Theory of Two-Dimensional Magnetopolarons

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Two-dimensional weak-coupling Fröhlich polarons are investigated theoretically in the presence of an external magnetic field normal to the two-dimensional plane. For a weak magnetic field ($\omega_c << \omega_{LO}$), the cyclotron mass approaches the zero-field polaron effective mass, whereas for large fields ($\omega_c >> \omega_{LO}$), the cyclotron mass approaches the bare band mass from below. In the resonance situation ($\omega_c \approx \omega_{LO}$), the cyclotron mass shows an enhanced polaronic renormalization effect. Connection is made with recent data on magnetoabsorption in semiconductor space-charge layers, explaining why polaron effects show up only in some experiments.

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Much attention[1-9] has recently been focused on the polaronic aspects of two-dimensional electronic systems in the context of space-charge layers in semiconductor heterostructures made of III-V or II-VI materials. All such quasi two-dimensional systems fabricated so far (e.g., GaAs-$Al_xGa_{1-x}As$ and InAs-GaSb heterostructures and superlattices, InSb and $Cd_{1-x}Hg_xTe$ metal-insulator-semiconductor structures) are made of weakly polar materials in the sense that the Fröhlich[10] coupling constant (α) is of the order of 0.1. The important effect of the electron–LO-phonon interaction ("polaronic effect") on the one-electron energy spectrum in this weak-coupling ($\alpha < 1$) limit is to produce a negative shift ("polaron binding energy") of the effective band edge, to renormalize the electron effective mass from its band value to a slightly larger "polaron" value, and to give rise to a weak nonparabolicity in the electronic energy. The magnitudes of these effects have recently been calculated[9] for a quasi two-dimensional system in the absence of any external magnetic field. In this paper I consider the effect of an external magnetic field, B (applied normal to the two-dimensional plane), on two-dimensional polarons in the weak-coupling limit. This limit is expected to be fairly accurate for the actual systems of experimental interest.[1-6] The theory is used to extract a cyclotron resonance frequency ω_c^* which immediately defines a cyclotron mass $m = eB/\omega_c^* c$ that can be related to experimental observations.

In the presence of the external magnetic field B the energy of a bare two-dimensional electron is given by

$$E_n = (n + \tfrac{1}{2})\hbar\omega_c, \tag{1}$$

where $\omega_c = eB/mc$ is the cyclotron frequency with m as the bare (band) effective mass and n is the relevant Landau level index. All band nonparabolicity effects are neglected. In the presence of the electron–LO-phonon coupling the electronic self-energy shift, δE_n, is given by[8,11]

$$\delta E_n = \sum_{n'}\sum_{\vec{q}}[\,|u_{nn'}(\vec{q})|^2/\hbar\,][(n-n')\omega_c - \omega_{LO}]^{-1}. \tag{2}$$

In Eq. (2) ω_{LO} is the LO-phonon frequency and the coupling term is given by

$$|u_{nn'}(\vec{q})|^2 = (2\pi\alpha/q)(\hbar/2m\omega_{LO})^{1/2}\hbar^2\omega_{LO}^2 V_{nn'}(q), \tag{3}$$

with

$$V_{nn'}(q) = (n_2!/n_1!)(q^2l^2/2)^{n_1-n_2}e^{-q^2l^2/2}[L_{n_2}^{n_1-n_2}(q^2l^2/2)]^2, \tag{4}$$

where $l = (c\hbar/eB)^{1/2}$ is the Landau length and $n_1 = \max(n,n')$, $n_2 = \min(n,n')$. In Eq. (4) L is the associated Laguerre polynomial. In Eqs. (2)–(4) the wave vector $\vec{q}$ is a two-dimensional quantity (a sum over all possible q_z values has already been done where q_z is the component of the LO-phonon wave vector in the magnetic field direction).

