Inflation with SU(5)

Qaisar Shafi
Bartol Research Foundation of The Franklin Institute, University of Delaware, Newark, Delaware 19716

and

Alexander Vilenkin
Physics Department, Tufts University, Medford, Massachusetts 02155
(Received 18 November 1983)

A simple extension of the SU(5) Higgs system is presented and shown to yield a satisfactory inflationary scenario.

PACS numbers: 98.80.Bp, 12.10.En, 14.80.Gt

An inflationary scenario[1] may be termed successful if it satifies the following requirements: (1) The expansion factor during inflation is sufficiently large,

$$H\tau \gtrsim 60, \tag{1}$$

in order to solve the horizon and flatness problems. Here τ is the duration of the inflationary phase and H is the de Sitter Hubble constant,

$$H^2 = (8\pi/3)G\rho_v, \tag{2}$$

where ρ_v is the vacuum energy density. (2) The density fluctuations resulting from the quantum fluctuations of the scalar field, whose slow evolution leads to the inflationary phase, should be sufficiently small,

$$(\delta\rho/\rho)_{\rm hor} \lesssim 10^{-4}. \tag{3}$$

(3) The number density of the superheavy magnetic monopoles should be suppressed to satisfy the existing constraints.[2] (4) After inflation, there should be a way for the universe to reheat and generate the baryon asymmetry.

Constraints 1 and 2 can be satisfied if the effective scalar potential is very flat near the origin ($\varphi = 0$). A Coleman-Weinberg potential of the form

$$V(\varphi) = A\varphi^4[\ln(\varphi^2/M^2) + C] \tag{4}$$

is acceptable only if[3] $A \lesssim 10^{-12}$. (Here M is an arbitrary renormalization mass and C is a constant of order unity.) This rules out the possibility that the Coleman-Weinberg potential (4) is due to gauge interactions, since in that case $A \sim g^4 \gtrsim 10^{-2}$. Some authors[4] have invoked supersymmetry to find an acceptable scenario, but it now appears that in supersymmetric models the constraints 1–4 cannot be satisfied in a natural way; besides, supersymmetry introduces additional complications with the cosmological term.[5] In this Letter we suggest a simple extension of the Higgs system of the minimal SU(5) model which gives a successful inflationary scenario. It requires introducing a very weakly coupled SU(5) singlet scalar field.

The basic idea behind our approach is rather straightforward. We know that the Coleman-Weinberg mechanism can give flat effective potentials, but we also know that the gauge coupling g^2 [~ 0.3 in SU(5)] is too large for our purposes. On the other hand, Higgs and Yukawa couplings are largely arbitrary, and it is not unusual for them to be very small. We therefore introduce a SU(5) singlet scalar field φ which develops a Coleman-Weinberg potential from its weak couplings to the adjoint and fundamental Higgs fields, Φ and H_5, thereby acquiring a nonzero vacuum expectation value. In the spirit of Coleman-Weinberg philosophy, we require the tree potential to be scale invariant. For simplicity we also impose the discrete symmetries, $\varphi \to -\varphi$ and $\Phi \to -\Phi$. (The last assumption can be removed without altering the main conclusions of the paper.)

The tree-level scalar potential is given by

$$V = \tfrac{1}{4}a(\mathrm{Tr}\Phi^2)^2 + \tfrac{1}{2}b\,\mathrm{Tr}\Phi^4 + \alpha(H_5^\dagger H_5)\,\mathrm{Tr}\Phi^2 + \tfrac{1}{4}\lambda(H_5^\dagger H_5)^2 + \beta H_5^\dagger\Phi^2 H_5 + \tfrac{1}{4}\lambda_1\varphi^4 - \tfrac{1}{2}\lambda_2\varphi^2\,\mathrm{Tr}\Phi^2 + \tfrac{1}{2}\lambda_3\varphi^2 H_5^\dagger H_5. \tag{5}$$

We assume that the coefficients a, b, α, and λ are $\sim g^2$, so that most radiative corrections in the (Φ, H_5) sector can be neglected. Below we shall assume a somewhat smaller value for the coefficient β, which requires a small amount of fine tuning (see later). We also take $\lambda_2, \lambda_3 \ll 1$ ($\lambda_2, \lambda_3 > 0$) and $\lambda_1 \lesssim \max(\lambda_2^2, \lambda_3^2)$.

Radiative corrections due to the couplings $\varphi^2\,\mathrm{Tr}\Phi^2$ and $\varphi^2 H_5^\dagger H_5$ induce a Coleman-Weinberg potential for φ, which is given by Eq. (4) with

$$A=(64\pi^2)^{-1}(24\lambda_2^2+10\lambda_3^2). \quad (6)$$

To produce density fluctuations of magnitude (3) we need $A\sim 10^{-12}$ and $\lambda_2,\lambda_3\sim 10^{-5}$–$10^{-6}$.

