Observable Neutrino Dirac Mass and Supergrand Unification

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A new mechanism for generating an ultralight Dirac neutrino is given. A discussion is given of how it might possibly arise in a supergrand unified theory (SUSY GUT) with a geometric type of a hierarchy. The neutrino is scaled in mass relative to its charge-$\frac{2}{3}$ family member by $M_{\mathrm{SUSY}}/M_{\mathrm{GUT}}$.

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A tiny mass of the neutrino has always been a possibility shrouded in deep mystery. This mystery has now deepened further. The recent remeasurement[1] (with a much improved energy resolution) by Boris *et al.* of the near-end-point shape of the β spectrum in tritium decay has yielded the lower limit $m(\bar{\nu}_e) > 20$ eV. This is to be compared with an older upper limit[2] of 55 eV. On the other hand, the lack of observation[3] of the neutrinoless nuclear double β decay $\mathrm{Ge} \to \mathrm{Se} + e^- + e^-$ has implied an upper limit of 10 eV for the Majorana mass of ν_e. If these results are taken at face value,[4] the simplest[5] inference is that ν_e has a Dirac mass somewhere in the range 20 to 55 eV.

The above conclusion is puzzling to followers of unified gauge theories. The standard $\mathrm{SU}(2) \otimes \mathrm{U}(1)$ electroweak and the minimal SU(5) grand unified theories (GUT's) do not admit any neutrino Dirac mass in view of the absence of the right-handed neutrino. The latter can be introduced as an additional singlet under the unifying group; but then the natural expectation would be for a neutrino Dirac mass in the same ballpark as the masses of its charged family members, i.e., $m(\nu_e) = O(\mathrm{MeV})$. An inexplicably small Yukawa coupling and/or Higgs vacuum expectation value (VEV) would be required to reduce it to the electronvolt range. Left-right–symmetric $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)$ as well as grand unified SO(10) and E_6 theories, which necessarily have nonsinglet right-handed neutrinos, naturally admit mechanisms yielding smaller neutrino masses anywhere between $O(10\text{ eV})$ and $O(10^{-5}\text{ eV})$. However, this is always a Majorana mass for the physical neutrino.[6] To our knowledge, there is at present no scheme within the unified-theory framework leading naturally (i.e., without either of the two vices mentioned earlier) to an ultralight Dirac neutrino as suggested by experiment.[1,3] This is what constitutes the aforestated puzzle. We aim in this Letter to try to solve this puzzle.

A massive Dirac neutrino can emerge[7] from two degenerate Majorana neutrinos. This idea was made use of some time ago by Georgi and Nanopoulos.[8] Recently, Wyler and Wolfenstein[9] have given a detailed and somewhat different treatment of a similar scheme. However, this mechanism, using three neutral fermions, leads to a very heavy Dirac neutrino (along with a massless one) rather than an ultralight Dirac particle, as presently desired. A slight adjustment[10] of the model enables the massless left-handed neutrino to pick up a small mass, but it becomes a Majorana and not a Dirac particle. Thus the problem of naturally generating an ultralight Dirac neutrino has remained pending. We propose a new scheme which offers a solution to this problem. In this scheme there are *two Dirac neutrinos: an ultralight one and another that is extremely heavy.* For the lowest-lying fermionic generation, the former can be interpreted as ν_e while the latter is as heavy as the GUT scale and hence phenomenologically inconsequential.

Four chiral neutral fermions are needed at the start to manufacture two physical Dirac neutrinos. We designate the corresponding left-handed fields as n_L, s_L, n_L', and s_L' and consider their right-handed charge conjugates as well. The mass matrix, pertaining to these fields, that we would like to have is

$$\begin{array}{c|cccc} & n_L & s_L & n_L' & s_L' \\ \hline n_R{}^C & 0 & 0 & A & 0 \\ s_R{}^C & 0 & 0 & B & C \\ n'_R{}^C & A & B & 0 & 0 \\ s'_R{}^C & 0 & C & 0 & 0 \end{array} \qquad (1)$$

(Here the elements A, B, and C can be chosen to be real and positive without loss of generality by redefining the chiral fermion fields—so long as CP nonconservation is neglected.) (1) is a rather simple-looking extension of the mass matrix of Wyler and Wolfenstein[9] having an extra row and column with just one pair of nonvanishing elements. It is of the form

$$\begin{pmatrix} 0 & F \\ F^T & 0 \end{pmatrix},$$

where F is a square matrix. Thus the necessary and sufficient condition[9] for the absence of massless physical neutrinos is met. The four nonzero eigenvalues of (1) split into two pairs, each of which consists of degenerate members (with opposite signs), yielding two Dirac masses given by

$$(A^2+B^2+C^2)^{1/2} \times [1 \pm \{1 - 4A^2C^2(A^2+B^2+C^2)^{-2}\}^{1/2}]^{1/2}/\sqrt{2}.$$

