Stable Grand-Unified Monopoles with Multiple Dirac Charge

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Magnetic monopoles with multiple Dirac charge are found to be stable in grand unified theories with symmetry breaking by an adjoint Higgs field under certain conditions. In the SU(5) model, the double, triple, quadruple, and sextuple monopoles are stable for a range of parameters in the Higgs potential. The effects of electroweak symmetry breaking on multiply charged monopoles are discussed. Evidence is also presented for the existence of a stable nonspherically symmetric quadruple monopole.

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Most studies of magnetic monopoles in grand unified theories have concentrated on monopoles with unit Dirac charge $g_D = 1/2e$. These "single" monopoles are typically the lightest that appear as classical solutions to the field equations. It is usually argued[1] that monopoles with multiple Dirac charge have sufficient energy to decay quickly into single monopoles. Thus, it is argued, only single monopoles should exist in the present universe. We will show that monopoles with multiple Dirac charge are stable in a large class of grand unified theories under certain conditions. We verify the stability argument explicitly in the SU(5) theory.

Stable multiply charged monopoles can exist in grand unified theories for purely topological reasons.[2] Our stability argument does not apply to these multiply charged monopoles, but our discussion of the effects of electroweak breaking on monopoles with weak fields is applicable.

The main difference between our work and previous treatments of multiply charged monopoles is in our treatment of electroweak breaking effects. The techniques of Coleman[1] and Brandt and Neri[3] can be used to show that for $r < 1/M_w$, multiple-charge monopoles satisfy the Brandt-Neri stability conditions with regard to SU(3) ⊗ SU(2) rather than just SU(3) as in previous treatments.[4-6] Thus we claim that calculations of monopole masses or monopole catalysis of nucleon decay should use *Ansätze* which look like stable SU(3) ⊗ SU(2) ⊗ U(1) monopoles for distances less than $1/M_w$ and like stable SU(3) ⊗ U(1) monopoles only for $r > 1/M_w$. We will first discuss SU(3) ⊗ SU(2) ⊗ U(1) monopoles and then analyze the effects of electroweak breaking.

The stable SU(5) monopoles have topological magnetic charges[4] g_D ("single monopole"), $2g_D$ ("double"), $3g_D$ ("triple"), $4g_D$ ("quadruple"), and $6g_D$ ("sextuple"), where the electronic charge is $e = (\frac{3}{8})^{1/2} g$ and g is the SU(5) coupling constant. In the Prasad-Sommerfield (PS) limit[7] (in which the Higgs particles are massless), the masses of the monopoles are integer multiples of M_x/α: $M_1 = M_x/\alpha$ (single monopole), $M_2 = 2M_1$ (double), $M_3 = 3M_1$ (triple), $M_4 = 4M_1$ (quadruple), and $M_6 = 6M_1$ (sextuple). (At the grand-unification scale M_x, $\alpha = \frac{1}{45}$.) The multiply charged monopoles are neutrally stable against decay into the lightest, and a zero-mode analysis[8] in the PS limit indicates that the multiply charged monopoles are composed of superpositions of the lightest monopole at a point.

Asymptotically the magnetic field of the monopole on the positive z axis is given by $B = Q/2gr^2$, where Q is a linear combination of unbroken generators. In unitary gauge, the upper 3×3 corner of Q represents SU(3), the lower 2×2 corner represents SU(2), while U(1) is represented along the diagonal. Since Q is Hermitian, Q can be diagonalized along the positive z axis by an SU(3) ⊗ SU(2) ⊗ U(1) gauge transformation. Diagonalizing Q diagonalizes both the asymptotic form of Φ_{PS} (since in the PS limit $B = d\Phi/dr$ asymptotically on the z axis) and the asymptotic form of Φ to first order (since Φ is matched against Φ_{PS} asymptotically) where Φ is the adjoint Higgs field. In this gauge, Q is constructed out of the generators $Y = \mathrm{diag}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2})$ of U(1) hypercharge, $Y_8 = \mathrm{diag}(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}, 0, 0)$ of color hypercharge, and

$T_3 = \text{diag}(0,0,0,\frac{1}{2},-\frac{1}{2})$ of SU(2). The magnetic fields stable against emission of gluons and W particles satisfy the Brandt-Neri[3] stability condition in the SU(3) and SU(2) subgroups. The solution of the quantization and Brandt-Neri conditions is $Q_n = nY + n_8 Y_8 + n_3 T_3$, where n is an integer, $n_8 = 0, \pm 1$, and $n_3 = 0, -1$ [$+1$ is SU(2)-gauge-equivalent to -1]. n labels the U(1) hypermagnetic charge of the monopole. After the phase transition to SU(3) ⊗ $U_{em}(1)$, the diagonal generators are Y_8 and $Q_E = Y - T_3 = \text{diag}(\frac{1}{3},\frac{1}{3},\frac{1}{3},-1,0)$. As we will show later, the monopoles adjust their T_3 charge so as to give a magnetic charge equal to ng_D in the SU(3) ⊗ U(1) phase. The monopole quantum numbers and masses are tabulated in Table I. The $SU(2)/Z_2$ quantum numbers are conserved modulo 2, while the $SU(3)/Z_3$ quantum numbers are conserved modulo 3 [since $\pi_1(SU(n)/Z_n) = Z_n$].

