Sustained Self-Reversal in the Reversed-Field Pinch

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(Received 25 August 1983)

Spontaneous reversal of the toroidal field in a reversed-field pinch as a result of low-β (small $J_\perp/J_\parallel$) resisitive kink mode activity is investigated with use of a three-dimensional magnetohydrodynamics code. Helical and three-dimensional steady reversed states are obtained. In three dimensions quasisteady fluctuating states are observed above a critical value of the pinch parameter θ.

PACS numbers: 52.55.Ez, 52.30.+r, 52.65.+z

The reversed-field pinch[1] (RFP), like the tokamak, is an axisymmetric toroidal device with both toroidal, B_ζ, and poloidal B_ϑ, magnetic fields. In contrast to the tokamak, however, B_ϑ and B_ζ are of the same order, and the safety factor $q=rB_\zeta/RB_\vartheta$ (r minor, R major toroidal radii) is less than 1 everywhere in the plasma. Magnetohydrodynamic (MHD) stability is provided by a close-fitting conducting wall and high magnetic shear associated with a q profile that reverses near the wall. The RFP state has been observed to arise spontaneously and persist for times longer than τ_r, the resistive diffusion time, when the pinch parameter $\theta=B_{\vartheta w}/\langle B_\zeta\rangle$ is large enough, where $B_{\vartheta w}$ is the surface averaged poloidal field at the wall, and the brackets denote the volume average. A cylindrically symmetric steady-state RFP is inconsistent with resistive diffusion.[2] Nonsymmetric plasma dynamics is required to sustain the magnetic fields, and in this respect, the RFP observations are closely related to the classical dynamo problem.

Taylor[3] showed that dissipative relaxation of an isolated system in the presence of a conjectured global conserved quantity leads to a force-free metastationary state in which the field is given by the Bessel-function model (BFM). This model predicts a universal relation between θ and $F=B_{\zeta w}/\langle B_\zeta\rangle$ for such systems. A driven MHD system was shown to undergo self-reversal by Sykes and Wesson[4] who simulated only the initial phase of a very dissipative system.

In this Letter, results of numerical simulations of driven systems with more realistic resistive dissipation and for times on the order of τ_r are reported. The results indicate that low-β resistive kink modes lead to self-reversal which may be sustained for many times τ_r.

A recently developed algorithm[5] is used to provide time advancement of the resistive incompressible MHD equations. The simulation domain is a cylinder with ends periodically identified (cylindrical torus). The initial state is a zero-β, nonreversed, force-free equilibrium. The resistivity is taken to be constant in time and increasing in r. Boundary conditions are chosen to simulate a driven system. The poloidal electric field $E_{\vartheta w}$ vanishes so that toroidal flux is conserved. The toroidal electric field $E_{\zeta w}$ is a function of time. It is zero until the current resistivity decays to a specified value. When $\theta(t)$ reaches $\theta_{\min}$ (specified), $E_{\zeta w}$ is applied to keep the current constant subsequently. Typical simulations are carried out with a magnetic Reynolds number $S(r)=\max(S_0(1-r^4)^2, 10)$, where $S_0=5\times10^3$. The viscous Reynolds number is $\sim10^4$, and the aspect ratio is 1. Fourier expansion is used to represent the poloidal and toroidal variations, with sixteen modes in the two-dimensional (2D) and sixty modes in the 3D calculations. A radial grid of 64 points is employed. The aspect ratio and S were chosen to limit the number of Fourier modes and the radial grid points required.

Initial toroidal current produces a safety-factor profile which is unstable to various $m=1$ modes, where m is the poloidal mode number. For the typical q profile of Fig. 1, $m/n=\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ modes are unstable with growth rates decreasing with increasing n, the toroidal mode number. The nonlinear evolution of a single mode can be followed in a single-helicity (2D) calculation in which only modes with a fixed helicity m_0/n_0 are retained. Resonant modes such as $m/n=\frac{1}{3}$ with a rational surface in the plasma saturate in a reversed state in a flux-conserving system, after the double reconnection previously reported.[6] The helically deformed current channel increases the toroidal field on axis. A reversed $B_{\zeta w}$ then develops to conserve flux.[7] Nonresonant modes such as $m/n=\frac{1}{2}$ exhibit qualitatively similar behavior, although they go through at most a single reconnection. The time