A complete numerical evaluation of Eq. (2) for an arbitrary value of B (i.e., ω_c) is not very interesting since the main goal of this paper is to obtain the qualitative analytic features of the magnetopolaron behavior. Three different regimes of physical interest are considered explicitly: (I) weak magnetic field ($\omega_c \ll \omega_{LO}$); (II) strong magnetic field ($\omega_c \gg \omega_{LO}$); and (III) resonant magnetic field ($\omega_c \approx \omega_{LO}$). In each of these three regimes the right-hand side of Eq. (2) can be evaluated analytically to give the leading-order polaronic correction for the ground and the first excited states of the system.

(I) *Weak magnetic field* ($\omega_c \ll \omega_{LO}$).—If we put $n=0$ in Eq. (2), the ground-state energy correction is given by

$$\delta E_0 = -\sum_{n'}\sum_{\vec{q}}[|u_{0n'}(\vec{q})|^2/(n'\omega_c+\omega_{LO})\hbar]. \tag{5}$$

By use of Eqs. (3) and (4) and the fact that ω_c/ω_{LO} is a small expansion parameter for the problem, it is straightforward to show that

$$\delta E_0 \simeq -\hbar\omega_{LO}\left[\frac{\pi\alpha}{2}+\frac{\pi\alpha}{16}\left(\frac{\omega_c}{\omega_{LO}}\right)+\frac{9\pi\alpha}{512}\left(\frac{\omega_c}{\omega_{LO}}\right)^2\right], \tag{6}$$

where terms up to $(\omega_c/\omega_{LO})^2$ have been retained. Similarly, the correction δE_1 to the first excited Landau level is given by

$$\delta E_1 \simeq -\hbar\omega_{LO}\left[\frac{\pi\alpha}{2}+\frac{3\pi\alpha}{16}\left(\frac{\omega_c}{\omega_{LO}}\right)+\frac{81\pi\alpha}{512}\left(\frac{\omega_c}{\omega_{LO}}\right)^2\right]. \tag{7}$$

The relevant cyclotron resonance frequency for the $0\rightarrow 1$ transition is given by[12]

$$\begin{aligned}\omega_c^* &= (E_1^* - E_0^*)/\hbar = [E_1+\delta E_1-(E_0+\delta E_0)]/\hbar \\ &= \omega_c - \left(\frac{\pi\alpha}{8}\right)\omega_c - \left(\frac{9\pi\alpha}{64}\right)\frac{\omega_c^2}{\omega_{LO}} = \omega_c\left[1-\frac{\pi\alpha}{8}-\frac{9\pi\alpha}{64}\frac{\omega_c}{\omega_{LO}}\right].\end{aligned} \tag{8}$$

Equation (8) implies a renormalized cyclotron mass given by

$$m^* = m\left(1+\frac{\pi\alpha}{8}+\frac{9\pi\alpha}{64}\frac{\omega_c}{\omega_{LO}}\right) \simeq m_p\left(1+\frac{9\pi\alpha}{64}\frac{\omega_c}{\omega_{LO}}\right), \tag{9}$$

where $m_p = m(1+\pi\alpha/8)$ is the two-dimensional polaron effective mass[9] in the absence of any external magnetic field. In the limit of a very weak magnetic field ($\omega_c\rightarrow 0$) Eq. (9) gives a cyclotron mass which is approximately the same as the zero-field polaron mass with a weak magnetic field dependence implied by the second term on the right-hand side of Eq. (9).

Following Bajaj's semiclassical arguments[13] it can easily be shown[14] that the above results [Eqs. (8) and (9)] for the cyclotron resonance are really valid for an arbitrary cyclotron transition ($n\rightarrow n+1$) in the sense that the polaronic correction to an arbitrary Landau level in the weak-field limit is given by

$$\delta E_n = -\hbar\omega_{LO}\left[\frac{\pi\alpha}{2}+(n+\tfrac{1}{2})\frac{\pi\alpha}{8}\left(\frac{\omega_c}{\omega_{LO}}\right)+(n+\tfrac{1}{2})^2\frac{9\pi\alpha}{128}\left(\frac{\omega_c}{\omega_{LO}}\right)^2\right], \tag{10}$$

so that $\omega_c^* = (E_{n+1}^* - E_n^*)/\hbar$ is given by Eq. (8).