In addition to monopoles, the SU(5) particle spectrum consists of the superheavy gauge particles X and Y; the massless gluons, W's, and U of SU(3) ⊗ SU(2) ⊗ U(1); and the octet ϕ_8, triplet ϕ_3, and singlet ϕ_0 components of the adjoint Higgs field[9] (with masses μ_8, μ_3, and μ_0, respectively). (For the moment we will ignore the fundamental Higgs field.) The Higgs particles are massless in the PS limit. Since we are interested in corrections to the PS limit, we will suppose $0 < \mu_8, \mu_3, \mu_0 << M_x$. In the SU(5) theory, $\mu_3 = 2\mu_8$.

We imagine trying to form a double monopole by bringing two single monopoles together. Consider two single monopoles with magnetic charges $Q = \text{diag}(0,0,1,-1,0)$ and Q' separated by a distance r. Q' may differ from Q by an SU(3) ⊗ SU(2) gauge transformation, i.e., by a permutation of the eigenvalues of T_3 and Y_8 to give T_3' and Y_8'. Massless gauge-boson exchange gives an interaction energy

$$\text{Tr}(QQ')/4\alpha r = [n^2 \text{Tr}(Y^2) + n_3^2 \text{Tr}(T_3 T_3') + n_8^2 \text{Tr}(Y_8 Y_8')]/4\alpha r$$

while Higgs exchange gives an interaction energy

$$-[n^2 \text{Tr}(Y^2)\exp(-\mu_0 r) + n_3^2 \text{Tr}(T_3 T_3')\exp(-\mu_3 r) + n_8^2 \text{Tr}(Y_8 Y_8')\exp(-\mu_8 r)]/4\alpha r$$

which cancels gauge exchange in the PS limit $\mu_i = 0$.[10] Outside of the PS limit, and for $r >> 1/\mu_i$, the monopoles will orient themselves so as to minimize the gauge interaction energy. The minimum interaction energy occurs for $Q' = \text{diag}(0,1,0,0,-1)$ with $\text{Tr}(QQ') = 0$. In this gauge orientation, the repulsive U exchange between the monopoles is exactly cancelled by the attractive W and gluon exchange.

As the monopoles are brought to within a distance $\sim 1/\mu_i$, the effects of ϕ_i exchange become important. First suppose $\mu_0 << \mu_3, \mu_8$. Then for $1/\mu_3 < r < 1/\mu_0$, attractive ϕ_0 exchange will cancel repulsive U exchange and the net force due to W and gluon exchange will be attractive. For $1/M_x < r < 1/\mu_3$ the monopoles behave essentially like PS monopoles and therefore feel no force. Since the force between single monopoles vanishes except in the range $1/\mu_3 < r < 1/\mu_0$ where it is attractive, we expect the double monopole to be stable with a binding energy $(\frac{5}{24}\mu_0 - \frac{1}{3}\mu_8)/\alpha$.

The same argument may be repeated to construct a stable triple monopole from a single and a double monopole. In this case, the long-range force is repulsive since $\text{Tr}(QQ') > 0$. However, the force between the monopoles is attractive for $1/\mu_3 < r < 1/\mu_0$ and we expect a binding energy $(\frac{5}{12}\mu_0 - \frac{1}{6}\mu_8)/\alpha$. Stable quadruple and sextuple monopoles may be constructed in a similar way, with binding energies $(\frac{5}{6}\mu_0 - \frac{1}{12}\mu_8)/\alpha$ and $(\frac{5}{3}\mu_0 - \frac{1}{6}\mu_8)/\alpha$, respectively. The quintuple

TABLE I. Monopole quantum numbers.

Monopole	Dirac Charge	$Q = 2gr^2 B$	$SU(2)/Z_2$	$SU(3)/Z_3$
M_1	1	$\text{diag}(0,0,1,-1,0) = Y - T_3 - Y_8$	1	1
M_2	2	$\text{diag}(1,1,0,-1,-1) = 2Y + Y_8$	0	2
M_3	3	$\text{diag}(1,1,1,-2,-1) = 3Y - T_3$	1	0
M_4	4	$\text{diag}(1,1,2,-2,-2) = 4Y - Y_8$	0	1
M_6	6	$\text{diag}(2,2,2,-3,-3) = 6Y$	0	0

monopole, however, is always unstable. If the quintuple monopole existed, its magnetic charge would equal $Q_5 = \mathrm{diag}(2, 2, 1, -3, -2) = Q_2 + Q_3$. The force between a double and a triple monopole, though, is proportional to $\mathrm{Tr}(Q_2 Q_3') = 6\,\mathrm{Tr}(Y^2) > 0$ and thus depends only on ϕ_0 and U exchange. This exchange is repulsive for $r > 1/\mu_0$ and neutral for $r < 1/\mu_0$.