We would like to emphasize that the SU(5)-symmetry breaking in our model is not of the Coleman-Weinberg type. Once the singlet field φ develops a vacuum expectation value, we get an effective SU(5) potential for Φ and H_5 with mass terms $\sim\lambda_2^{1/2}\langle\varphi\rangle$. (A potential of this kind has been analyzed in detail in Ref. 6.[7]) The SU(5) symmetry is broken to $SU(3)\otimes SU(2)\otimes U(1)$ when Φ acquires a vacuum expectation value

$$\langle\Phi\rangle=[v/(15)^{1/2}]\,\mathrm{diag}(1,1,1,-\tfrac{3}{2},-\tfrac{3}{2}), \quad (7)$$

where

$$v^2=[30/(15a+7b)]\lambda_2\varphi^2. \quad (8)$$

With Φ given by Eq. (7), the (φ,Φ) sector of the effective potential can be written as

$$V=\frac{15a+7b}{240}v^4-\frac{1}{4}\lambda_2\,\varphi^2v^2+A\varphi^4\,(\ln\frac{\varphi^2}{M^2}+C). \quad (9)$$

In Eq. (9) M is an arbitrary renormalization mass; different choices of M correspond to different values of the constant C. If we choose $C=-\frac{1}{2}+15\lambda_2^2/4A(15a+7b)$, then the zero-temperature minimum of the potential is at $\varphi=M$ and $v=v_0$, where v_0 is given by Eq. (8) with $\varphi=M$. The scale of the SU(5)-symmetry breaking is $M_x\sim\lambda_2^{1/2}M$. Requiring that $M_x\sim 10^{15}$ GeV gives $M\sim 10^{18}$ GeV. We fine tune the vacuum energy ρ_v to zero at present; then the energy density of the false vacuum ($\varphi=\Phi=0$) is of order M_x^4.

At high temperatures the scalar fields develop temperature-dependent mass terms,[8]

$$V_T=\gamma T^2\varphi^2+\sigma T^2\,\mathrm{Tr}\Phi^2+\nu T^2H_5^\dagger H_5, \quad (10)$$

where

$$\gamma=-\lambda_2+\tfrac{5}{12}\lambda_3,$$

$$\sigma=\tfrac{1}{120}(75\,g^2+130a+94b+100\alpha+10\beta),$$

$$\nu=\tfrac{1}{120}(72\,g^2+30\lambda+240\alpha+48\beta).$$

If we want the expectation value of φ to vanish as $T\to\infty$, we must require that $\gamma>0$ or

$$\lambda_3>\tfrac{12}{5}\lambda_2. \quad (11)$$

It has recently been suggested[9] that the universe could tunnel quantum mechanically from a state of pure space-time foam ("nothing") directly to the inflationary phase with the field φ at the top of the effective potential. In this scenario the preinflationary hot period is absent, and the condition (11) is unnecessary. In this paper we shall take a more conventional approach assuming a hot big bang, although most of the following discussion and all the conclusions of the paper apply to both versions of the inflationary scenario.

At $T\sim\rho_v^{1/4}\sim M_x$ the universe enters an inflationary phase of exponential expansion with a Hubble constant $H\sim M_x^2/M_P\sim 10^{11}$ GeV. (Here $M_P=1.2\times 10^{19}$ GeV is the Planck mass.) The initial thermal energy is rapidly reshifted, but the gravitational effects in de Sitter space are rather similar to those of finite temperature,[10] and the terms given by Eq. (10) are still present with $T\sim T_H=H/2\pi$, the "Hawking temperature."[11] Early in the de Sitter phase the false vacuum gets destabilized by quantum fluctuations, and the field starts "rolling down." The rollover time is[12]

$$\tau\sim\pi^2(8\lambda_0)^{-1/2}H^{-1}, \quad (12)$$

where $\lambda_0=A\ln(\lambda_2M^2/H^2)\sim 20\lambda_2^2$.

Initially, the evolution of φ is dominated by quantum fluctuations and[13]

$$\langle\varphi^2\rangle\simeq H^3t/4\pi^2. \quad (13)$$

Strictly speaking, this equation applies only for a massless field. In our case φ has a small effective mass $\sim\lambda_2^{1/2}T_H$, but it is easily verified that the effect of this mass on the evolution of φ is negligible. When the classical evolution takes over, φ is given by (for $\tau-t\gg H^{-1}$)

