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Large-N QCD Baryon Dynamics—Exact Results from Its Relation to the Static Strong-Coupling Theory

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Starting from the large-N power counting which suggests that the baryons are QCD solitons, the authors derive an exact large-N equation identical to the so-called bootstrap condition of static strong-coupling theory. This equation determines the group structure of the baryon multiplets at $N\to\infty$. One solution is the standard nonrelativistic quark model.

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Some time ago Witten[1] suggested that baryons can be considered as solitons in the large-N limit of QCD. This was partially realized in more recent works.[2] These works suggest that the large-N limit could be studied as a sort of semiclassical approximation where $\hbar$ is replaced by $1/N$. However, it seems impossible to derive from first principles a concrete baryonlike solution, which is needed as the starting point of the corresponding $1/N$ expansion. Fortunately we already know one example of a theory where the essential results of semiclassical expansion were derived without making use of the classical solution. This is the so-called static strong-coupling theory of the meson-nucleon interactions. Many well-known physicists[3] have attached their names to the corresponding semiclassical expansion. On the other hand most of the results were later derived by Goebel[4] by means of an S-matrix bootstrap strong-coupling approach, in which no concept of field appears, not to mention any classical solution. Looking back at this approach[4,5] and in view of later studies of the strong-coupling theory[6] and of the more recent developments of semiclassical methods[7] in general, we realized that Goebel's viewpoint is certainly quite general. It provides an alternative route to semiclassical expansions where no classical solution is needed, which we plan to follow in this paper to study the large-N QCD baryon dynamics.

We shall follow closely the method of Ref. 5, where one of the essential ingredients is the behavior of various physical quantities in the strong-coupling limit. We now point out that this behavior is precisely the one which follows from the general arguments of Witten[1] in the large-N QCD. First the baryon mass is proportional to N so that we can use the nonrelativistic kinematics for baryons. On the other hand the meson mass is finite and mesons are fully relativistic. We further need the order of magnitude of the meson-baryon Yukawa coupling. By a simple quark counting one can see that the corresponding nonrelativistic overlap integral of meson-baryon-baryon wave functions is of order $\sqrt{N}$. This agrees with the standard behavior of the meson-soliton vertex in the semiclassical expansion.[7]

Consider the meson-baryon scattering: "α" + "i" $\to$ "β" + "j", where α and β indicate the initial and the final mesons, respectively, while i and j are baryon states. The dispersion relation of the corresponding scattering amplitude can be written as

$$T_{\beta\alpha}{}^{ji}(\omega) = -N\sum_k\left[\frac{(\tilde{A}_\beta)^{jk}(\tilde{A}_\alpha)^{ki}}{M_k - M_i - \omega + R_{ijk}} + \frac{(\tilde{A}_\alpha)^{jk}(\tilde{A}_\beta)^{ki}}{M_k - M_j + \omega + R_{ijk}'}\right] + \tilde{T}_{\beta\alpha}{}^{ji}(\omega), \tag{1}$$

where ω is the meson energy, M_i is the mass of baryon i, and R_{ijk} and R_{ijk}' are the baryon recoil corrections which are of order $1/N$. The matrix element $(\tilde{A}_\alpha)^{ij}$, which specifies the meson-baryon coupling, is of order 1 according to our previous discussion. $\tilde{T}_{\beta\alpha}{}^{ij}(\omega)$ includes all the scattering terms and meson poles. It is at most of order 1 according to Witten's discussion, which agrees with the general features of semiclassical expansions about soliton solutions.

In general we can write

$$M_i = M + \delta M_i, \tag{2}$$

where M is of order N. Since δM_i is the collective excitation energy of a very heavy object of mass M, it is of order $1/M \sim 1/N$. This is of course consistent with the soliton picture.