Suppose, on the other hand, $\mu_0 >> \mu_3, \mu_8$. Then in the intermediate range $1/\mu_3 < r < 1/\mu_8$, the force between two monopoles is repulsive because of U exchange. We therefore do not expect stable multiply charged monopoles in this case. We also note that there is an intermediate range of parameters where the monopoles will be classically stable but quantum mechanically unstable because of tunneling.

This argument applies to any unified gauge group [except SU(2)] which is spontaneously broken by an adjoint Higgs field. Symmetry breaking by an adjoint Higgs field always leaves an unbroken U(1) factor. Thus PS monopole solutions exist. If the Higgs scalars are lighter than the superheavy gauge bosons, then our stability argument is applicable, and we expect stable monopoles with multiple Dirac charge whenever the lightest scalar is the singlet.

Our stability argument can be explicitly checked in the SU(5) model by calculating the monopole masses near the PS limit by the technique of asymptotic matching.[11, 12]

The most general Higgs potential for the adjoint Higgs field with the discrete symmetry $\Phi \to -\Phi$ is[9]

$$\lambda V = \lambda[-\tfrac{1}{2}\mu^2 \mathrm{Tr}\Phi^2 + \tfrac{1}{4}a(\mathrm{Tr}\Phi^2)^2 + \tfrac{1}{2}b\,\mathrm{Tr}\Phi^4], \qquad (1)$$

where a and b are of order 1, $\lambda = 0$ in the PS limit, and $\lambda << 1$ near the PS limit. A cubic term in $V(\Phi)$ would alter the masses μ_i and the numerical factors in the stability criterion, but would not change our results qualitatively. In unitary gauge, $\langle\Phi\rangle = v\,\mathrm{diag}(1, 1, 1, -\tfrac{3}{2}, -\tfrac{3}{2})$ breaks SU(5) $\to$ SU(3) $\otimes$ SU(2) $\otimes$ U(1). v is determined by the condition $\mu^2 = (\tfrac{15}{2}a + \tfrac{7}{2}b)v^2$. For $b > 0$ and $a > -7b/15$, the global minimum of V occurs at the asymmetric minimum $\langle\Phi\rangle$.[13] In the further breaking of SU(3) $\otimes$ SU(2) $\otimes$ U(1) $\to$ SU(3) $\otimes$ $U_{\mathrm{em}}(1)$ discussed below, $\langle\Phi\rangle = v\,\mathrm{diag}(1, 1, 1, -\tfrac{3}{2} - \epsilon/2, -\tfrac{3}{2} + \epsilon/2)$ acquires a part that breaks SU(2) and $\langle H\rangle = v_0(0, 0, 0, 0, 1)$ gives masses to $W_\pm$ and Z_0, where $5gv/2 \sim M_x$ and $gv_0/2 \sim 100$ GeV for realistic theories. In calculating the first-order corrections to the monopole masses, $\lambda^{1/2}v$ will be assumed to be $< 10^{13}$ GeV $<< M_x$, H will be set equal to $\langle H\rangle$, and the terms in the potential for H and coupling Φ to H will be set equal to zero. The first-order corrections are on the order of $\lambda^{1/2}v$. The effects of H would be to add corrections on the order of $v_0 << \lambda^{1/2}v$. In addition, ϵ will be set equal to 0. The effects of ϵ would be to add corrections on the order of $v_0^2/\lambda^{1/2}v$.

To calculate the mass corrections, the static SU(5) monopole solutions[14] in the PS limit which are[15, 16] spherically symmetric with respect to $J + T$ [where T are the generators of an SU(2) embedding] are matched against the $\exp(-Mr)/r$ long-distance behavior of the fields near the PS limit. All fields will be evaluated on the positive z axis. Fields in any other direction will differ by a gauge transformation. (Once the existence of a spherically symmetric B is established, the long-range behavior of Φ_{PS} can be found from the Bogomol'nyi equation $B = d\Phi_{\mathrm{PS}}/dr$.)

