Efficiency Enhancement in Free-Electron Lasers Using a Tapered Axial Guide Field

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A scheme for efficiency enhancement in free-electron lasers is described which employs a tapered axial guide field. An analytical description of this process is given in the strong-pump regime. The efficiency enhancement occurs in such a way that the axial velocity remains constant as kinetic energy is extracted from the beam. Results indicate that efficiency enhancements of as much as 100% are possible with only modest gradients in the axial field.

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Free-electron lasers (FEL's) offer the promise of a continuously tunable source of extremely high-power coherent radiation from millimeter waves through visible wavelengths; however, the intrinsic efficiency of the FEL is low, ranging from $<1\%$ at visible wavelengths up to $\sim 10\%$ for millimeter waves. For that reason, much attention has been given in the literature to schemes for FEL efficiency enhancement. The efficiency enhancement schemes which have been proposed to date[1] have been concerned principally with FEL's at infrared wavelengths which typically employ relatively high-energy ($\gtrsim 30$ MeV) and low-current ($\lesssim 1$ A) electron beams propagating through a linearly polarized wiggler magnetic field. As a consequence, it is technically convenient to taper either the period or amplitude of the wiggler field, and successful experiments have been conducted using each of these approaches to enhancement of the extraction efficiency. In contrast, millimeter-wave FEL's[2-4] make use of relatively-low energy (~ 1 MeV) but high-current (~ 1 kA) electron beams, and an axial guide field is often required in order to confine the beam against the effects of self-fields. In addition, the bulk of the experiments in this regime have employed helically polarized wiggler fields generated by means of a bifilar helical current winding. For these reasons, it is a technically simpler procedure to taper the axial guide field than it is to taper the wiggler field. Indeed, preliminary experiments in efficiency enhancements by axial field taper have been performed[5] and increases in the output power of 100% over the observations for the uniform guide field have been measured. It is our attention in this paper to explain the fundamental nature of the interaction, and to provide a quantitative description of the process in strong-pump (Compton) regime in which collective effects may be ignored.

The nature of the interaction can best be understood from a consideration of the single-particle dynamics. We shall assume an idealized one-dimensional wiggler and a slowly varying guide field, so that the external magnetic field can be represented as

$$\vec{B}(z) = [B_0 + \delta B_0(z)]\hat{e}_z + B_w(\hat{e}_x \cos k_w z + \hat{e}_y \sin k_w z), \tag{1}$$

where B_w is the amplitude of the wiggler field, k_w ($=2\pi/\lambda_w$, where λ_w is the wiggler period) is the wiggler wave number, B_0 is a constant denoting a bulk amplitude of the guide field, and $\delta B_0(z)$ describes the taper of the guide field such that $|\delta B_0/B_0| << 1$. The radiation field is represented by a plane wave

$$\delta\vec{A}(z,t) = \delta A(z)[\hat{e}_x \cos(kz-\omega t) - \hat{e}_y \sin(kz-\omega t)], \tag{2}$$

where $\delta A(z)$ denotes the slowly varying amplitude of the vector potential, ω is the frequency, and k is the wave vector. The equations of motion, therefore, are of the form

$$\begin{aligned}
\dot{p}_1 &= -(1/\gamma)(\Omega_0 + \delta\Omega_0 - \gamma k_w v_z)p_2 + (e\delta A/c)(\omega - k v_z)\sin\psi, \\
\dot{p}_2 &= (1/\gamma)(\Omega_0 + \delta\Omega_0 - \gamma k_w v_z)p_1 - (1/\gamma)\Omega_w p_z + (e\delta A/c)(\omega - k v_z)\cos\psi, \\
\dot{p}_3 &= (1/\gamma)\Omega_w p_2 + (k/\omega)mc^2\dot{\gamma}, \quad \dot{\gamma} = (e\delta A/\gamma m^2c^2)(p_1\sin\psi + p_2\cos\psi),
\end{aligned} \tag{3}$$

where $\Omega_{0,w} = |eB_{0,w}/mc|$, $\delta\Omega_0 = |e\delta B_0/mc|$, $\gamma = (1 + p_2/m^2c^2)^{1/2}$ is the relativistic factor, $\psi = (k+k_w)z - \omega t$, and (p_1, p_2, p_3) are the components of the momentum in the frame defined by the basis vectors $\hat{e}_1$

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$= \hat{e}_x \cos k_w z + \hat{e}_y \sin k_w z$, $\hat{e}_2 = -\hat{e}_x \sin k_w z + \hat{e}_y \cos k_w \times z$, and $\hat{e}_3 = \hat{e}_z$.

