Measurement of the Lamb Shift in Muonium

C. J. Oram, J. M. Bailey, and P. W. Schmor
TRIUMF, Vancouver, British Columbia V6T 2A3, Canada

and

C. A. Fry, R. F. Kiefl,[a] and J. B. Warren
Physics Department, University of British Columbia, Vancouver, British Columbia V6T 2A6, Canada

and

G. M. Marshall and A. Olin
Physics Department, University of Victoria, Victoria, British Columbia V8W 2Y2, Canada
(Received 29 August 1983)

The Lamb shift in muonium has been measured for the first time by a beam-foil-spectroscopy radio-frequency technique. The value obtained is 1070^{+12}_{-15} MHz, with a systematic uncertainty of 2 MHz, compared with the present theoretical value of 1047.03 MHz.

PACS numbers: 36.10.Dr, 12.20.Fv, 13.10.+q

The measurement of the Lamb shift in hydrogen provided the first experimental verification of quantum electrodynamics (QED) and continues to be one of the most rigorous tests of the theory. However, recent measurements of the Lamb shift in hydrogen show a possible discrepancy with theory at the level of 40 ppm,[1,2] which may be explained by the difficulty in estimating proton structure effects.[3] The Lamb shift has been calculated[4] to the few parts per million level for the hydrogenlike muonium ($\mu^+ e^-$) atom, where finite nuclear size effects are absent, while similar quantum effects are involved. A muonium experiment with comparable precision could thus test QED without the ambiguity present for hydrogen. We report here the first radio-frequency (rf) measurement of the muonium Lamb shift. This measurement, with precision at the percent level, is intended to explore the feasibility of a QED test. Such a test would compliment the present measurements of energy level differences in positronium,[5] hydrogen, and high-Z atoms.[6] The muonium energy level diagram is shown in Fig. 1.

Experiments at TRIUMF and LAMPF have established the flux and estimated the velocity distribution of 1S (Ref. 7) and 2S (Ref. 8) muonium from foils bombarded with muons. The fraction emerging in the 2S muonium state[9] is comparable to that for hydrogen[10] in the velocity range of $c/50$ to $c/200$. These rates are confirmed in the present measurement, where the resonant deexcitation of the 2S state provided a distinctive signature of 2S muonium.

A 15.4-MeV/c positive muon beam with 7% momentum acceptance (full width at half maximum) was transported by the TRIUMF M13 beam line[11] into the evacuated apparatus shown in Fig. 2 through a 50-μm Mylar window. A 63-μm plastic scintillator (X) detected particles incident on a

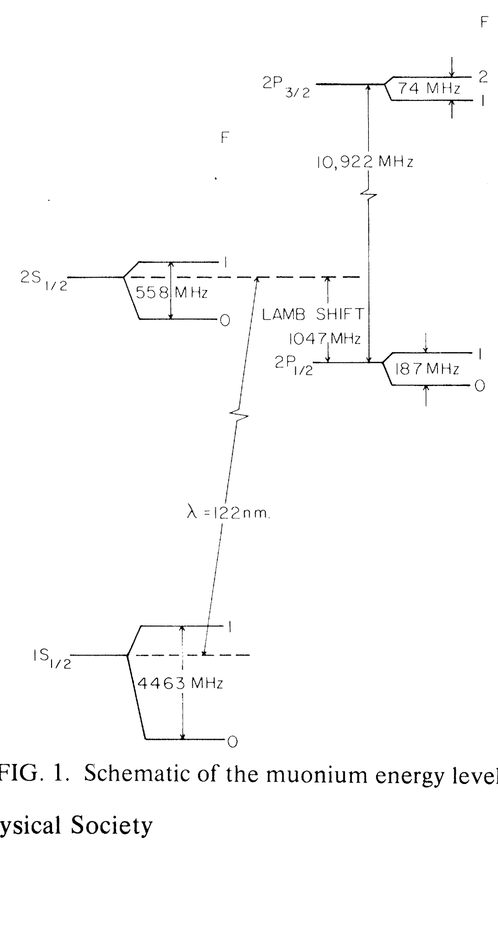

FIG. 1. Schematic of the muonium energy levels.