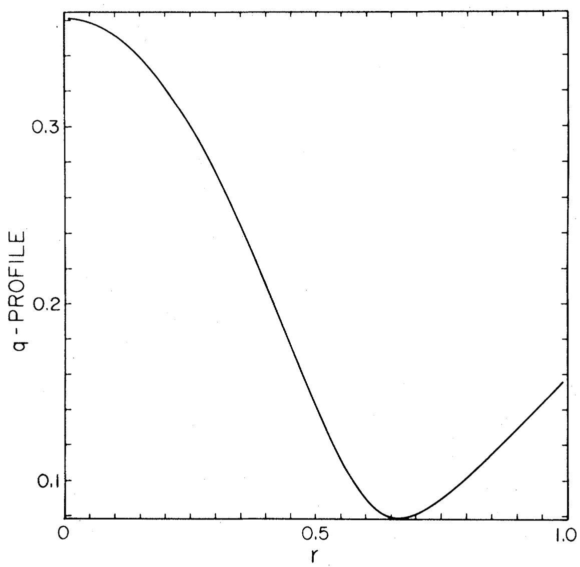

FIG. 1. Initial q profile.

evolution of $B_{\zeta w}$ for two different boundary conditions, $\theta_{\min}=0$, and $\theta_{\min}=1.6$, is shown in Fig. 2. The initial reduction in $B_{\zeta w}$ in both cases parallels the growth and saturation of the instability in a helical state. With no applied $E_{\zeta w}$ ($\theta_{\min}=0$), there is no further decrease in $B_{\zeta w}$. In the second case, however, $B_{\zeta w}$ is driven negative after the current is clamped by a further increase in the amplitude of the helix. This behavior can be explained as follows in terms of the F-θ relation. With $\theta_{\min}=0$, after the saturation of the instability the plasma relaxes towards a point on the F-θ curve (not necessarily that of the BFM[3]) along a trajectory with an approximately constant F and decreasing θ (Fig. 3). Ultimately resistive decay would carry the system along the F-θ curve to the point $(F,\theta)=(1,0)$.

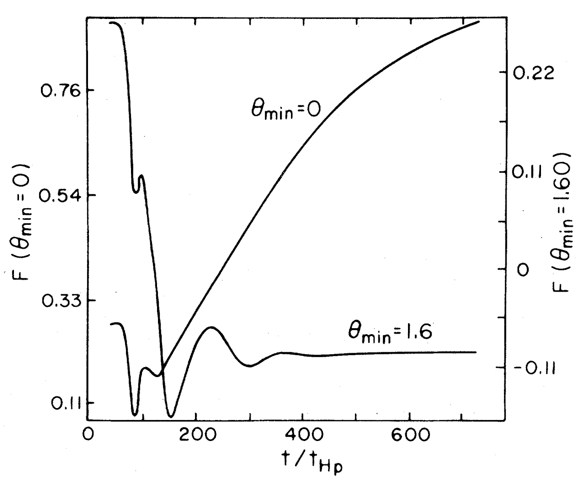

FIG. 2. Field-reversal parameter in helical calculations with helicity $\frac{1}{3}$.

With the constraint $\theta \geqslant 1.6$, however, the plasma can approach the preferred F-θ curve only along a trajectory with decreasing F, thus decreasing $B_{\zeta w}$. In this case, with a positive $E_{\zeta w}$ maintaining the toroidal current, the plasma eventually reaches a steady state at $(F,\theta)=(-0.10, 1.60)$. Similar calculations have been carried out for $1\times\tau_r$. Extrapolation of the results indicates that the observed state is steady for at least $100\tau_r$. The final equilibrium is characterized by large velocity flows maintaining the reversal against resistive diffusion. It differs from the BFM equilibrium in that the fluid velocity is not zero, and the currents vanish near the wall.

In a second, fully three-dimensional series of calculations, the equilibrium is perturbed with two modes of different helicities, $m/n=\frac{1}{2}$ and $\frac{1}{3}$, and the evolution of these modes and others nonlinearly generated by them is followed. While the 2D simulations always lead to a final steady state, the interaction of many helicities in 3D produces steady solutions only for $\theta < 1.55$. For these steady solutions, the modes with the highest linear growth rate, $m/n=\frac{1}{2}, \frac{1}{3}$, dominate throughout the calculation, and the final state is basically a superposition of these two modes. The total toroidal field on the wall shows a strong $m=1$ variation. The average field $B_{\zeta w}$ is reversed only for $\theta > \theta_r$, where $\theta_r \cong 1.45$, the exact value depending on the resistivity profile, and the initial conditions. The time evolutions of $B_{\zeta w}$ and kinetic energy for $\theta_{\min}=1.5$