The monopole mass corrections δM_m are most easily calculated[12] by differentiating M_m with respect to λ:

$$dM_m/d\lambda = 4\pi\int_0^\infty dr\, r^2[V(\Phi) + \tfrac{15}{16}(15a + 7b)v^4]. \qquad (2)$$

To evaluate the mass corrections to first order, the limits of integration in Eq. (2) may be replaced by $1/M_x << r < \infty$ and Φ may be replaced by its asymptotic form. Φ_{PS} is matched against the diagonal part of Φ near the PS limit: Φ is gauge equivalent to

$$\langle\Phi\rangle + \tfrac{6}{5}^{1/2}\phi_0 Y + \tfrac{3}{2}^{1/2}\phi_8 Y_8 + 2^{1/2}\phi_3 T_3.$$

The masses[9] of the Higgs particles are $\mu_8 = (10b \times \lambda)^{1/2}v/2$, $\mu_0 = [(15a + 7b)\lambda]^{1/2}v$, and $\mu_3 = (10b\lambda)^{1/2}v = 2\mu_8$. Asymptotically, $\phi_i = A_i \times \exp(-\mu_i r)/r$, where the A_i are determined by matching Φ_{PS} against Φ in the region $1/M_x << r << 1/\mu_i$. Then substituting Φ into Eq. (2) yields the mass corrections:

$$\delta M_m = [(\tfrac{1}{6}n_8^2 + \tfrac{1}{4}n_3^2)\mu_8 + \tfrac{5}{24}n^2\mu_0]/2\alpha.$$

The mass corrections near the PS limit are tabulated in Table II.

The spherically symmetric (see Ref. 16) quadruple monopoles are unstable against emission of gluons or W's. Furthermore the force between two double monopoles or a triple and single monopole is attractive for distances in the range $1/\mu_3 < r < 1/\mu_0$, so that the quadruple monopole cannot decay classically into widely separated mono-

TABLE II. Monopole mass corrections.

Monopole	$\alpha/\delta M_m$	Stability criterion
M_1	$\frac{5}{24}\mu_8+\frac{5}{48}\mu_0$	
M_2	$\frac{1}{12}\mu_8+\frac{5}{12}\mu_0$	$\mu_0<\frac{8}{5}\mu_8$
M_3	$\frac{1}{8}\mu_8+\frac{15}{16}\mu_0$	$\mu_0<\frac{2}{5}\mu_8$
M_6	$\frac{15}{4}\mu_0$	$\mu_0<\frac{1}{10}\mu_8$

poles with smaller charge. Thus we expect a stable quadrupole monopole to exist (for $\mu_0 << \mu_8$) which cannot be put in the spherically symmetric form of Ref. 16.

In conclusion, we consider the effects of the symmetry breaking $SU(2)\otimes U(1)\rightarrow U_{em}(1)$ on the monopoles with weak fields. The magnetic charge of the single monopole is $Q_1=Q_E-Y_8$, where Q_E is the generator of $U_{em}(1)$. The long-range fields of the single monopole thus lie in the unbroken $SU(3)\otimes U_{em}(1)$ directions. On the other hand, the double, triple, quadruple, and sextuple monopoles carry weak magnetic fields. Since these fields acquire a mass when the electroweak theory is spontaneously broken, it has been argued[2,17] that these monopoles should be confined by flux tubes in analogy with the confinement of monopoles in a superconductor. However, flux tubes formed during the symmetry breaking $G\rightarrow H$ are topologically stable only if $\pi_1(G/H)\neq 0$.[18] For an ordinary superconductor, $\pi_i(U(1))=Z$, while in our case $\pi_1(SU(2)\otimes U(1)/U_{em}(1))=0$.

This argument suggests that there is a lower energy configuration than a monopole plus flux tube. We rewrite the magnetic charge of the monopoles with weak fields as $Q_n(r)=2gr^2B=Q_n-f(r)T_3$, where $f(r)\rightarrow 0$ for $r << 1/M_w$ and $f(r)\rightarrow n+n_3$ for $r >> 1/M_w$ with M_w the weak breaking scale. The modification of the magnetic fields requires kinetic and Coulomb energy on the order of M_w which is much less than the energy required to create a flux tube (with energy per unit length $\sim M_w^2$) or to continue the pure electromagnetic field down to the origin ($\delta E\sim\mu_8/\alpha$). This energy is a small correction to the masses calculated above and therefore does not change our conclusions regarding the existence of stable multiply charged monopoles.

The screening of weak magnetic fields at a distance $1/M_w$ is analogous to the screening of color magnetic fields at a distance $1/\Lambda_{QCD}$. In fact, the Higgs phase and the confining phase for the electroweak theory are not expected to be distinct,[19] and the discussion in Ref. 1 of the effects of confinement on the Dirac quantization condition can be applied to this problem.

The presence of weak magnetic fields will alter the branching ratios for catalysis of nucleon decay and lead to catalysis of weak processes. Work on weak catalysis, cosmological production of multiply charged monopoles. and numerical calculations of monopole masses is currently in progress.

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