Mass of the Higgs Boson in the Canonical Realization of the Salam-Weinberg Theory

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It is shown that, for a wide range of top-quark masses, the Higgs mass in the canonical realization of the Salam-Weinberg theory must be $\lesssim$ 125 GeV. The considerations are predicated on the premise that the pure $\lambda\phi^4$ theory is a free-field theory. The bound emerges as a necessary condition, albeit within a specific scenario, to avoid the trap of a trivial Higgs sector.

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We reexamine the question of the consistency of the canonical realization—with spontaneous symmetry breaking triggered by elementary scalars— of the Salam-Weinberg theory,[1] in the context of the possible triviality of the $\lambda\phi^4$ theory.[2,3] Motivated by recent work of Dashen and Neuberger,[4] we consider a possibility suggested by some early work of Gross and Wilczek[5]: In the coupled Higgs–gauge-field system, the difficulties associated with the quartic coupling—usually deemed to find their resolution via vanishing of the renormalized coupling constant and thus a trivial theory—may be matched and compensated by similar difficulties associated with asymptotically nonfree gauge-field couplings. More precisely, we constrain the theory in such a way that the ratio $y=\bar{\lambda}(t)/\bar{g}_1(t)^2$ of running coupling constants does not diverge or become unreasonably large ($>>1$) for large t. Here $2t=\ln(p^2/m_W^2)$, m_W being the W-boson mass and p the momentum variable customarily used in renormalization-group caluclations[5]; the precise meaning of "large t" will be given later; g_1 is the coupling associated with the U(1) factor in the gauge group of the standard model, U(1) $\otimes$ SU(2)$_L$ $\otimes$ SU(3)$_C$. In what follows, g_2 and g_3 denote the coupling constants associated with the other two groups. (Possible effects of grand unification, gravity, etc., are not considered in this paper.)

The requirement of a reasonable magnitude for y appears to be a minimal condition for the consistency of the theory. If it is not met, there will be an energy regime in which $\bar{\lambda}$ is much larger than all gauge couplings and the purely scalar sector can be decoupled from the rest of the system; the usual arguments for the triviality of the $\lambda\phi^4$ theory will then go through.[6]

To implement the above requirement, we retain only one-loop contributions to the β functions and demand that y be driven to an ultraviolet-stable fixed point; that this indeed leads to $y\lesssim O(1)$ for large t will be demonstrated below. In the meantime, we note that there is a measure of uniqueness attached to y; the other available ratio, $\bar{\lambda}/\bar{g}_2^2$, cannot go to a fixed point.[5] Furthermore, if the theory is to be in the domain of attraction of the fixed point, the parameters of the theory cannot be arbitrary; in particular, the initial value of y— which determines the mass of the Higgs boson—must lie in a bounded interval. We distinguish three cases:

(a) If m_t, the mass of the top quark (or any other heavy quark), is less than a determinable value, say m^*, then

$$0<y(t=0)\leqslant Y_{\max}(m_t)\quad(m_t\leqslant m^*). \tag{1}$$

(b) If m_t exceeds m^*, there is also a nontrivial lower bound for $y_{\rm init}$ which arises from the requirement that y remain positive definite for all t:

$$Y_{\min}(m_t)\lesssim y(t=0)\lesssim Y_{\max}(m_t)\quad(m_t>m^*). \tag{2}$$

(c) If m_t equals M, the maximal value allowed in our formulation, the upper and lower bounds on $y(t=0)$ coalesce into one; the Higgs mass is then determined rather than bounded.

We are now in a position to state our numerical results for m_H, the Higgs-boson mass, obtained in the manner explicated below.

Case (a): For $m_t<m^*\simeq 80$ GeV, we find that $(m_H/m_W)^2\lesssim 2.376$, corresponding to $m_H\lesssim 125$ GeV. It is obvious that this is the most interesting case, the bound being the lowest of the many upper bounds on Higgs mass that have been published in the literature.[7]

Case (b): For $m_t>80$ GeV, the bounds vary quite sharply with m_t. Thus, 65 GeV $<m_H<122$ GeV for $m_t=120$ GeV, while 140 GeV $<m_H$

< 148 GeV for $m_t = 150$ GeV.

Case (c): For $m_t = 168$ GeV, the largest possible value, $m_H \simeq 175$ GeV.

We proceed to sketch a derivation of our results. To establish our notation and normalization, we display explicitly the Lagrangian density corresponding to the Higgs sector[8]:

$$\mathscr{L}_\phi = (\partial_\mu \phi^\dagger - \frac{i}{2} g_1 \phi^\dagger B_\mu - \frac{i}{2} g_2 \phi^\dagger \vec{\tau} \cdot A_\mu)(\partial^\mu \phi + \frac{i}{2} g_1 B^\mu \phi + \frac{i}{2} g_2 \vec{\tau} \cdot \vec{A}^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 + G \bar{\psi}_L \tilde{\phi} \psi_R + \dots \quad (3)$$

Here B and A are the U(1) and $SU(2)_L$ gauge fields, respectively; the τ are the Pauli matrices; $\mu^2 < 0$, for spontaneous symmetry breaking; $\lambda < 0$, for stability; and the Higgs field is assigned hypercharge $Y = +1$. The ϕ's are Fermi fields; we have displayed only one of the many possible Fermi-Higgs Yukawa couplings $[\tilde{\phi} \equiv i\tau_2(\phi^\dagger)^T]$; the dots indicate the others.