The lowest-order solutions (we take the term in δB_0 and δA to be first order) have been amply discussed in the literature.[6,7] Steady-state solutions exist for which $p_1 = p_w \equiv \Omega_w p_\parallel / (\Omega_0 - \gamma_0 k_w v_\parallel)$, $p_2 = 0$, and $p_3 = p_\parallel$ where $p_\parallel =$ constant and $v_\parallel = p_\parallel / \gamma_0 m$. The choice of $p_\parallel$ is not arbitrary, however, and varies with γ_0, B_0, B_w, and k_w as determined by the conservation of energy $p_\parallel^2 + p_w^2 = (\gamma_0^2 - 1) m^2 c^2$. This equation constitutes a quartic equation for the zeroth-order parallel momentum which defines four roots each bearing a functional relationship of the form $p_\parallel = p_\parallel(\gamma_0, \Omega_0, \Omega_w, k_w)$. One root has $p_\parallel < 0$ and will be ignored, while the remaining three roots can be divided into two classes characterized by $\Omega_0 < \gamma_0 k_w v_\parallel$ (group I) and $\Omega_0 > \gamma_0 k_w v_\parallel$ (group II). It should be remarked here that group-I orbits are unstable whenever

$$(1+\beta_w^2)^{-1} \gamma_0 k_w v_\parallel < \Omega_0 < \gamma_0 k_w v_\parallel,$$

where $\beta_w \equiv p_w / p_\parallel$. These orbit classes are shown in Fig. 1(a) where we plot $v_\parallel / c$ vs $\Omega_0 / \gamma_0 k_w c$ for $\gamma_0 = 3.5$ and $\Omega_w / \gamma_0 k_w c = 0.05$, and in which the unstable orbits are denoted by a dashed line.

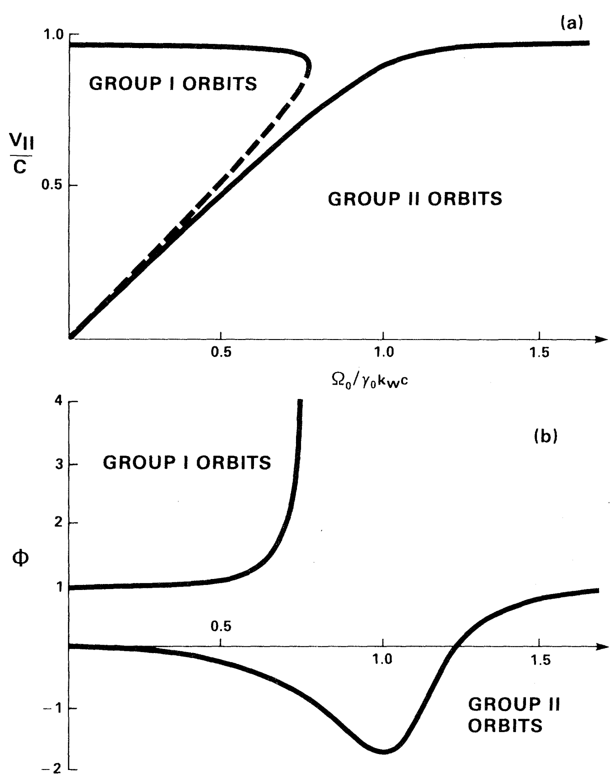

FIG. 1. Graphs of (a) the axial velocity and (b) Φ vs the axial guide field for $\Omega_w / \gamma_0 k_w c = 0.05$ and $\gamma_0 = 3.5$.

We write $p_1 = p_w + \delta p_1$, $p_2 = \delta p_2$, $v_3 = v_\parallel + \delta v_3$, and $\gamma = \gamma_0 + \delta\gamma$ to first order. Since $\dot{v}_3 = v_3^3 \omega^{-1} \psi''$ (where the prime denotes d/dz), the orbit equations reduce to

$$\psi'' = K^2 (\sin\psi - \sin\psi_{\text{res}}) \tag{4}$$

in the limit in which $\omega \simeq (k + k_w) v_\parallel$, where $\psi' = k + k_w - \omega / v_3$,

$$K^2 \equiv \frac{e\,\delta A}{\gamma_0 m c} \beta_w \frac{(k+k_w)^2}{\gamma_\parallel^2 v_\parallel} \Phi, \tag{5}$$

$$\sin\psi_{\text{res}} \equiv (\Phi - 1) \frac{k + k_w}{\gamma_\parallel^2 K^2} \delta\kappa_0, \tag{6}$$

$$\Phi = 1 - \frac{\beta_w^2 \gamma_\parallel^2 \Omega_0}{(1+\beta_w^2)\Omega_0 - \gamma_0 k_w v_\parallel}, \tag{7}$$

