Modification of Predictions of Grand Unified Theories in the Presence of Spontaneous Compactification

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There are speculations that grand unified theories (GUT's) may arise from higher-dimensional models of gravity in which the extra dimensions are spontaneously compactified. The GUT predictions for proton decay and $\sin^2\theta_w$ can be significantly modified when nonrenormalizable interactions, scaled by inverse powers of the compactification scale M_c, are added to the standard GUT Lagrangian. For example, the decay rate for $p \rightarrow e^+\pi^0$ in minimal SU(5) can be lowered by one (two) orders of magnitude if M_c is on the order of 10^{17} GeV. However, $\sin^2\theta_w$ then decreases by 0.005 (0.01). The SO(10) model is also discussed.

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One of the most enduring and elegant ideas regarding the unification of gravity and gauge theories goes as far back as 1921, when Kaluza showed[1] that unification of gravity and electromagnetism can be achieved within Einstein's general relativity in five dimensions. Spontaneous compactification of one dimension to a size sufficiently small as to be presently unobservable is assumed to take place.[2] The ground state $M^4 \times S^1$ possesses a local U(1) gauge symmetry to be identified with the electromagnetic gauge invariance, with some of the extra components of the metric tensor constituting the gauge vector potential. In a theory of this kind scalar fields necessarily make an appearance. They constitute the remaining components of the metric tensor.

Attempts to extend the Kaluza idea to incorporate the low-energy gauge group $SU(3) \otimes SU(2) \otimes U(1)$[3] and grand unified theories[4] (GUT's) have recently been made. A potential stumbling block is the apparent difficulty encountered in placing the observed fermions in chiral representations of the low-energy group. The simplest attempts lead to vectorlike theories. Among the proposals put forward for overcoming this, we may mention the following. Since the theory apparently wants to be vectorlike let it be so, and hope that mirror fermions will be discovered in future experimental searches. This is probably the least attractive scenario. Another approach asserts that even if the underlying theory is vectorlike, it still may yield an effective GUT theory which has chiral representations.[5] This approach is adopted, for instance, by the proponents of $N = 1$ supergravity in eleven dimensions, who hope to deliver a realistic SU(5) GUT theory after descending to four dimensions. Finally, there is some hope that the superstring theory in ten dimensions[6] may yield a realistic gauge theory in four dimensions.

Since most recent attempts at unifying gravity with the gauge interactions follow the Kaluza idea, we decided to pursue possible implications that could ensue as a consequence of such unification for grand unified theories. In this Letter we show that the standard GUT predictions for proton decay and $\sin^2\theta_w$ can undergo significant modifications in the presence of spontaneous compactification. In theories with spontaneous compactification, the gauge couplings at the compactification scale can be calculated as functions of the characteristic length scale(s) of the internal space (for a general formula see Wetterich[4] and Weinberg[7]). In addition, these theories always generate nonrenormalization couplings involving gauge fields and one or more scalar fields [as in Eqs. (3) and (17)]. The coefficients of these interactions are scaled by one or more powers of $M_c^{-1} = R_c$, where R_c determines the size of the internal space. It is shown that the inclusion of these terms modifies the boundary conditions that are usually imposed on the gauge couplings at the GUT scale, with the consequence that the predictions for the GUT scale and $\sin^2\theta_w$ are altered.

In SU(5),[8] for instance, the new value for the GUT scale can be larger than its standard value,[9] thus increasing the proton lifetime.[10] However, and this is a surprising conclusion, $\tau(p \rightarrow e^+\pi^0)$ cannot be made arbitrarily large. An increase in the value of the GUT scale necessarily implies a corresponding decrease in $\sin^2\theta_w$. Thus, if $\tau(p \to e^+\pi^0)$ is increased by one (two) orders of magnitude, $\sin^2\theta_w$ decreases by 0.005 (0.01).

The SO(10) model[11] is also briefly considered. It is shown that if the symmetry breaking proceeds via the subgroup $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$,[12] $\tau(p \to e^+\pi^0)$ is easily arranged to be one or two orders of magnitude beyond the present IBM limit, with $\sin^2\theta_w$ close to 0.22.