We are now in a position of repeating the discussion of Ref. 5. To the leading order N we obtain

$$T_{\beta\alpha}{}^{ji}(\omega) = -(N/\omega)([A_\beta, A_\alpha])^{ji}, \tag{3}$$

where A_α is the leading term of $\tilde{A}_\alpha$. Because of the unitarity relation, $T_{\beta\alpha}{}^{ji}(\omega)$ is of order 1. Thus,

$$[A_\alpha, A_\beta] = 0. \tag{4}$$

This equation is known to be the static bootstrap equation,[4] since it was derived in the static model by generating the baryon resonance poles from the driving force due to the exchange of the very same one-particle states.[8]

Equation (4) is an exact equation for QCD in the large-N limit. We show, following Ref. 5, how it can be used to obtain the baryon multiplet structure.

Let us call K the group of invariance of static baryon dynamics, i.e., $SU(2)_{spin} \otimes SU(n)_{flavor}$. We note that in the semiclassical method K is the group of symmetries which are broken by the classical soliton solution. The baryon states form a basis of a unitary representation of K. Let X_a be the infinitesimal generators of this representation. Then

$$[X_a, X_b] = C_{ab}{}^c X_c, \quad [X_a, A_\alpha] = D(a)_\alpha{}^\beta A_\beta, \quad [A_\alpha, A_\beta] = 0, \tag{5}$$

where $D(a)_\alpha{}^\beta$ is the infinitesimal generator of the representation to which the mesons belong. Altogether formula (5) gives a representation of the Lie algebra of the semidirect product $G = K \times T$ where the Abelian group T is generated by A_α. Since T is Abelian and $G \supset T$, G is noncompact and hence its unitary representations are infinite dimensional. The number of baryon states is therefore infinite.

There are two ways to obtain the representations. The first one is the induced representation method[9] which is naturally related to the soliton picture where the matrix A_α is given by the classical solution, Fourier transformed over the group K. The second method, which we follow after Ref. 5, is based on the notion of group contraction.[10]

In the present case if we consider only pseudoscalar mesons interacting in P states with baryons, the matrix A_α belongs to the adjoint representation both of the ordinary spin group and of the flavor group SU(n). Then G is obtained by contraction from SU($2n$). Accordingly a representation of G is obtained from an infinite-dimensional representation of SU($2n$). By reducing this infinite-dimensional representation according to $SU(2)\otimes SU(n)$ we obtain a tower of SU(n) baryon multiplets.

At this point one can make contact with the naive nonrelativistic quark model as follows. The representations of SU($2n$) are specified by Young tableaux which are labeled by $2n-1$ integers: $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{2n-1}$. Let us choose $\lambda_1 = \lambda$, $\lambda_2 = \lambda_3 = \ldots = \lambda_{2n-1} = 0$, which corresponds to the representation of SU($2n$) by a completely symmetric tensor with λ indices. The reduction of this representation to $SU(2)\otimes SU(n)$ is exactly the same as the reduction of symmetric state of λ nonrelativistic quarks. To obtain the representation of G we have to let λ be infinite and if we identify $\lambda = N$ we obtain exactly the naive nonrelativistic quark model for large-N QCD baryons.

In particular if we include up, down, and strange quarks, G is obtained by the contraction of SU(6) for $N=\infty$. For physical $N=3$ choosing the completely symmetric representation (3,0,0,0,0) we recover exactly the SU(6) quark model.[11] For completeness we note that one can also treat interactions through any partial wave.[12] Regge-type towers of baryon states come out.

Finally, if one neglects the recoil by setting $R = R' = 0$ in formula (1), one can derive mass formulas[4,5] in terms of the Casimir operators of K by considering expression (1) at order zero in N. Here, QCD being fully relativistic, we depart from the static strong-coupling theory. For large N, the recoil is nonnegligible. By taking it into account one may derive useful information. This is beyond the scope of the present paper but neglecting the recoil one recovers, in particular, the static results recently derived from the Skyrme solution.[2]

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Note added.—The relation between large-N QCD baryon dynamics and strong-coupling theory was also noticed by Bardakci[13] by use of the chiral-soliton model of baryons.

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