$\gamma_\parallel^2 \equiv (1 - v_\parallel^2 / c^2)^{-1}$, and $\delta\kappa_0 \equiv B_0^{-1}\, d(\delta B_0)/dz$ is the inverse scale length for variation of the axial guide field. We remark that $\psi = \psi_0 + \delta\psi$, where ψ_0 is a constant equal to the initial phase ($\psi_0' = k + k_w - \omega / v_\parallel \simeq 0$), and $\delta\psi' = -\omega \delta v_3 / v_\parallel^2$. Hence $\psi'' = \delta\psi''$, and Eq. (4) describes the first-order variation in the phase and describes the trapping of electrons in the ponderomotive potential formed by the beating of the wiggler and electromagnetic wave. It is formally identical to the dynamical equation derived in the analysis of tapered-wiggler FEL's.[1] In particular, the term in $\sin\psi_{\text{res}}$ describes the bulk acceleration or deceleration of electrons due to the inhomogeneity of the axial field. It should be remarked that the function Φ is important in determining the dynamics of the interaction,[8,9] and is plotted in Fig. 1(b) for each orbit group. Evidently the sign of Φ can radically alter the phase of the interaction in that the signs of both K^2 and $\sin\psi_{\text{res}}$ are governed by this function. In order to illustrate this, we can integrate (4) to obtain

$$\psi'^2 + U(\psi) = W, \tag{8}$$

where W is the integration constant, and $U(\psi) \equiv 2K^2 (\cos\psi + \psi \sin\psi_{\text{res}})$ is a trapping potential.

In order to determine the efficiency enhancement implied by the tapered guide field, we observe that by application of the chain rule $\psi'' = (d\psi' / dv_3)(dv_3 / d\gamma)\gamma'$. To lowest order $d\psi'/dv_3 \simeq \omega / v_\parallel^2$ and $dv_3 / d\gamma \simeq v_\parallel / d\gamma_0$, where from the functional form of the zeroth-order trajectories we find that $dv_\parallel / d\gamma_0 = c^2 \Phi / \gamma_0 \gamma_\parallel^2 v_\parallel$. Thus, Eq. (4) is equivalent to

$$\gamma' = \frac{e\,\delta A}{mc^2} \beta_w \frac{\omega}{c} (\sin\psi - \sin\psi_{\text{res}}). \tag{9}$$

In the analysis of the efficiency enhancement process, the radiation field amplitude, δA, must be sufficiently large to trap the beam electrons. This may occur in several ways. In some schemes a large-amplitude radiation field is injected into the interaction region in synchronism with the electron beam, and trapping occurs at the outset (i.e., the electrons are prebunched). As a result, the taper of the guide field (or of the wiggler field) may begin at or near the entrance to the wiggler and be effective. In contrast, if the radiation field is grown from noise, then it would be ineffective to begin the taper near the wiggler entrance. The taper should begin downstream from the wiggler entrance at a later stage in the interaction when the bulk of the electron beam is trapped—specifically, when the interaction is near saturation. For simplicity, therefore, we shall assume in what follows that all the particles are trapped within the separatrices shown in Fig. 2 and at saturation. As a result, the phase-space distribution of particles is such that there is no net energy transfer between the electron beam and the radiation field (i.e., zero gain), and one may write $\langle \sin\psi \rangle = 0$ which describes a uniform phase distribution.

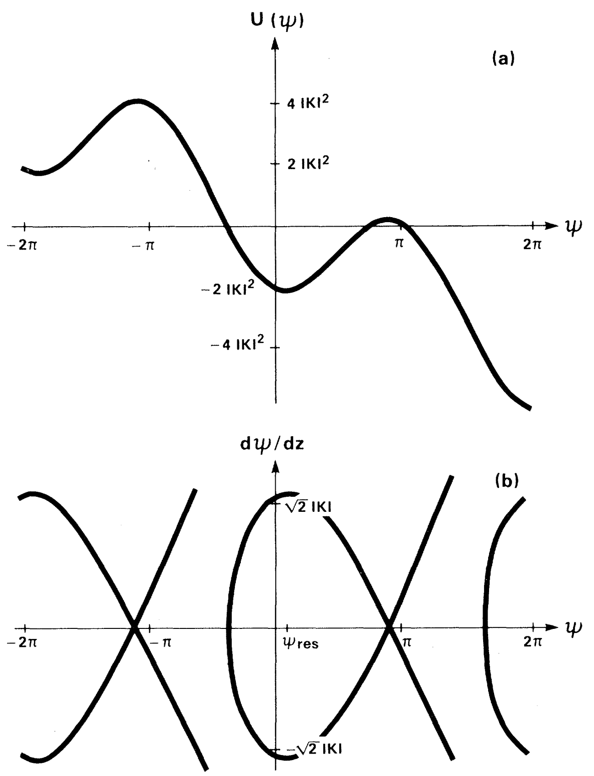

FIG. 2. Graphs of (a) the trapping potential $U(\psi)$ and (b) the separatrices in phase space for $K^2 < 0$ and $\psi_{\mathrm{res}} = 0.1\pi$.