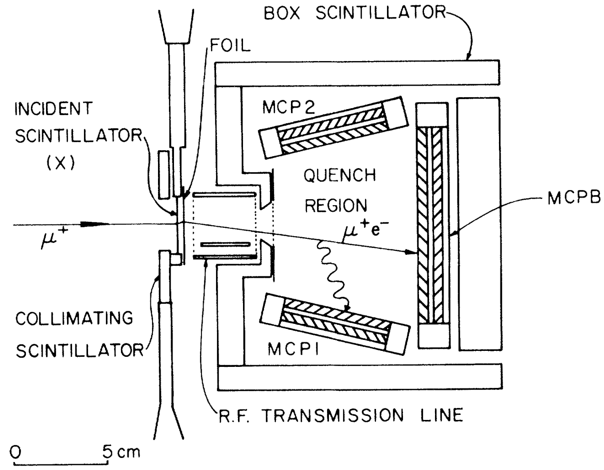

FIG. 2. A schematic of the apparatus, showing a good event in which a Lyman α photon is detected in microchannel plate (MCP1) from deexcitation of $\mu^+e^-(2S)$ in the quench region.

0.75-μm aluminum foil. Muons and muonium emerging from the foil passed through an rf transmission line into a static electric quench field (~ 300 V/cm) and were detected by a rectangular (92×75 mm) microchannel plate (MCPB). Transitions from $2S$ to $2P$ states induced in the 3.4-cm-long rf transmission line region depopulate the $n=2$ state. At resonance with 12.5 W of rf power, approximately one-third of the $2S$ muonium moving at $c/70$ would survive the rf region. The lifetime of the surviving $2S$ muonium in the quench region was reduced to about 8 ns due to Stark mixing. The resulting Lyman α radiation was detected by two (40-mm-diam) CsI- coated microchannel plates (MCP1 and MCP2). Positrons in the beam or from muon decay were detected, with an efficiency of about 98%, by a rectangular box of thick plastic scintillators (BOX). Quartz halogen light bulbs were mounted near the MCP's but outside the beam path to facilitate baking of the MCP's and testing of their photon efficiency and pulse-height response.

After adjusting beam parameters for optimum flux of particles with velocity near $c/70$ between X and MCPB, data were accumulated in runs of approximately eight hours over an eight day period. The rf power was maintained at either 2, 12.5, or 25 W during a run and, to average over possible slow drifts in beam conditions and detection efficiency, the frequency was changed every few minutes. An event was defined by a count in either MCP1 or MCP2 as well as in MCPB within 450 ns of detection of an incident muon in the X scintillator. Events were veteod for which any MCP fired in fast coincidence with a BOX scintillator.

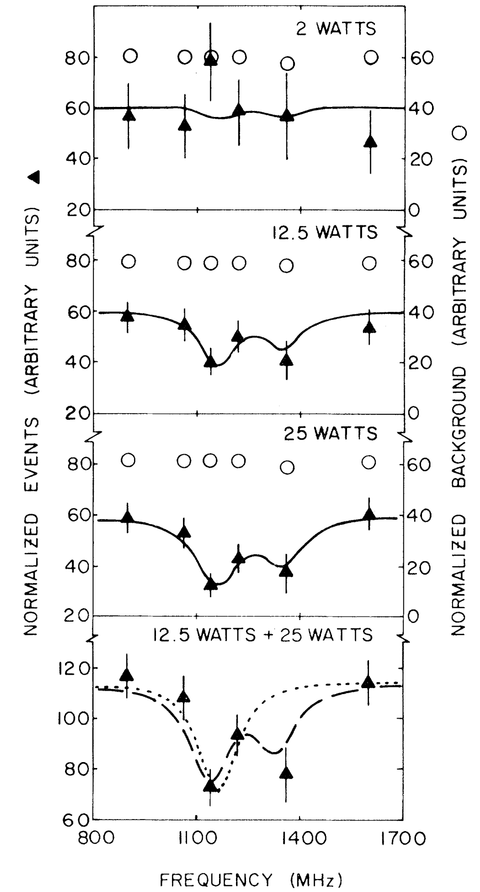

FIG. 3. The intensity of the muonium $2P$-$1S$ signal is plotted (triangles) for the measured rf frequencies and powers. The circles, representing the background intensities (before imposing timing criteria), give an independent test of the normalization (error bars are smaller than the circles). The smooth curve is the fit with three free parameters: signal amplitude, background amplitude, and the Lamb shift (see Table I, fit III). The two resonances correspond to the transitions $2S_{1/2}(F=1) \rightarrow 2P_{1/2}(F=1)$ at 1140 MHz and $2S_{1/2}(F=1) \rightarrow 2P_{1/2}(F=0)$ at 1327 MHz. The dashed curve is a fit with two free parameters: signal amplitude and background amplitude. The dotted curve is a fit with four free parameters: signal amplitude, background amplitude, peak width, and peak position.