At the one-loop level, the renormalization-group equations for the gauge couplings are

$$d\bar{g}_i / dt = \epsilon_i b_i \bar{g}_i^3 / 16\pi^2, \quad (4)$$

where i $(=1,2,3)$ labels the gauge group, $b_i > 0$, $\epsilon_1 = +1$, and $\epsilon_2 = \epsilon_3 = -1$. Before we write down the one-loop β functions for the other couplings, we introduce variables

$$x \equiv \bar{g}_2^2/\bar{g}_1^2; \quad \zeta = \ln[\bar{g}_1^2/\bar{g}_1^2(t=0)];$$

$$z \equiv \bar{G}^2/\bar{g}_1^2; \quad u = \bar{g}_3^2/\bar{g}_1^2,$$

where G is now identified as the Yukawa coupling associated with the top quark.

Note that x, ζ, and u are simply related by virtue of Eq. (4). We have

$$u(x) = (b_1/b_3) C b_2 x / [b_1 + x b_2 (1 - C)], \quad (5a)$$

$$x = \frac{(b_1/b_2) C' \exp(-\zeta)}{1 - C' \exp(-\zeta)}, \quad (5b)$$

where C and C' can be determined from our knowledge of the coupling constants at $t = 0$, corresponding to $\zeta = 0$ and $x = 1/\tan^2\theta_w$, θ_w being the electroweak angle. (The domain of ζ, $0 \lesssim \zeta < \infty$, corresponds to the interval $\tan^{-2}\theta_w \geqslant x > 0$.) Note further that at the tree level,

$$y(t=0) = m_H^2/(8 m_W^2 \tan^2\theta_w), \quad (6a)$$

$$z(t=0) = m_t^2/(2 m_W^2 \tan^2\theta_W). \quad (6b)$$

The renormalization-group equations for the quartic and Yukawa couplings[9] can be put in the form

$$\frac{dy}{dx} = -\frac{192 y^2 - 8y(3 + 2b_1 + 9x - 12z) + 3(1 + 2x + 3x^2) - 48 z^2}{16 x (b_1 + b_2 x)}, \quad (7a)$$

$$dz/dx = -z[9z - 2b_1 - 16u(x)]/2x(b_1 + b_2 x). \quad (7b)$$

[Some electroweak contributions, small in comparison with $2b_1 + 16u(x)$, have been neglected in Eq. (7b).]

Both equations are of the Riccati type.[10] The second can be solved exactly; using ζ as the independent variable, we have

$$z(\zeta) = \frac{z(0)\exp[\chi(\zeta)]}{1 + z(0)(-9/2b_1)\int_0^\zeta \exp[\chi(\zeta)]\, d\zeta}, \quad (8)$$

where $\chi(\zeta) = -b_1^{-1} \int_0^\zeta (b_1 + 8u)\, d\zeta$.

To avoid the singularity—in other words, have a solution that tends to the fixed point $z = 0$ as $\zeta \to \infty$—that would otherwise develop in $z(\zeta)$, $z(0)$ must be bounded from above; this leads to the upper bound $M (\simeq 168$ GeV) on the mass of the top quark.

Equation (7a) can be solved analytically for small x and z; this is adequate, however, only to show (in a constructive way!) that there exists a nonsingular solution which is driven to the fixed point

$$y^* = [1 + \tfrac{2}{3} b_1 - \{(1 + \tfrac{2}{3} b_1)^2 - 4\}^{1/2}]/16$$

as $x \to 0$. To proceed further, it is necessary to integrate Eqs. (7a) and (7b) numerically; this was done with use of a computer program based on the Runge-Kutta method,[10] with the following values

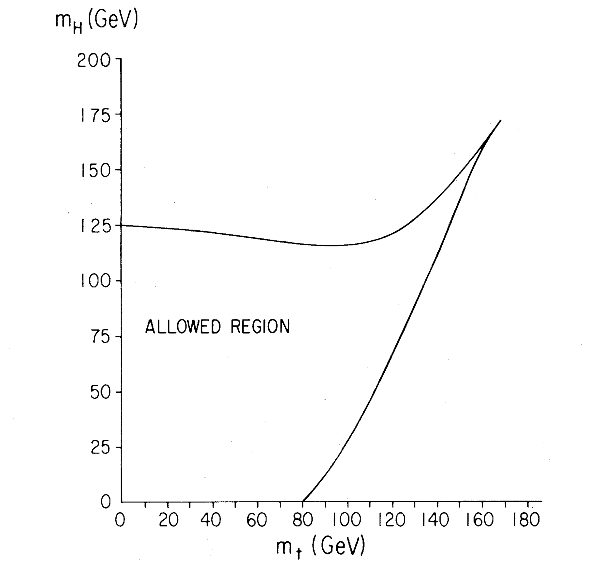

FIG. 1. Upper and lower bounds on the Higgs-boson mass plotted as a function of the top-quark mass. [The lower bound of Weinberg and Linde ($m_H \gtrsim 7$ GeV; see, for example, Ref. 1) is not depicted.]