When considered within a Kaluza-Klein framework, any GUT Lagrangian is expected to be modified through the addition of nonrenormalizable terms whose form is dictated by the appropriate local and global symmetries present. Let us discuss the SU(5) case in detail. The standard Lagrangian contains the pure gauge boson term

$$-\tfrac{1}{2}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}), \qquad (1)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu],$$

g being the SU(5) gauge coupling,

$$(A_\mu)_a^b = A_\mu^i(\lambda_i)_a^b,$$

and λ_i are SU(5) generators, normalized such that

$$\mathrm{Tr}(\lambda_i\lambda_j) = \tfrac{1}{2}\delta_{ij}. \qquad (2)$$

We now introduce the following SU(5)-invariant nonrenormalizable (NR) (dimension-five) interaction term

$$\mathscr{L}_{\mathrm{NR}} = (\eta/M_c)(-\tfrac{1}{2})\mathrm{Tr}(F_{\mu\nu}\Phi_{24}F^{\mu\nu}), \qquad (3)$$

where Φ_{24} denotes the Higgs 24-plet, η is a dimensionless parameter, and M_c is the compactification scale. Now let Φ_{24} acquire a nonzero vacuum expectation value,

$$\langle\Phi_{24}\rangle = (\tfrac{1}{15})^{1/2}\phi_0\,\mathrm{diag}(1,1,1,-\tfrac{3}{2},-\tfrac{3}{2}). \qquad (4)$$

The SU(5) gauge symmetry breaks to $SU(3)\otimes SU(2)\otimes U(1)$ and the gauge bosons that mediate baryon-number–nonconserving processes acquire masses $M_x \simeq (\tfrac{5}{24})^{1/2}g\phi_0$, g being the SU(5) gauge coupling.

The presence of the nonrenormalizable coupling in (3) modifies the usual kinetic energy terms of the SU(3), SU(2), and U(1) gauge bosons contained in (1). To order ϵ, the gauge boson part of the low-energy Lagrangian is given by

$$-\tfrac{1}{2}\{(1+\epsilon)\mathrm{Tr}(F_{\mu\nu}^{(3)}F^{(3)\mu\nu}) + (1-\tfrac{3}{2}\epsilon)\mathrm{Tr}(F_{\mu\nu}^{(2)}F^{(2)\mu\nu})\} - \tfrac{1}{4}(1-\tfrac{1}{2}\epsilon)F_{\mu\nu}^{(1)}F^{(1)\mu\nu}, \qquad (5)$$

where the superscripts 3, 2, and 1 refer to the gauge field strengths of SU(3), SU(2), and U(1), respectively, and ϵ is defined by

$$\epsilon = (\tfrac{1}{15})^{1/2}\eta\phi_0/M_c. \qquad (6)$$

After appropriate rescaling of the field variables, one finds that the SU(3), SU(2), and U(1) gauge couplings are related at the scale M_x as follows:

$$(1+\epsilon)g_3^2(M_x) = (1-\tfrac{3}{2}\epsilon)g_2^2(M_x) = (1-\tfrac{1}{2}\epsilon)g_1^2(M_x). \qquad (7)$$

Thus, the presence of the nonrenormalizable term (3) modifies the boundary conditions usually imposed on the gauge couplings which are $g_1 = g_2 = g_3$ at scale M_x. To order ϵ, the value of the weak mixing angle at M_x is given by

$$\sin^2\theta_w(M_x) = \tfrac{3}{8}(1-\tfrac{5}{8}\epsilon). \qquad (8)$$

Next, we employ the renormalization-group equations for the gauge couplings g_3, g_2, g_1, and by standard manipulations[13] obtain the following relations for the SU(5)-breaking scale M_x and $\sin^2\theta_w(M_w)$, where M_w denotes the weak scale:

$$\ln(M_x/M_w) \simeq (1+\epsilon K)\ln(M_5/M_w), \qquad (9)$$

$$\sin^2\theta_w(M_w) \simeq \sin^2\theta^{(5)}(M_w) + \delta(\sin^2\theta_w). \qquad (10)$$