We are interested in the limit $B \gg C, A$. Then, to leading approximation, one obtains a very heavy and an ultralight Dirac neutrino of masses $B+(C^2+A^2)/2B$ and AC/B, respectively. The corresponding normalized Dirac states can be described as $|N\rangle$ and $|\nu\rangle$ where we find their left- and right-handed components to be

$$|N_L\rangle \simeq (1+A^2/B^2)^{-1/2}(A/B|n_L\rangle),$$

$$|N_R\rangle \simeq (1+C^2/B^2)^{-1/2}(|n_R{'}^C\rangle + C/B|s_R{'}^C\rangle),$$

$$|\nu_L\rangle = (1+A^2/B^2)^{-1/2}(|n_L\rangle - A/B|s_L\rangle),$$

and

$$|\nu_R\rangle \simeq (1+C^2/B^2)^{-1/2}(C/B|n_R{'}^C\rangle - |s_R{'}^C\rangle).$$

Hence for $B \sim 10^{16}$ GeV, $A \sim 10^{10}$ GeV, and $C \sim 10$ MeV, the ultralight Dirac mass is ~ 10 eV which is the desired order of magnitude.

In realizing the above scheme within a unified-theory framework, we adopt the philosophy of Ref. 9: To wit, our purpose is not the construction of a realistic gauge model but the illustration of the possible origin of (1). We take an SO(10) grand unified theory[11] with supersymmetry, broken at an intermediate scale[12] $M_{\rm SUSY}$ suggested by a geometric type of a hierarchy. A word here about the reasoning behind this choice. Evidently, $B \sim 10^{16}$ GeV suggests a grand unified scale $M_{\rm GUT}$ while $A \sim 10$ MeV indicates the relevance of the charged quark masses of the lightest generation. These considerations naturally lead to an SO(10) GUT (with n_L and $n_L{'}$—which is identified with $n_L{}^C$—in the lowest spinorial hexadecaplet) where the mass of the charge-$\frac{2}{3}$ quark gets inducted[11] into the neutrino mass matrix. On the other hand, $C \sim 10^{10}$ GeV suggests an intermediate scale not far from the geometric mean between the weak and the grand unification masses. This fits in perfectly with $M_{\rm SUSY}$. Moreover, we shall need a gauge-singlet scalar field with a VEV given by C divided by a Yukawa coupling strength, and two gauge-singlet left-handed fermion fields to be identified with s_L and $s_L{'}$; all these are inspired by supersymmetry.[13] Thus although the matter content of this SO(10) model is a minor extension of that of Wyler and Wolfenstein[9] (having two additional singlets: a scalar and a fermion), it has entirely different physical consequences now as a result of the crucial roles played by $M_{\rm SUSY}$ and $M_{\rm GUT}$.

Let ψ stand for the left-handed hexadecaplet of fermion fields and S, S' for the singlet ones. For the moment we confine ourselves to the lightest fermion family. Three Higgs fields are needed: Φ in the usual (complex) $\underline{10}$ representation, χ in the $\underline{16}^*$ representation, and Σ—the singlet[13] referred to earlier. Our Yukawa interaction is given in terms of three couplings as

$$\mathcal{L}_Y = h_1 \overline{\psi}^C \Phi \psi + h_2 \overline{\psi}^C \chi S + h_3 \overline{S}'^C \Sigma S + \text{H.c.} \tag{2}$$

This interaction is chosen so as to respect three global U(1) symmetries. Those rule out other possible Yukawa terms and are respectively designated $U_a(1)$, $U_b(1)$, and $U_c(1)$: (a) $\psi \to e^{i\alpha}\psi$, $\Phi \to e^{-2i\alpha}\Phi$, $S \to e^{-i\alpha}S$, $S' \to e^{i\alpha}S'$, (b) $S \to e^{i\beta}S$, $\chi \to e^{-i\beta}\chi$, $S' \to e^{-i\beta}S'$, and (c) $S' \to e^{i\gamma}S'$, $\Sigma \to e^{-i\gamma}\Sigma$. Here α, β, and γ are arbitrary real parameters and all fields whose transformations are not explicitly displayed are taken not to transform at all. Thus $U_a(1)$ and $U_b(1)$ are the same as those introduced by Wyler and Wolfenstein,[9] except that S' has the corresponding charges equal and opposite to those of S. Only $U_c(1)$ is an additional global symmetry involving just our new singlet fields.