(II) *Strong magnetic field* ($\omega_c \gg \omega_{LO}$).—In this limit one can neglect all the off-diagonal ($n\neq n'$) terms in the right-hand side of Eq. (2) yielding

$$\delta E_n = -\sum_{\vec{q}}|u_{nn}(\vec{q})|^2/\hbar\omega_{LO}. \tag{11}$$

To leading order in the coupling constant α the cyclotron mass in the strong-field limit can then be shown to be given

$$m^* = m[1-\tfrac{1}{2}\pi^{1/2}\alpha(\omega_{LO}/\omega_c)^{1/2}]. \tag{12}$$

There are three important differences between the strong-field result [Eq. (12)] and the weak-field result [Eq. (9)]. These are the following: (i) In the high-field ($B\rightarrow\infty$) limit the cyclotron mass m^* approaches the bare mass m whereas in the weak-field ($B\rightarrow 0$) limit it approaches the renormalized polaron mass (m_p); (ii) for strong fields ($\omega_c > \omega_{LO}$) the cyclotron mass is *lower* than the bare mass (approaching it as $\omega_c\rightarrow\infty$), i.e., the mass renormalization is negative for large magnetic fields whereas for low fields ($\omega_c < \omega_{LO}$) the mass renormalization is positive (i.e., the cyclotron mass is heavier than the bare mass); (iii) the first-order mass correction in the high- and the low-field situations have different power-law dependences on ω_c (and hence on B)—the correction in the weak-field situation [Eq. (9)] is linear in B whereas that in the high-field situation [Eq. (12)] goes as $1/\sqrt{B}$.

This high-frequency result [Eq. (12)] of a negative polaronic mass renormalization is both new and qualitatively surprising because one is used to think[10] in terms of the static, or the low-frequency, result where an electron becomes heavier as it polarizes the lattice [Eq. (9), for example]. However, the high-frequency polaronic behavior is qualitatively different as has been explicitly shown in this paper. The physical reason for this is the "pinning behavior" associated with the resonance condition $\omega_c = \omega_{\mathrm{LO}}$ and the fact that the polaronic effects are qualitatively different below and above the resonance condition. It will be very interesting actually to observe this predicted negative mass renormalization by using reasonably high magnetic fields (>10 T) in InSb space-charge layers[6] (in GaAs heterostructure the field needed would be in excess of 20 T).

(III) *Resonant magnetic field* ($\omega_c \approx \omega_{\mathrm{LO}}$).—It is clear from the above analysis of the high- and the low-field situations that the nature of the polaronic correction to the electron effective mass changes qualitatively as the cyclotron frequency sweeps through the LO-phonon frequency. This is also obvious from the singular structure of the electronic self-energy due to the electron–LO-phonon interaction at the resonance ($\omega_c = \omega_{\mathrm{LO}}$) situation. The corresponding effect[15] in a three-dimensional system is a much weaker phenomenon because of the spread of electronic kinetic energy in the z direction.

The electron–LO-phonon coupling lifts the singularity and gives rise[8] to split cyclotron resonance peaks around $\omega_c \approx \omega_{\mathrm{LO}}$ as one would expect in a level-crossing situation. For the $0 \rightarrow 1$ cyclotron transition (i.e., when the Fermi level is in the lowest Landau level) the two cyclotron masses corresponding to the split peaks are given by

$$m^*_{\pm} = m\left[1 \mp \left(\tfrac{1}{8}\pi^{1/2}\alpha\right)^{1/2} + \lambda/2\omega_c\right], \tag{13}$$

where $\lambda = \omega_{\mathrm{LO}} - \omega_c \ll \omega_{\mathrm{LO}}$ or ω_c (i.e., $\omega_{\mathrm{LO}} \approx \omega_c$). The $\pm$ signs in Eq. (13) refer, respectively, to the split cyclotron peaks above and below the bare cyclotron frequency ω_c ($\approx \omega_{\mathrm{LO}}$). In deriving Eq. (13) resonance situation (i.e., $\lambda/\omega_c \ll 1$) has been explicitly assumed so as to allow one to consider only the degenerate self-energy term (which couples Landau levels 0 and 1) as the dominant one. The usual diagonal self-energy term contributes an $O(\alpha)$ renormalization of the cyclotron mass which can be neglected in the weak-coupling limit in comparison with the $O(\sqrt{\alpha})$ contribution of the resonant off-diagonal term of Eq. (13).