Here

$$\ln\frac{M_5}{M_w} = \frac{1}{\alpha}\frac{\pi}{11}\left[1 - \frac{8}{3}\frac{\alpha}{\alpha_s}\right]$$

is the standard SU(5) prediction[13] for the superheavy scale to one loop, with $\alpha \simeq 1/128.5$ ($\alpha_s \simeq 0.11$) the electromagnetic (QCD) coupling at scale M_w. $\delta(\sin^2\theta_w)$ is the difference between the new and standard ($\sin^2\theta_w^{(5)} = \tfrac{1}{6} + \tfrac{5}{9}\alpha/\alpha_s$) one-loop prediction[13] for the weak mixing angle θ_w:

$$\delta(\sin^2\theta_w) \simeq [-\tfrac{5}{6} + \tfrac{10}{99}n_g + (\alpha/\alpha_s)(\tfrac{5}{9} - \tfrac{80}{297}n_g)]\epsilon. \qquad (11)$$

K is given by

$$K \simeq \tfrac{5}{2} - \tfrac{10}{33}n_g + 5(\alpha/\alpha_s)(1-\tfrac{8}{3}\alpha/\alpha_s)^{-1}, \qquad (12)$$

n_g being the number of fermion generations. Note that $K > 0$ for any realistic value of n_g, including 3 and 4.

It is instructive to express $\delta(\sin^2\theta_w)$ as a function of M_x/M_5 as follows:

$$\delta(\sin^2\theta_w) \simeq -(11\alpha/3\pi)\ln(M_x/M_5) \quad (13)$$

which clearly shows that $\delta(\sin^2\theta_w) < 0$ (>0) for $M_x > M_5$ ($< M_5$). Thus, a decrease in the decay rate of $p \to e^+\pi^0$ (which is predominantly mediated by the superheavy gauge bosons) is certainly possible through the mechanism that we are considering. However, the same mechanism then also causes a decrease in the predicted value of $\sin^2\theta_w(M_w)$. Because of its generality we find this to be a rather remarkable result and it should provide a useful constraint on model building.

For definiteness consider the case $n_g = 3$. Then, from (9), (11), and (12),

$$M_x/M_5 \simeq (M_5/M_w)^{\epsilon K}, \quad (14)$$

$$\delta(\sin^2\theta_w) \simeq -\frac{35+\frac{50}{3}\alpha/\alpha_s}{66}\epsilon, \quad (15)$$

$$K \simeq \tfrac{35}{22} + 5(\alpha/\alpha_s)(1-\tfrac{8}{3}\alpha/\alpha_s)^{-1}. \quad (16)$$

For $\epsilon \simeq 10^{-2}$ ($\frac{1}{50}$), $M_x \simeq 1.78 M_5$ [$(1.78)^2 M_5$], which corresponds to an increase in the lifetime of the process $p \to e^+\pi^0$ by one (two) orders of magnitude from its standard value. The corresponding value of $\delta(\sin^2\theta_w)$ is -0.005 (-0.01) which presumably is still acceptable. If one imposes the constraint $|\delta(\sin^2\theta_w)| \leq 0.015$, the conclusion one may draw is that the decay rate of $p \to e^+\pi^0$ in SU(5) cannot be suppressed by more than three orders of magnitude through the introduction of the nonrenormalizable term (3). However, this is comfortably outside the range of present experimental limits. Spontaneous compactification can save minimal SU(5)!

The value of the compactification scale M_c can be estimated from the above considerations if η is known. The precise value of η will depend on the specific theory (which remains to be constructed!), but presumably a value of order unity is not unthinkable. The scale M_c turns out then to be on the order of 10^{17} GeV.