SO(10) invariance is broken at $M_{\rm GUT} \sim 10^{16}$ GeV by the VEV's of the neutral members of χ in the subrepresentation[14] $(\underline{1}, \underline{2}, \underline{4}^*)$ of the Pati-Salam subgroup $SU(2)_L \otimes SU(2)_R \otimes SU(4)$. Further, $\langle\Sigma\rangle$ is associated[13] with supersymmetry breaking at $M_{\rm SUSY} \sim 10^{10}$ GeV. Finally, the neutral members of the $(\underline{2}, \underline{2}, \underline{1})$ component of Φ acquire VEV's to break electroweak symmetry and generate masses for fermions. It is now clear how the matrix (1) is obtained with $h_1\langle\Phi\rangle$, $h_2\langle\chi\rangle$, and $h_3\langle\Sigma\rangle$ leading to A, B, and C, respectively. While $U_{a,b,c}(1)$ are violated, a different global $U_K(1)$ is left[9] invariant. The latter combines $U_b(1)$ with the gauged $U_L(1) \otimes U_R(1)$ subgroup of SO(10) and has the charge $Q_K = Q_b + 2(I_{3L} + I_{3R})$ conserved since $Q_K(\langle\chi\rangle) = Q_K(\langle\Phi\rangle) = Q_K(\langle\Sigma\rangle) = 0$. [Even if the full $U_K(1)$ symmetry is not respected by the Higgs self-interactions, it suffices[9] to have a discrete subgroup left invariant.]

The Q_K values for the chiral fermion fields constituting the mass matrix (1) are $Q_K(n_L)=1$, $Q_K(n_L')=-1$, $Q_K(s_L)=1$, $Q_K(s_L')=-1$, and opposite amounts for the corresponding charge-conjugate fields. Therefore protection from nonzero contributions induced by symmetry breaking (both at the tree level and to all loops) is granted by $\mathrm{U}_K(1)$ invariance to all the zeros in the matrix (1) except the off-diagonal pair corresponding to $s_R'^C n_L = n_R^C s_L'$. Thus we expect an induced contribution $ds_R'^C n_L$ + H.c. in the neutral-fermion mass term from higher-order corrections. This violates $\mathrm{U}_{a,b,c}(1)$ simultaneously. Thus, if we expand in powers of M_{GUT}^{-1}, the leading term in d must be proportional to $h_1 h_2 h_3 M_{\mathrm{GUT}}^{-2}\langle\Phi\rangle\langle\chi^*\rangle\langle\Sigma\rangle$ which is of order AC/B. (One can easily convince oneself that M_{WEAK} or M_{SUSY} cannot come into the denominator since the d term has to vanish in the limit when $M_{\mathrm{WEAK}}/M_{\mathrm{GUT}}$ or $M_{\mathrm{SUSY}}/M_{\mathrm{GUT}}$ tends to zero.) Thus though d cannot be made to vanish, its induced magnitude is comparable to the ultralight Dirac mass. This conclusion also follows explicitly from mass analyses of relevant higher-order diagrams. Hence, if we define $d \equiv D^2/B$, D^2 is of the same order as AC. Now the F submatrix of (1) is changed to

$$F = \begin{pmatrix} A & D^2/B \\ B & C \end{pmatrix}.$$

To leading approximation, in the limit $B \gg A, C, D$, the ultralight Dirac mass consequently becomes $|AC - D^2|/B$, but not much else is changed. Thus the possibility of a neutrino Dirac mass ~ 10 eV remains at the same level.

The above results, derived with only one fermionic generation, readily admit generalization to the k-generation case. Now A, B, and C of the matrix (1) become $k \times k$ matrices in generation space. Their eigenvalues characterize the charge-$\frac{2}{3}$ fermion mass, the GUT mass, and the SUSY-breaking mass scales, respectively. We have verified[15] that there are k extremely heavy and k ultralight physical neutrinos with masses given approximately by the square roots of the eigenvalues of the matrices $BB^\dagger$ and $AB^{-1}C(AB^{-1}C)^\dagger$, respectively. Any mixing between the two classes of neutrinos is strongly suppressed by M_{GUT}^{-1}, but substantial mixing among members within each class is in principle possible. However, we cannot calculate the corresponding angles without additional assumptions. The masses of the ultralight neutrinos in different generations are not expected to be spaced closely. Instead, unless the generation hierarchies in the matrices B and C are too strong, it is reasonable to expect[16] $m(\nu_e):m(\nu_\mu):m(\nu_\tau) = O(m_u):O(m_c):O(m_t)$. The consequent cosmological difficulty arising from such high-mass neutrinos may be evaded by making ν_μ and ν_τ decay as well as scatter into ν_e and/or light neutral yet unobserved pseudo-Goldstone bosons[16] (e.g., majoron) which are natural in our scenario in view of the presence of spontaneously broken global U(1) symmetries. With such masses, the Glashow-Iliopoulos-Maiani type of suppression makes flavor-changing processes such as $\mu \to e + \gamma$, μ + nucleus $\to e$ + nucleus, etc., unobservable. The neutrino oscillation lengths are predicted to be very small for laboratory energies so that present experiments are sensitive only to averages over many oscillatory cycles. For our neutrino masses, experiments looking for depletion of neutrino flux (relative to calculation) imply[17] that the ν_μ mixing angles are $\lesssim 0.05$ while the ν_e-ν_τ one can be as large as 0.2. A verifiable consequence of the present scenario is the order of magnitude of the ν_μ mass which is predicted to be in the 10-kev range; this can be checked by bringing down the present upper limit—obtained from study of the $\pi \to \mu\nu_\mu$ decay—by a factor of 50 (perhaps careful study of reactions such as $K \to \mu\nu_\mu$, $K \to \pi\mu\nu_\mu$ at a K-meson factory will yield dividends). One might also be able to detect neutrino flux depletion due to oscillations. Our work provides a rationale to improve further the already excellent present experiments in these directions.