The two most important aspects of the resonant magnetopolaron absorption are the splitting of the cyclotron resonance around $\omega_c \approx \omega_{\mathrm{LO}}$ and a significant enhancement of the polaronic effect (within the weak-coupling $\alpha < 1$ theory) which now shows up as a $\sqrt{\alpha}$ effect rather than the normal linear-in-α correction [e.g., compare Eq. (13) with Eqs. (9) and (12)] in the nonresonant situation.

The enhancement of the polaronic correction at the resonance situation makes it possible for a resonant experiment like the one carried out[6] recently by Horst *et al.* on the InSb inversion layer to unambiguously show polaronic effects whereas other very careful[1] nonresonant experiments have failed to detect any polaron correction in quasi two-dimensional electronic systems. In particular for InSb with its very weak polar coupling ($\alpha \approx 0.03$) the nonresonant or the zero-field polaron mass correction would be negligibly small (about 1%) whereas Eq. (13) gives about 8% resonant polaron mass correction. Similarly for GaAs ($\alpha \approx 0.07$), nonresonant polaron mass correction is about 3% whereas the resonant correction is considerably higher (about 12%). Experiments should be carried out on GaAs heterostructure in the resonant situation ($\omega_c \approx \omega_{\mathrm{LO}}$) which corresponds to a magnetic field of about $B \approx 20$ T to test the validity of this analysis.

It should be emphasized that the use of *purely* two dimensional approximation for the electronic wave function is *not* an essential restriction of this work—it has been done only for the sake of analytic simplicity and clarity of the final results. Introduction of the finite width of the electronic wave function in the z direction into the above formalism is straightforward and it will reduce[9] the effective electron-phonon coupling, changing the α in the final results [Eqs. (8), (9), (12), and (13) for example] to some effective $\alpha' < \alpha$. The actual reduction in the effective coupling will obviously depend on the details of the system involved and can only be obtained numerically for specific systems. These calculations are underway and will be reported in a forthcoming longer publication[11] on the subject. However, the qualitative features and the unifying aspects of the theory developed in this Letter are independent of this two-dimensional approximation because even though the specific numbers quoted above would change (go down in magnitude), their relative magnitudes remain unaffected by these nonessential approximations.

In summary, this paper treats within a unified theory the problem of electron–LO-phonon interaction effect on the cyclotron mass of two-dimensional electrons in semiconductor space-charge layers. The theory completely explains the hitherto puzzling experimental data, some of which clearly show the polaronic effects (and some of which do not), as a manifestation of the polaronic enhancement at the resonance situation. The theory is a weak-coupling one which should be valid for the existing experimental systems and it neglects a number of effects (e.g., screening, finite width of the electron wave function, etc.) which are present in the actual experimenatal situation, but vary quantitatively from system to system. Fortunately these neglected effects have no bearing on the qualitative conclusions of this paper (since these affect resonant and nonresonant situations equally). In view of the great current experimental[1–6,16–18] and theoretical[7–9] interest in these systems (including the II-VI materials[16] which are even more strongly polar than the usual III-V systems), it is hoped that this paper would stimulate more experimental work in the high-field ($\omega_c \gtrsim \omega_{\mathrm{LO}}$) region which will be helpful in a better understanding of the role of electron–LO-phonon interaction in two-dimensional systems. This is of considerable fundamental interest in view of the questions that have been raised[8] with respect to the validity of the Fröhlich continuum model in semiconductor microstructures.

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