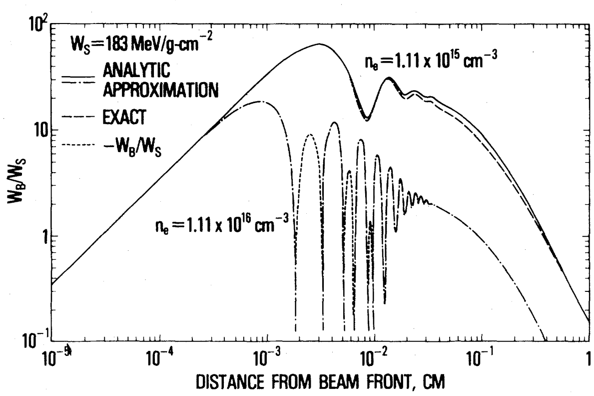

FIG. 1. The ratio of the beam-density contribution W_B to the single-particle energy loss W_S vs the distance of a beam particle from the beam front. The curves were generated for a 5-MeV proton beam with beam density $n_b = 6.44 \times 10^{14}$ cm^{-3} interacting with partially ionized H_2 containing n_e free electrons per cubic centimeter.

Eqs. (12) and (13) with $L_1 \rightarrow L_2$. The curves were calculated for a 5-MeV proton beam of 10 kA with a Gaussian radial parameter $b_0 = 1$ mm, i.e., $n_b = 6.44 \times 10^{14}$ cm^{-3}. The beam interacts with weakly ionized H_2 having a density of 2.67×10^{19} cm^{-3}, corresponding to $\hbar\omega_p = 0.272$ eV.[15,16] A free-electron density of $n_e = 1.11 \times 10^{16}$ cm^{-3}, which occurs for H_2 with $kT \approx 0.75$ eV, was assumed, yielding $\hbar|\alpha_e \omega_p| = 3.92 \times 10^{-3}$ eV.

The dash-dotted curve in Fig. 1 for $n_e = 1.11 \times 10^{16}$ cm^{-3} exhibits an oscillatory behavior from 0 to 0.01 cm from the beam front, while at larger distances W_B / W_S is positive. The oscillations come from the resonant contribution W_R, Eq. (12), which is dominant at small distances. When n_e is reduced by an order of magnitude (solid curve), the positive, nonresonant term W_N, Eq. (13) is dominant. The qualitative behavior of these results is not sensitive to beam energy. The figure shows that for this case the stopping power of a particle near the beam front is increased by nearly two orders of magnitude. This large value of W_B at the beam head mirrors the behavior of the phenomenon of return currents which are driven by the electric fields induced by the rapidly varying current at the head and tail of a finite pulse.[18,19] The dashed curve in the case of $n_e = 1.11 \times 10^{15}$ cm^{-3} was obtained by using a numerical integration of Eq. (11) along contours on the positive and negative imaginary axis to obtain W_N. This indicates that the error in using the analytic approximation does not exceed $\sim 20\%$.

We have shown that the nonresonant terms play a significant role in determining the long-range behavior of the two-particle cooperative energy loss in contrast to the single-particle energy loss for which the resonances of the inverse of the dielectric function are dominant in the nonrelativistic regime. The dipolelike behavior of the vicinage function for larger distances has not been noticed previously; however, a dipole back-flow current around a charged particle traveling in a conducting medium has been discussed by Pines and Noizeres.[18] It can be shown that the beam-density effect encompasses wakes[7,9] behind charged particles in solids, multiparticle vicinage effects on collisional energy loss,[8,10-14] and also the fields that drive the return currents giving rise to Ohmic loss (for a review see Ref. 19). These may all be considered as different aspects of the inductive energy loss caused by the interaction of a current with the electric field that it induces in the medium.

The results of the present study should contribute to the understanding of the energy deposition process of intense beams in applications such as beam transport in gases, the interaction of beams with ablation plasmas from fusion targets, and their interaction with the transient conduction electrons that they produce in ionic solids.

This work was supported by the Directed Energy Program Office of the Naval Sea Systems Command and by the Naval Surface Weapons Center's Independent Research Program. The Oak Ridge National Laboratory is operated for the U.S. Department of Energy by Union Carbide Corporation under Contract No. W-7405-eng-26. We are grateful to Dr. R. H. Ritchie for many helpful discussions on wake effects and related phenomena. We also wish to express our gratitude to Dr. George Hudson for numerous beneficial conversations on the subject of this research.

[1]Alan J. Toepfer, Adv. Electron. Electron Phys. 53, 1 (1980).

[2]K. Imasaki *et al.*, Phys. Rev. Lett. 43, 1937 (1979), and Appl. Phys. Lett. 37, 533 (1980).

[3]E. Nardi and Z. Zinamon, Phys. Rev. A 18, 1246 (1978).

[4]Thomas A. Mehlhorn, J. Appl. Phys. 52, 6522 (1981).

[5]F. C. Young *et al.*, Phys. Rev. Lett. 49, 549 (1982).

[6]R. A. McCorkle and G. J. Iafrate, Phys. Rev. Lett. 39, 1263 (1977).

[7]W. Brandt, A. Ratkowski, and R. H. Ritchie, Phys. Rev. Lett. 33, 1325 (1974), and 35, 130(E) (1975); Werner Brandt and R. H. Ritchie, Nucl. Instrum. Methods 132, 43 (1976).

[8]N. R. Arista and V. H. Ponce, J. Phys. C 8, L188 (1975).

[9]P. M. Echenique, R. H. Ritchie, and Werner Brandt, Phys. Rev. B 20, 2567 (1979).

[10]J. Steinbeck and K. Dettmann, J. Phys. C 11, 2907 (1978).

[11]George Basbas and R. H. Ritchie, Phys. Rev. A 25, 1943 (1982).

[12]Donald S. Gemmell, Nucl. Instrum. Methods 194, 255 (1982).

[13]D. W. Rule and M. H. Cha, Phys. Rev. A 24, 55 (1981).

[14]D. W. Rule, in *High Power Beams '81, Proceedings of the Fourth International Topical Conference on High-Power Electron- and Ion-Beam Research and Technology, Palaiseau, France,* edited by H. J. Doucet and J. M. Buzzi (Ecole Polytechnique, Palaiseau, 1981).

[15]R. M. Sternheimer, Phys. Rev. 88, 851 (1952), and 91, 256 (1953).

[16]R. M. Sternheimer, Phys. Rev. 103, 511 (1956).

[17]D. W. Rule and O. H. Crawford, to be published.

[18]David Pines and Philippe Nozières, *The Theory of Quantum Liquids* (Benjamin, New York, 1966), Vol. 1.

[19]G. Wallis *et al.*, Usp. Fiz. Nauk 113, 435 (1974) [Sov. Phys. Usp. 17, 492 (1975)].