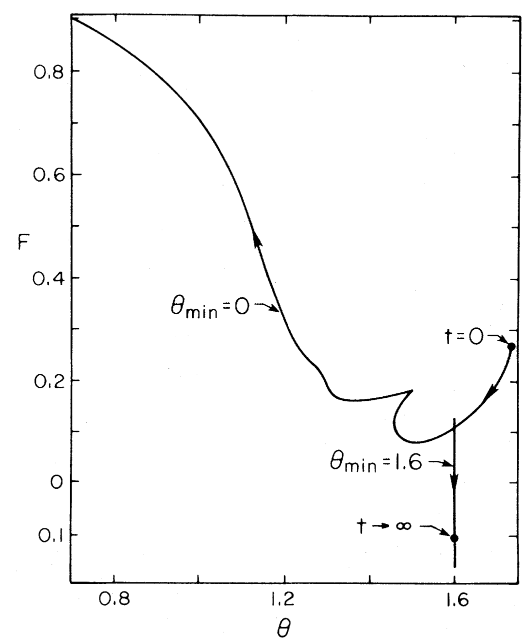

FIG. 3. F-θ diagram for helical calculations.

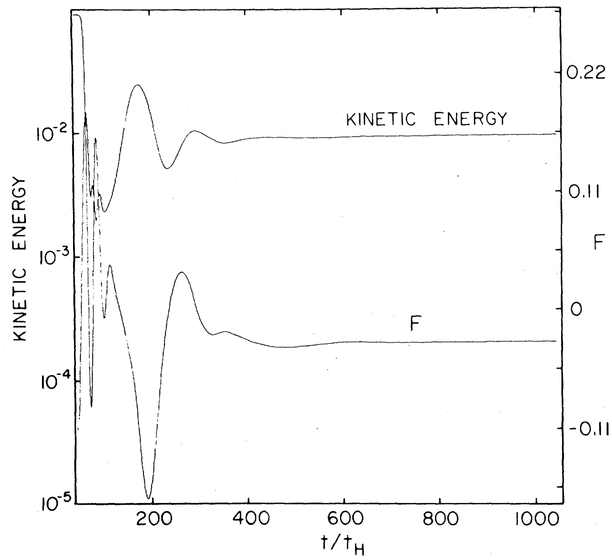

FIG. 4. Kinetic energy and F for $\theta_{min} = 1.5$ (3D).

are shown in Fig. 4 and clearly indicate a steady reversed state.

For $\theta > 1.55$, the evolution of the plasma is markedly different in that the state variables, instead of becoming constant in time, exhibit fluctuations about steady-state mean values. The $\theta_{min} = 1.6$ case shown in Fig. 5 has the same initial conditions as that pictured in Fig. 4. For this case an interesting period doubling occurs after $t = 800$. As seen in Fig. 5, reversal is lost around $t = 700$, which is accompanied by the return of the current to a more symmetric state. However, in this nonreversed state, the $m/n = \frac{1}{2}$ mode becomes unstable again [Fig. 6(a)], which drives the plasma

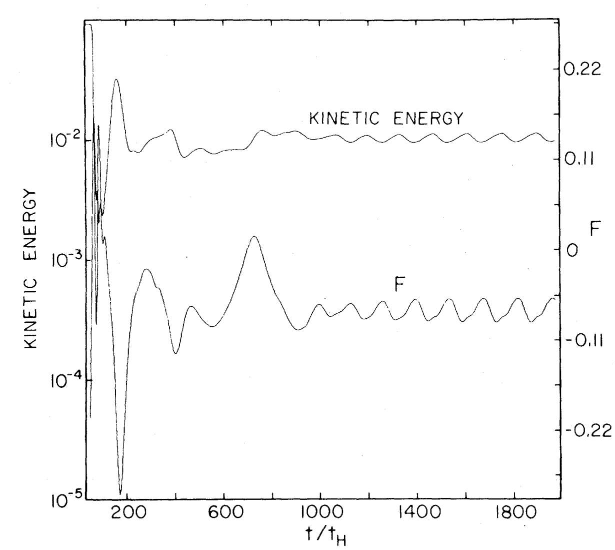

FIG. 5. Kinetic energy and F for $\theta_{min} = 1.6$ (3D).

into a state with half its original periodicity length. This is evident in the plots of mode amplitudes which show, for $t > 900$, periodic fluctuations in the energy of the modes with even toroidal mode number [Fig. 6(a)], while the energy in the modes with odd toroidal mode number decays exponentially [Fig. 6(b)]. For $t > 1000$, the plasma is in a quasisteady state in which the reversal is maintained by low-level mode activity which displays limit cycles about a mean state. As θ_{min} in increased, these fluctuations become larger in amplitude and aperiodic.