The apparatus was tested in a similar configuration with the beam line tuned to produce an optimum flux of $c/70$ protons between X and MCPB. This allowed estimation of the efficiencies of MCP1 and MCP2 for Lyman α detection, using measured neutral fractions for hydrogen[10] and assuming 10% were in the $2S$ state. An efficiency of about 11% was obtained, in reasonable agreement with a quoted value of 14% for similar detectors.[12]

Off-line analysis started by requiring the following criteria for the time relationship between X, MCP1 or MCP2, and MCPB:

(1) The time of flight between X and MCPB corresponded to a velocity betweeen $c/55$ and $c/200$.

(2) MCP1 or MCP2 fired after the particle entered the quench region, but more than 5 ns prior to striking MCPB.

(3) The interval between entry into the quench region and detection of a Lyman α photon was less than 20 ns.

(4) No muon arrived in X before the preceding muon had reached MCPB.

These requirements removed 99.98% of the recorded events. Restrictions on the MCP pulse heights removed less than 20% of the remaining events. The time distribution of the remaining events displayed a signal with a quench lifetime of about 8 ns, plus a background consistent with the random coincidence rates.

The remaining number of events was normalized by the $X \cdot$MCPB coincidences at each frequency and power. To check for the effect of stray rf fields on the efficiencies of MCP1 and MCP2 and to check the normalization of the data, the total (i.e., before imposing time requirements) number of events at each frequency and power was normalized in the same way. The data are shown in Fig. 3. Also shown are fits consisting of a power-independent flat background plus the resonance curve determined from the $2S$ muonium velocity distribution and the total transition rate to the $2P$ states. The total transition rate at each frequency and power was the sum of the individual transition rates between the $2S$ and $2P$ hyperfine levels. The individual rates are functions of the Lamb shift because they depend upon the energy differences between the initial and final levels.[13] Since the transition rate is proportional to the rf power, the data at all three power levels were fitted simultaneously with that requirement. The frequencies for this measurement were chosen to optimize our sensitivity to the position and amplitude of the known double peaked line shape, in the data collection time available. The velocity distribution of $2S$ muonium was taken to be the product of the observed velocity distribution between X and MCPB and the fraction of the beam in the $2S$ state as calculated from hydrogen data.[10] The Lamb shift obtained from the fit was found to be quite insensitive to the exact velocity distribution. Consistent values were obtained when the data for MCP1 and MCP2 were analyzed separately. The results of the fitting process under a variety of conditions are summarized in Table I. The contribution to χ^2 from the data at different power levels is given separately as only the 12.5- and 25-W data contain significant frequency information. The $2P_{1/2}$ level splitting and mean life converge to reasonable values when freed in the fit. Our final results are based on fit III.

In order to study the sensitivity of the data to the line shape, the sum of the 12.5- and 25-W data was fitted. The fit to this data with a single resonance is shown in Fig. 3 along with the fit using the theoreti-

TABLE I. Results of fits to the data.

Signal amplitude	Background amplitude	Lamb shift (MHz)	Hyperfine splitting $2P_{1/2}$ level (MHz)	$2P_{1/2}$ Mean life (ns)	Total χ^2 contribution 2 W	12 W	25 W	Degrees of freedom
Fixed 0	51.1±1.7	-	-	-	5.8	6.3	20.0	17
40.7±8.1	19.8±6.0	1047.0[a]	187[b]	1.6[c]	4.5	2.3	3.0	16
43.3±8.3	17.2±6.3	1069.8 ±13.3	187[b]	1.6[c]	4.4	1.3	2.7	15
43.7±8.3	17.2±6.3	1068.0 ±13.6	205 ±35	1.6[c]	4.4	1.2	1.6	14
41.2±11.6	19.4±10.1	1069.6 ±13.6	203 ±36	1.8 ±0.8	4.5	1.1	1.4	13

[a]Theoretical Lamb shift of muonium.

