Doubly Strange Dibaryon in the Chiral Model

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It shown that the chiral model with SU(3) flavor symmetry predicts a dibaryon state of low mass M ($M \approx 2.2$ GeV). It is electrically neutral and is an SU(3) singlet with $J^P = 0^+$. It corresponds to a six-quark state found in the MIT bag model by Jaffe. It is also shown that there is no stable particlelike state of baryon number 2 which is based on Skyrme's spherically symmetric *Ansatz* for the chiral field.

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It is believed that the low-energy properties of QCD are effectively reproduced by the chiral model. The order parameter in this model when we consider only the light quarks is a field U where $U(x)$ is a 3×3 SU(3) matrix. Skyrme pointed out many years ago[1] that this model admits solitons characterized by an integer-valued topological number and proposed to interpret the states with the unit value of this number as the nucleon and its excitations. He also suggested that the topological number t is the baryon number B of the nucleon. This conjecture was confirmed in all essential respects by Balachandran, Nair, Rajeev, and Stern[2,3] who showed that $b = \text{const}\times t$, where the constant is completely determined by the detailed assumptions in the treatment of the fermions in the model. Further studies of the chiral model[4,5] which include in particular the topological effects of the Wess-Zumino term also suggest that the $|t| = 1$ states are indeed fermions. There is thus good support to Skyrme's conjecture that the $|t| = 1$ solitons are baryons and t is related to the baryon number. The conservative assumption at this point would be to identify these states with the baryon octet and assume that t is exactly equal to B. Following Skyrme[1] and Witten,[4] we shall adopt this interpretation for the purposes of this paper[6]; our conclusions can, however, be readily modified if, as has been suggested,[2] the topological excitations represent a novel family of states.

The stable static solutions with $|B| = 1$ in the Skyrme model are described by a "spherically symmetric" configuration. In this context, spherical symmetry is understood in a generalized sense and depends on the choice of an SU(2) subgroup of the flavor SU(3). There are, however, spherically symmetric configurations which involve instead the SO(3) subgroup of real orthogonal matrices of SU(3) [7] and the major results of this note pertain to these configurations. The associated topological excitations are characterized by $|B| = 0, 2, 4, \ldots$. We show in this note that the lightest dibaryon states in this sequence with $B = \pm 2$ have a mass of the order of 2.2 GeV. They are also expected to be SU(3) singlets with $J^P = 0^+$. In this note, we shall also briefly study the $|B| = 2$ states based on the SU(2) subgroup and show that the corresponding static configurations are not stable even classically. Therefore we do not expect a dibaryon resonance identifiable with such a configuration.[8]

We shall first briefly review the relevant aspects of Skyrme's model for three flavors. It is based on the Lagrangian density

$$\mathscr{L} = -\tfrac{1}{2} f_\pi^2 \operatorname{Tr}(\partial_\mu U^\dagger \partial_\mu U) + (1/32e^2)\operatorname{Tr}\{[\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2\}, \quad (1)$$

where $f_\pi \simeq 67$ MeV. (Here we omit the Wess-Zumino term since it does not contribute to the energy.) For static configurations, the energy functional for this $\mathscr{L}$ is

$$E(U) = \int d^3x \left(\tfrac{1}{2} f_\pi^2 \operatorname{Tr}(\partial_i U^\dagger \partial_i U) - (1/32e^2)\operatorname{Tr}\{[\partial_i U U^\dagger, \partial_j U U^\dagger]^2\}\right). \quad (2)$$

Finiteness of $E(U)$ requires that U approaches a constant as $|\vec{x}| \to \infty$. After a chiral rotation if necessary, we may thus assume that $U \to 1$ as $|\vec{x}| \to \infty$. The topological or baryon number of the configuration U is the integral

$$B(U) = (1/24\pi^2)\epsilon_{ijk} \int d^3x \operatorname{Tr}(\partial_i U U^\dagger \partial_j U U^\dagger \partial_k U U^\dagger). \quad (3)$$

The spherically symmetric *Ansatz* for U based on the I-spin SU(2) subgroup is

$$U(\vec{x}) = \begin{bmatrix} \cos\theta(r) + i\vec{\tau}\cdot\hat{x}\sin\theta(r) & 0 \\ 0 & 1 \end{bmatrix},$$
$$r = |\vec{x}|, \quad \hat{x} = \frac{\vec{x}}{r}, \tag{4}$$

where τ_i are the Pauli matrices. It is spherically symmetrical in the sense that

$$-i(\vec{x}\times\nabla)_i U(\vec{x}) + [\lambda_i/2, U(\vec{x})] = 0, \quad i = 1,2,3, \tag{5}$$