A characteristic of the 3D calculations that has a great bearing on confinement is the rapid annihilation of flux surfaces in all sustained reversal cases. In Fig. 7(a), the shift of the flux surfaces is due to the saturated, nonresonant $m/n = \frac{1}{2}$ mode. The resonant $m/n = \frac{1}{3}$ mode that is still growing has

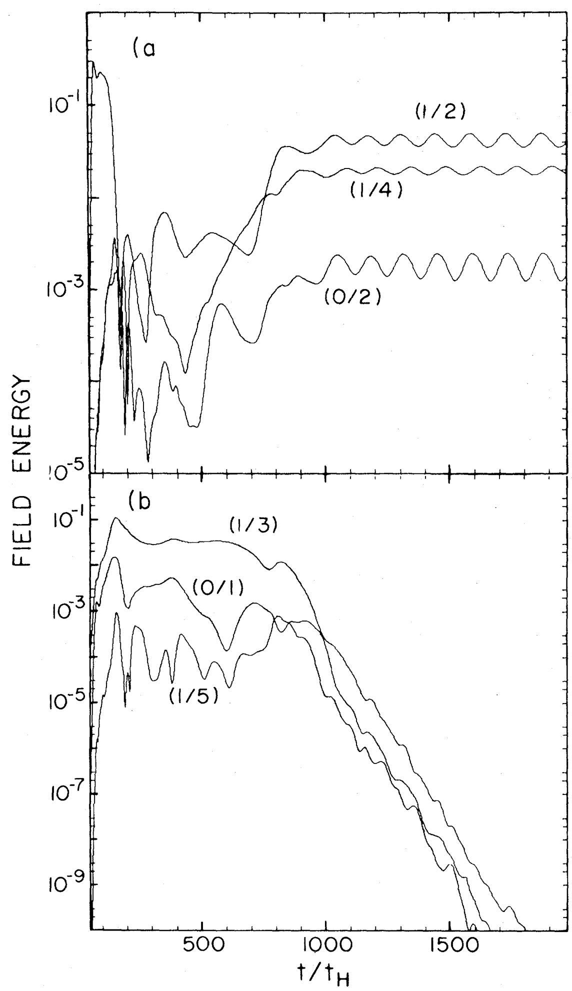

FIG. 6. Selected mode energies: (a) even toroidal mode numbers, (b) odd toroidal mode numbers ($\theta_{min} = 1.6$, 3D).

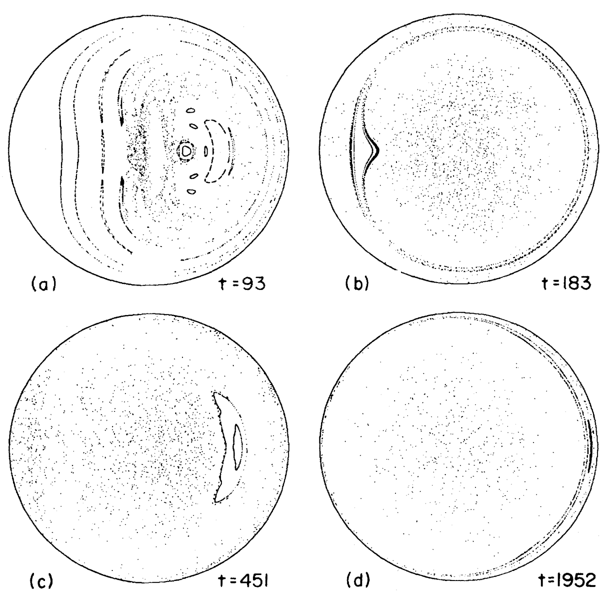

FIG. 7. Field-line traces ($\theta_{min} = 1.6$, 3D).

produced an island chain, and a region of stochastic field lines around it. As the mode amplitude increases, this region quickly grows and essentially fills the whole plasma volume [Fig. 7(b)]. At $t = 451$ [Fig. 7(c)] when $\frac{1}{3}$ is the dominant mode, except for a $\frac{1}{3}$ island, there are no good surfaces left, although some reemerge near the wall after the plasma reaches a quasisteady state [Fig. 7(d)].

In conclusion, low-β resistive kink activity leads to reversal in both 2D helical and full 3D calculations. If the plasma current is maintained, it is always possible to find 2D steady states in which dynamo action of large helical flows maintains the fields against resistive diffusion. In 3D calculations, there is a critical value of the pinch parameter above which there are only quasisteady solutions with fluctuation levels increasing with θ. However, low-level mode activity still maintains the mean fields against resistive diffusion when external sources maintain the toroidal current. The fluctuations are analogous to other nonlinear systems in which the increase of a parameter leads from a stationary state to a limit cycle to aperiodic fluctuations. Finally, the implications of the observed stochastic magnetic field on confinement need to be investigated.

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