[b]Theoretical hyperfine splitting of the $2P_{1/2}$ level.

[c]Theoretical $2P_{1/2}$ mean life.

TABLE II. Estimated magnitudes of effects caused by systematic uncertainties.

Source of systematic uncertainty	Resulting uncertainty (MHz)
25% uncertainty in rf power	1.4
1% rf power variations per 100 MHz for data at same nominal power	0.3
5° uncertainty in alignment of rf cavity with beam direction	1.3
Uncertainty in velocity distribution	0.1
1 G uncertainty in magnetic field	0.1
2% nonlinearity in normalization	1.0

cal line shape. The χ^2 per degree of freedom is 5.5 and 1.0, respectively, for these two fits, showing that the theoretical line shape is strongly preferred.

The sensitivity of the measurement to systematic effects is shown in Table II. These are believed to be uncorrelated and so are added in quadrature giving a total systematic error of 2 MHz.

It is useful to estimate the yield of $2S$ muonium from the aluminum foil in order to aid in the design of future experiments. However, the detection efficiency of MCPB for muonium is unknown and extrapolation from electron and ion data[12] is difficult. Assuming an average efficiency of 25% we find that about 31 $2S$ muonium emerge from the foil for an incident μ^+ flux of 4×10^4 per second, in agreement with the $1S$ data of Bolton *et al.*[7] (after correcting for our reduced stopping rate at the surface of the foil and assuming one-tenth of the neutrals were in $2S$ state), and slightly less than Oram *et al.*[8]

In conclusion, we have measured the Lamb shift of the $2S$ state of muonium to be $1070^{+12}_{-15}\pm2$ MHz, within two standard deviations of the calculation by Owen[6] of 1047.03 MHz. The main limitation in this type of experiment is the low stopping density of available muon beams. Using present beams, the statistical uncertainty in future experiments could in principle be reduced, for instance, by a larger angular acceptance from the foil; however, it appears that such improvements result in larger systematic uncertainties.

We would like to thank T. Oka and F. M. Pipkin for their help in the initial design of the rf transmission line. The assistance of G. A. Ludgate and G. Morris was very much appreciated. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

(a)Present address: Physik-Institut de Universität Zürich, Schönberggasse 9, CH-8001 Zürich, Switzerland.

1S. R. Lundeen and F. M. Pipkin, Phys. Rev. Lett. 46, 232 (1981).

2P. J. Mohr, Phys. Rev. Lett. 34, 1050 (1975).

3E. Borie, Phys. Rev. Lett. 47, 568 (1981).

4D. A. Owen, Phys. Lett. 44B, 199 (1973). This paper contains an error which will be corrected in forthcoming work (D. A. Owen, private communication).

5S. Chu and A. P. Mills, Jr., Phys. Rev. Lett. 48, 1333 (1982).

6P. J. Mohr, At. Data Nucl. Data Tables 29, 453 (1983).

7P. R. Bolton, A. Badertscher, P. O. Egan, C. J. Gardner, M. Gladisch, V. W. Hughes, D. C. Lu, M. Ritter, P. A. Sounder, J. Vetter, G. zu Putlitz, M. Eckhause, and J. Kane, Phys. Rev. Lett. 47, 1441 (1981).

8C. J. Oram, C. A. Fry, J. B. Warren, R. F. Kiefl, and J. H. Brewer, J. Phys. B 14, L789 (1981).

9C. A. Fry, J. B. Warren, R. F. Kiefl, C. J. Oram, G. A. Ludgate, P. W. Schmor, A. Olin, G. M. Marshall, B. Erickson, and G. Morris, in Proceedings of the Yamada Conference on Muon Spin Rotation and Associated Problems, Shimoda, Japan, April 1983 (to be published).

10S. K. Allison, Rev. Mod. Phys. 30, 1137 (1958).

11C. J. Oram, J. B. Warren, G. M. Marshall, and J. Doornbos, Nucl. Instrum. Methods 179, 95 (1981).

12E. A. Kurz, private communication.

13W. E. Lamb, Jr., and R. C. Retherford, Phys. Rev. 79, 549 (1950).