where λ_i are the Gell-Mann matrices. In (4), the condition $\sin\theta(0)=0$ is required to have a well defined U at $r=0$, while since $U \to 1$ as $r\to\infty$, $\cos\theta(\infty)=1$. For the *Ansatz* (4), $B(U)$ reduces to

$$B(U) = (1/\pi)[\theta(0) - \theta(\infty)]. \tag{6}$$

For $B=1$, the energy can be minimized for the choice $\theta(r) = \pi\exp(-\xi)/(\lambda\xi^2+1)$, where $\xi = r/R$ and R and λ are variational parameters. The minimum is

$$m_N \approx 104 f_\pi/e. \tag{7}$$

The spherically symmetrical *Ansatz* based on the SO(3) subgroup of SU(3) has been studied in detail elsewhere. The SO(3) Lie algebra is spanned by the generators Λ_α, where $\Lambda_1 = \lambda_7$, $\Lambda_2 = -\lambda_5$ and $\Lambda_3 = \lambda_2$. The most general such *Ansatz* which fulfills the constraint

$$-i(\vec{x}\times\nabla)_i U(\vec{x}) + [\Lambda_i, U(\vec{x})] = 0 \tag{8}$$

required by spherical symmetry reads

$$U(\vec{x}) = e^{i\psi(r)} + i\sin\chi(r) e^{-i\psi(r)/2}\vec{\Lambda}\cdot\hat{x} + [\cos\chi(r)e^{-i\psi(r)/2} - e^{i\psi(r)}](\vec{\Lambda}\cdot\hat{x})^2. \tag{9}$$

{It is easily checked [by substituting $\hat{x} = (0,0,1)$ for example] that $U(\vec{x}) \in \mathrm{SU}(3)$.} Since $U\to 1$ as $r\to\infty$, we have the condition[7]

$$\cos\chi(\infty) = 1, \quad e^{-i\psi(\infty)/2} = 1. \tag{10}$$

Since U must be well defined as $r\to 0$, we have either of the two conditions[7]

$$\cos\chi(0) = -1, \quad e^{-i\psi(0)/2} = -e^{i2\pi k/3}; \tag{11}$$

$$\cos\chi(0) = 1, \quad e^{-i\psi(0)/2} = e^{i2\pi k'/3}, \tag{12}$$

where k,k' are integers. It has also been shown[7] that $B(U) = 2[\chi(0) - \chi(\infty)]/\pi$ if U has the form (9). The analytic expression for $E(U)$ in terms of ψ and χ is

$$E(U) = f_\pi^2 R I_1 + (1/e^2R) I_2,$$
$$I_1 = 4\pi\int_0^\infty d\xi\, \xi^2[\tfrac{3}{4}\psi'^2 + \chi'^2 + (4/\xi^2)\{1 - \cos\chi\cos(\tfrac{3}{2}\psi)\}],$$
$$I_2 = 2\pi\int_0^\infty d\xi \Big\{\xi^{-2}\{3\sin^2\chi\sin^2(\tfrac{3}{2}\psi) + [1-\cos\chi\cos(\tfrac{3}{2}\psi)]^2\} + \tfrac{9}{4}[1-\cos\chi\cos(\tfrac{3}{2}\psi)]\psi'^2 + [1-\cos\chi\cos(\tfrac{3}{2}\psi)]\chi'^2 + 3\sin\chi\sin(\tfrac{3}{2}\psi)\chi'\psi'\Big\}. \tag{13}$$

Here we have introduced a length scale R and the dimensionless variable $\xi = r/R$ so that χ and ψ are functions of ξ. The prime denotes differentiation with respect to ξ. The boundary conditions on χ and ψ appropriate to the study of the minimum of $E(U)$ in the $B=2$ sector are $\chi(0)=\pi$, $\chi(\infty)=0$, $\psi(0) = 2\pi/3$, $\psi(\infty)=0$. These are consistent with (10) and (11). (The condition on ψ comes from the remark that we expect a ψ with the least total variation to be associated with the energy minimum.) The minimum M of $E(U)$ has been studied for a variety of forms of χ and ξ compatible with these boundary conditions. We find that $M \simeq 1.92 m_N$, where m_N is given in (7). The mean baryon octet mass is 1151.7 MeV. With this value for m_N, we thus have

$$M \simeq 2.21 \text{ GeV}. \tag{14}$$

Since the state in question has $B=2$, we shall interpret it as a six-quark state. The classical configuration (9) and hence the associated quantum state $|U\rangle$ are not invariant under SU(3) or rotations of $\hat{x}$. The lowest-energy configuration U_c with the mass quoted above is expected to be associated with a quantum state characterized by a high degree of symmetry. The six-quark interpretation shows that the state of highest symmetry is an SU(3) singlet with $J^P = 0^+$; it is the SU(3) singlet component of