Next, let us briefly discuss the SO(10) model.[11] The breaking of SO(10) to SU(3)⊗SU(2)⊗U(1) can proceed in various ways. For definiteness, consider the breaking via the Pati-Salam (PS) subgroup $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$ which can be achieved with a 54-plet of Higgs fields. The relevant nonrenormalizable coupling in this case is

$$\mathcal{L}_{NR} = (\eta/M_c)(-\tfrac{1}{2})\mathrm{Tr}(F_{\mu\nu}\Phi_{54}F^{\mu\nu}), \quad (17)$$

where $F_{\mu\nu}$ is an antisymmetric 10×10 matrix and Φ_{54} denotes a symmetric traceless 10×10 matrix. When Φ_{54} acquires a nonzero vacuum expectation value,

$$\langle\Phi_{54}\rangle = (\tfrac{1}{30})^{1/2}\Phi_0 \mathrm{diag}(1,1,1,1,1,1,-\tfrac{3}{2},-\tfrac{3}{2},-\tfrac{3}{2},-\tfrac{3}{2}),$$

the presence of the nonrenormalizable term in (17) modifies the usual gauge kinetic energy terms of the $SU(4)_c$, $SU(2)_L$, and $SU(2)_R$ gauge bosons.

Defining

$$\epsilon = (\tfrac{1}{30})^{1/2}(\eta/M_c)\phi_0 \quad (18)$$

the modified boundary conditions on the $SU(4)_c$, $SU(2)_L$, and $SU(2)_R$ gauge couplings at the SO(10) breaking scale M_x are

$$g_4^2(M_x)(1+\epsilon) = g_L^2(M_x)(1-\tfrac{3}{2}\epsilon) = g_R^2(M_x)(1-\tfrac{3}{2}\epsilon). \quad (19)$$

Note that, as in SU(5),

$$\sin^2\theta_w(M_x) = \tfrac{3}{8}(1-\tfrac{5}{8}\epsilon) + O(\epsilon^2). \quad (20)$$

The effects of the nonrenormalizable coupling (17) on the SO(10) predictions are exactly similar to the ones the coupling in (3) had on the SU(5) predictions, provided that one neglects nonrenormalizable couplings involving Higgs fields whose vacuum expectation values break the PS subgroup further. This is a reasonable thing to assume if the Pati-Salam breaking scale M_{PS} is sufficiently below the GUT scale. For definiteness, suppose that the PS subgroup directly breaks to SU(3)⊗SU(2)⊗U(1). Then, for $\alpha_s \simeq 0.11$, $M_{PS} \simeq 10^{13}$ GeV and in the absence of spontaneous compactification, $\sin^2\theta_w^{(10)} \simeq 0.23$ and $M_{10} \simeq 10^{15}$ GeV.[14] If we now switch on the coupling in (17), the GUT scale can be made to increase, with $\sin^2\theta_w$ correspondingly decreasing. Thus, it is easy to arrange

$$\tau(p \to e^+\pi^0)(\mathrm{SO}(10)) \gtrsim (10\text{–}100)\tau(p \to e^+\pi^0)(\mathrm{IBM}),$$

with $\sin^2\theta_w$ predicted to be about 0.22, in very good agreement with experiments.

To summarize, spontaneous compactification of higher-dimensional models of gravity may yield the standard GUT's, such as those based on gauge groups SU(5) or SO(10). Nonrenormalizable couplings scaled by inverse powers of the compactification mass scale M_c are then expected to be present. We have shown that these terms can significantly modify the standard GUT predictions if M_c lies between the GUT scale and the Planck mass.

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Note added.—After this paper had been submitted for publication Hill kindly sent us a copy of a paper[15] in which he also discusses the effects of dimension-five operators on GUT predictions. Where the two papers overlap, the results are essentially in agreement. However, in constrast to Hill, we conclude that the dimension-five operator by itself can save SU(5). The reason for our conclusion should be clear. In contrast to Hill's case, the dimension-five operator in our case is scaled by inverse powers of M_c rather than the Planck mass. Thus, even though nonrenormalizable operators are to be expected in any theory which incorporates gravitational effects, they can be especially important in the context of Kaluza-Klein theories where M_c can easily be two orders of magnitude smaller than the Planck mass (see, e.g., Freund[16]).

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