of the input parameters:

$$b_1 = \tfrac{20}{9} n_g + \tfrac{1}{6} = 6.833\,33;$$

$$b_2 = \tfrac{22}{3} - \tfrac{4}{3} n_g - \tfrac{1}{6} = 3.166\,66,$$

$$b_3 = 11 - \tfrac{4}{3} n_g = 7,$$

$$x_{\text{init}} \equiv x_0 = 3.545\,45,$$

$$u_{\text{init}} \equiv u_0 = 10.0987.$$

The numerical values correspond to n_g (number of generations) = 3, $\sin^2\theta_w = 0.22$, $\Lambda_{\overline{MS}} = 0.1$ GeV, and initialization at momentum $m_W \simeq 81$ GeV. ($\Lambda_{\overline{MS}}$ is the usual QCD parameter.[1])

The computer-generated solution of Eqs. (7a) and (7b) led to the bounds on m_H quoted earlier, and plotted as a function of m_t, in Fig. 1.

Remarks.—(i) To verify that our reasonable-magnitude requirement is indeed fulfilled, if the theory is in the domain of attraction of the fixed point, consider the solution of Eq. (7a) for $m_t = 31$ GeV. With $y(x_0) = 1.02$, the computer-generated solution satisfied $0.027 < y(x) < 1.02$ for $5 \times 10^{-5} < x < x_0$. (The lower bound on y may be compared to 0.023, the value of y^* for $n_g = 3$.) To investigate what happens if one steps outside the domain, we set $y(x_0) = 1.10$ and found that $y \sim 4 \times 10^8$ for $x \sim 0.16$!

(ii) A shortcoming of our discussion lies in our retention of only one-loop contributions to the β functions. For the U(1) coupling, g_1, this leads to a singularity of the type first discussed by Landau[11] for QED. Crossing this singularity would take us into a domain of the absurd in which probabilities take on negative values. We do not, however, cross this singularity; $x \to 0$, or $\zeta \to \infty$, corresponds to approaching this singularity, but takes us no further; in other words, all momenta are cut off at a value determined by the position of the ghost pole. The value of this cutoff,

$$\Lambda_G = m_W \exp[2\pi \cos^2\overline{\theta}_w(m_W)/b_1\overline{\alpha}(m_W)] \simeq 4 \times 10^{41}\ \text{GeV},$$

is so large that its finiteness need have no effect on the physics we explore.

Nonetheless, there is a problem that calls for careful examination; it is the breakdown of the loop expansion for β_1 in the neighborhood of the ghost pole. To stay within the domain of validity of this expansion, one must cut off all momenta at a value Λ smaller than Λ_G. Now we anticipate that perturbation theory will be good if we adhere to t values such that $\overline{\alpha}_1(t) \equiv \overline{g}_1^2/4\pi \lesssim 0.1$; this corresponds, however, to $\Lambda \lesssim 4 \times 10^{37}$ GeV ($x \gtrsim 0.14$), an enormous energy indeed. Furthermore, as illustrated in (i), the upper bound imposed on m_H, by the mathematical requirement that the solution y of the one-loop equations tends to a fixed point as $x \to 0$, differs only by a few percent from the maximum value of m_H allowed by the demand that $y(t) \lesssim O(1)$ for momenta $\lesssim \Lambda$. Thus the finiteness of Λ need also be no cause for alarm!

(iii) A prime purpose of this Letter is to encourage our experimental colleagues to look for the elusive Higgs particle at masses ~ 10–150 GeV. That this mass range may be of interest for other reasons (it is the abode of pseudo-Goldstone bosons in the hypercolor scenario) has been noted in Ref. 1.

If a Higgs particle is not found, one may have to adopt a viewpoint wherein the scalar particles of the canonical theory are regarded as phenomenological props.[8] New physics then has to emerge at energies in the teraelectronvolt regime, if not earlier.[1] The challenge posed by unsolved theoretical problems in dynamical symmetry-breaking scenarios would have to be met.

(iv) The limitations of the renormalization-group formalism, in determining low-energy parameters, will be discussed elsewhere.

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[5]D. J. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973): See also D. J. E. Callaway, CERN Report No. TH.3660, 1983 (to be published). The point of view underlying Callaway's interesting paper is similar to ours; however, the implementation and the results differ.

[6]One may, however, regard the Salam-Weinberg theory as an effective field theory, with momenta cut off at values much smaller than the parameter Λ introduced later in this paper. Such a scenario would not meet our definition of "canonical realization." See Ref. 1; cf. Ref. 4.

[7]Almost all bounds, including the ones quoted in the present paper, are based on some formulation of the principle that weak interactions lend themselves to a perturbative treatment. See B. W. Lee, C. Quigg, and H. B. Thacker, Phys. Rev. D 16, 1519 (1977); N. Cabibbo, L. Maiani, G. Parisi, and R. Petronzio, Nucl. Phys. B158, 295 (1979).

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