$|U_c\rangle$ with $J^P=0^+$. The Skyrme model with zero meson mass predicts a mass M for this state which makes it stable ($M < 2m_N$). The calculation has also been done by putting in the meson mass term $(f_\pi^2\mu^2/2)\mathrm{Tr}(U+U^\dagger-2)$, where $\mu^2=(408.2\text{ MeV})^2$ is the average of the squared meson octet masses. Here we readjust E so as to reproduce the value 1151.7 MeV for m_N and then recompute M. The meson mass hardly affects the value of M. Thus the model with or without the meson mass predicts this level to be stable. Note, however, that one does not know how accurate the Skyrme model is, so that the preceding value of M could easily be in error by 100–200 MeV. Thus the level may well be unstable, although for phase-space reasons, it would then be expected to have a narrow width.

Dibaryon SU(3)-singlet 0^+ states have been studied before[9] in the context of the MIT bag model. The level discussed here seems to correspond to the level called H by Jaffe.[9] He also predicts a dibaryon octet with $J^P=1^+$. As we shall discuss elsewhere, it is likely to correspond to the rotational excitation of the level that we find. Production and decay systematics of H have also been discussed.[9] Further, there is some experimental indication of the existence of these levels.[10]

We next turn to the $B=2$ state based on the SU(2) subgroup and assume that U is of the form (4) with $\theta(0)-\theta(\infty)=2\pi$. If the energy is minimized on such configurations, the minimum E_0 is known to be about $3m_N$.[1,5,8] This would suggest at first sight that this state is unstable against decay into baryons and not of interest. However, by general theorems,[11] if E_0 is a *minimum* for spherically symmetrical perturbations, it is an *extremum* for nonspherically symmetric perturbations. If this extremum is also a (local) minimum for nonspherical perturbations, the state in question will be absolutely stable classically, and will decay into two baryons only by barrier penetration in quantum theory. It may therefore be quite long lived and physically interesting. We find, however, that E_0 is a *maximum* for certain nonspherical perturbations so that in fact there is no such resonant state.

The stability of the state can be studied by the behavior of the energy when we split the *Ansatz* into two $B=1$ solitons and move the latter far apart. Such a splitting can be done as follows. Consider

$$V_{\vec{D}}=U_{\vec{S}=0}U_{\vec{S}=\vec{D}},\quad U_{\vec{S}}(\vec{x})=\begin{bmatrix}\cos\alpha(|\vec{x}-\vec{S}|)+\dfrac{i\vec{\tau}\cdot(\vec{x}-\vec{S})}{|\vec{x}-\vec{S}|}\sin\alpha(|x-\vec{S}|) & 0\\ 0 & 1\end{bmatrix},\tag{15}$$

where α corresponds to $B=1$ so that $\alpha(0)-\alpha(\infty)=\pi$. The field $V_{\vec{D}}$ corresponds to $B=2$. When $\vec{D}$ is zero, $V_{\vec{D}}$ becomes the *Ansatz* (4) with $\theta=2\alpha$. When $D\equiv|\vec{D}|$ becomes large, $V_{\vec{D}}$ describes two widely separated $B=1$ solitons. Thus $V_{\vec{D}}$ does the splitting for us and we wish to study $E(V_{\vec{D}})$ as a function of D. (Of course, here V_0 must be the configuration which minimizes the energy in the class of spherically symmetric configurations.) The details of our work on $E(V_{\vec{D}})$ need not be reproduced here in view of the investigations of Jackson, Jackson, and Pasquier.[8] We find that $E(V_{\vec{D}})$ shows no dip at $D=0$, and hence V_0 does not describe even a classically stable state.

A fuller account of our work will be published elsewhere.[12] There we shall in particular discuss the quantum numbers of the dibaryon level and its excitations using the collective-coordinate approach.[13] The wave functions of these levels will be shown to be given by $D^{(p,q)}_{\lambda,\mu}(g)$, where $g\in\mathrm{SU}(3)$ and $D^{(p,q)}(g)$ is the *triality-zero* representation (p,q) of SU(3). Flavor SU(3) acts on g by $g\rightarrow hg$ while spatial rotations act on g by $g\rightarrow gR$, where $R\in\mathrm{SO}(3)\subset\mathrm{SU}(3)$. The lowest-lying level has a constant wave function $[(p,q)=(0,0)]$ and has spin zero, while the next set of levels correspond to the wave functions $D^{(1,1)}_{\lambda\mu}(g)$ which describe octets of spin-one and spin-two states. These results are consistent with those based on the quark model. We also plan to discuss other properties of these levels accessible to the collective-coordinate approach in Ref. 12.

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