If an average of (9) is taken over such an ensemble of electrons, then we find that the extraction efficiency over a distance L relative to the reference point $z = z_0$ (at which saturation is found) is given by

$$\Delta\eta(L) \equiv \frac{\langle \gamma(z=z_0+L)\rangle - \langle \gamma(z=z_0)\rangle}{\langle \gamma(z=z_0)\rangle} = \beta_\parallel^2 L \left(\frac{1-\Phi}{\Phi}\right) \delta\kappa_0. \tag{10}$$

It should also be pointed out that in the latter scheme in which the system has come to saturation prior to the start of the taper, Eq. (10) describes an enhancement in the efficiency over that found in the uniform-B_0 regime. Hence, the total efficiency is the sum of the efficiencies in the uniform-B_0 region prior to saturation and the efficiency enhancement described in (10) due to the subsequent taper of the guide field. It is evident that efficiency enhancement occurs for $\delta\kappa_0 < 0$ (i.e., for decreasing axial fields) for group-I orbits ($\Phi > 1$) and for group-II orbits such that $(1-\gamma_\parallel^2 v_w^2/c^2)\Omega_0 < \gamma_0 k_w v_\parallel$ (i.e., $\Phi < 0$); and for $\delta\kappa_0 > 0$ corresponding to group-II orbits in the case where $0 < \Phi < 1$. This conclusion is consistent with the results of experiment.[5] Finally, we remark that this approximation corresponds to the neglect of the $\sin\psi$ term in Eq. (9) and, hence, the efficiency enhancement described by Eq. (10) measures the effect of the change in the resonant phase.

Observe that the conditions on the axial-field taper leading to efficiency enhancement are identical to those which maintain the resonance condition. In order to see this we observe that for the steady-state orbits $\partial v_\parallel/\Omega_0 = (1-\Phi)v_\parallel/\gamma_\parallel^2\Omega_0$ and $\partial v_\parallel/\partial\gamma = c^2\Phi/\gamma\gamma_\parallel^2 v_\parallel$. Hence, for group-I orbits where $\Phi > 1$ and group-II orbits where $\Phi < 0$, the resonant frequency [i.e., $\omega \simeq (k+k_w)v_\parallel$] is maintained by decreasing the axial field as the electrons lose energy. For group-II orbits where $0 < \Phi < 1$, however, the resonant frequency is maintained by increasing the axial field as the electron energy decreases.

The FEL efficiency-enhancement scheme outlined here requires a guide field that is of the order of $B_0 \sim (mc/e)\gamma_0 k_w v_\parallel$, and is most interesting at relatively low values of γ (< 10) since use of higher electron energies would require very high guide fields ($\gtrsim 30$ kG) for typical values of wiggler period (3–10 cm). While the specific theory described herein is directly applicable only to the strong-pump (or Compton) regime of operation in which space-charge fields are negligible, the fundamental mechanism is appropriate to the collective, Raman FEL as well. In particular, the conclusions regarding the direction of the axial guide-field taper are in agreement with recent experimental results on a Raman FEL.[5] In order to treat the efficiency enhancement quantitatively in the Raman regime, however, numerical simulation is required, as no satisfactory theory for the efficiency of a Raman FEL exists in the presence of even a uniform axial guide field. Such work is currently in progress.

The efficiency enhancements which can be described by Eq. (10) are inherently limited by the assumption of small variations in the guide field, i.e., $\delta B_0/B_0 \sim L\,\delta\kappa_0 \ll 1$. However, overall efficiencies even in the Raman regime for uniform guide fields are less than about 8% for typical parameters,[8] and would be somewhat lower than this in the Compton regime. Hence, a variation in the axial magnetic field of less than 10% can account for efficiency enhancements of a factor of 2 or greater over the uniform-B_0 regime.

Finally, we observe that an efficiency enhancement scheme based upon tapered guide fields may be of particular relevance to FEL experiments which make use of two-stage processes in which the wiggler field in the second stage is electromagnetic.[10] In such cases it does not currently appear to be technically feasible to control the taper of the wiggler field with the precision required for the efficiency enhancement.

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