Although we have not been able to calculate the neutrino Dirac masses precisely, we claim to have found a model where one can estimate their orders of magnitude with reasonable reliability. In spite of the theoretical speculations involved, this model appears interesting in the light of the present experimental situation on the ν_e mass. If the electron neutrino is indeed a Dirac particle in the 20–55-eV mass range, the above mass-generation mechanism is a serious contender for being the right scheme. Realistic SUSY GUT models, being attempted nowadays, do then need to implement it and obey its constraints. Could it be that an ultralight Dirac neutrino is already signaling supergrand unification with a geometric hierarchy?

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[1]S. Boris *et al.*, to be published, quoted by V. A. Lubimov, in Proceedings of the International Conference on High Energy Physics, Brighton, United Kingdom, 1983 (to be published). See also V. A. Lubimov *et al.*, Phys. Lett. 94B, 266 (1980).

[2]K. E. Bergkvist, Nucl. Phys. B39, 317 (1972).

[3]F. T. Avignone *et al.*, Phys. Rev. Lett. 50, 721 (1983); E. Bellotti *et al.*, to be published, quoted by V. Khovansky, in Proceedings of the International Conference on High Energy Physics, Brighton, United Kingdom, 1983 (to be published), claim an upper limit of 7 eV on the Majorana mass of ν_e.

[4]Instead of raising skeptical questions about the treatment of molecular effects by Boris *et al.* (Ref. 1) and the calculation of nuclear matrix elements used in Ref. 3, we shall choose to accept the claims of the experimentalists and explore their theoretical implications.

[5]We do not consider more complicated explanations such as the existence of two closely spaced physical Majorana neutrinos with opposite *CP* properties; L. Wolfenstein, Phys. Lett. 107B, 77 (1981).

[6]See, for example, T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe*, edited by O. Sawata and A. Sugamoto (National Institute for High Energy Physics KEK, Tsubaraki, Japan, 1979), as well as M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. Van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979), and also E. Witten, Phys. Lett. 91B, 81 (1980).

[7]For a discussion of the emergence of a neutrino Dirac mass from two degenerate Majorana masses see L. Wolfenstein, Nucl. Phys. B 186, 147 (1981).

[8]H. Georgi and D. V. Nanopoulos, as cited by P. Langacker, Phys. Rep. 72, 185 (1981); see the discussion around Langacker's Eq. (6.19).

[9]D. Wyler and L. Wolfenstein, Nucl. Phys. B218, 205 (1983).

[10]See Eq. (6.20) of Langacker, Ref. 8.

[11]H. Georgi, in *Particles and Fields—1974*, edited by C. E. Carlson, AIP Conference Proceedings No. 23 (American Institute of Physics, New York, 1975); H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) 93, 193 (1975).

[12]M. Dine and W. Fischler, Nucl. Phys. B204, 346 (1982), specifically have $M_{\rm SUSY} \sim 10^{10}$ GeV and $M_{\rm GUT} \sim 10^{16}$ GeV as used here. Many other models of this type, but slightly different in detail, exist in the literature. For instance, J. Ellis, L. Ibanez, and G. C. Ross, Phys. Lett. 113B, 283 (1982); R. Barbieri, S. Ferrara, and D. V. Nanopoulos, Phys. Lett. 116B, 6 (1982); S. Dimoupoulos and S. Raby, Nucl. Phys. B219, 479 (1982); J. Polchinski and L. Susskind, Phys. Rev. D 26, 3661 (1982).

[13]Our gauge-singlet fields are inspired by the works of Dine and Fischler, Ref. 12 and of Polchinski and Susskind, Ref. 12.

[14]Thus $\mathrm{SU}(2)_R$ is also broken at $M_{\rm GUT}$. Moreover, the right-handed components of our ultralight neutrinos are predominantly SO(10) singlets. Hence they cannot excite measurable weak right-handed currents.

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