$$\varphi^2\simeq 3H/2\lambda_0(\tau-t). \quad (14)$$

The SU(5) symmetry is broken to $SU(3)\otimes SU(2)\otimes U(1)$ when $\lambda_2\varphi^2/2\sim\sigma T_H^2$, that is when $H(\tau-t)\sim\lambda_2^{-1}$. The SU(5) phase transition is completed in a few Hubble times, but the initial scale of symmetry breaking is $\langle\Phi\rangle\sim T_H$. As φ continues rolling down, this scale increases like $\langle\Phi\rangle\sim\lambda_2^{1/2}\varphi$ and reaches its present value $\sim M_x$ by the end of inflation. On the other hand, the direction of the Higgs field is fixed in each horizon volume at the time of the phase transition, and since the phase transition occurs early in the inflationary era, the problem of primordial monopoles is solved. According to this scenario there are no monopoles in our present universe, even after reheating. By the same token, even though we imposed a discrete ($\Phi\to-\Phi$) symmetry on the potential (5) there is no domain-wall problem.

Note that the vacuum expectation value $\langle\Phi\rangle\sim\lambda_2^{1/2}\varphi$ has little effect on the subsequent evolution of φ. It gives rise to a term of order $\lambda_2^2\varphi^4$ in the effective potential for φ, which is similar in magnitude to the Coleman-Weinberg term. Thus, the rollover pace is not substantially altered and the model produces a huge expansion factor as desired.

When φ becomes $\sim M$, it starts oscillating about the minimum of the effective potential. In fact, because of the coupling between φ and Φ, we have coherent oscillations of both fields. The frequency m_φ is found by diagonalizing the mass matrix of the potential (9) at the minimum: $m_\varphi = (8A)^{1/2}M \sim 10^{12}$ GeV. Since the frequency of oscillations is small ($m_\varphi << M_x$), the Higgs field Φ follows φ adiabatically, so that v is always given by Eq. (8) $v \propto \lambda_2^{1/2}\varphi$.

The oscillations of φ can be described by the equation

$$\varphi - M = a(t)M \sin(m_\varphi t). \tag{15}$$

Here $a(t)$ is a dimensionless amplitude which is of order 1 when the oscillations start. The amplitude $a(t)$ decreases with time because of the expansion of the universe and because of particle production.

The effective equation of state of the oscillating field φ is that of a pressureless gas,[14] its energy density is $\rho \sim \frac{1}{2}a^2(t)M^2 m_\varphi^2$, and so, as long as particle production is unimportant, the universe expands like $R(t) \propto t^{2/3}$ and the amplitude decreases like $a(t) \propto t^{-1}$. (Here and below we set $t=0$ at the end of the inflationary era.)

Efficient reheating is possible only through creation of particles with mass smaller than m_φ. In our model φ couples only to the Higgs fields, Φ and H_5. The mass of Φ bosons is $\sim M_x >> m_\varphi$, and so their production is exponentially suppressed. The Higgs 5-plet H_5 splits into a SU(2) doublet H_2 and a SU(3) triplet H_3; the corresponding mass terms are given by

$$m_2^2 = (\alpha + 0.3\beta)v^2 + \lambda_3\varphi^2,$$
$$m_3^2 = m_2^2 - (\beta/6)v^2. \tag{16}$$

Since $v \propto \varphi$, both mass terms can be expressed as φ^2 multiplied by a combination of coupling constants. The mass of the doublet has to be fine tuned to be close to zero ($|m_2| \sim M_w$). Then the mass of the triplet is determined solely by the coefficient β ($\beta < 0$)[6]:

$$m_3^2 = (|\beta|/6)v^2 = (2|\beta|/5g^2)M_x^2. \tag{17}$$

Because of the fine tuning, the effective coupling of the doublet to φ is extremely small, and the corresponding reheating rate is negligible.[15] To have a reheating through the production of H_3 particles, we must require that $2m_3 < m_\varphi$, that is $|\beta| \lesssim 10^{-6}$. [Note that the SU(3) triplet H_3 mediates proton decay which imposes the constraint $|\beta| \gtrsim 10^{-8}$, corresponding to $m_3 \gtrsim 10^{11}$ GeV.] Radiative corrections to β are $O(\alpha^2) \sim 10^{-4}$, and some fine tuning is therefore required to implement these constraints.

To estimate the reheating rate, we note that the relevant term in the potential takes the form

$$\xi M^2 a(t) \sin(m_\varphi t) H_3^\dagger H_3,$$

where $\xi \sim \beta\lambda_2$. To the lowest order of perturbation theory in ξ, the energy of H_3 particles produced per unit space-time volume is

$$\dot{\rho} = (3/8\pi)\xi^2 M^4 a^2(t)(m_\varphi^2 - 4m_3^2)^{1/2}. \tag{18}$$

The rate at which the energy of the oscillating field φ is dissipated is

$$\Gamma = \dot{\rho}/\rho \sim \beta^2\lambda_2 M. \tag{19}$$

The oscillations are damped out when the Hubble time t becomes $\sim \Gamma^{-1}$, and the universe reheats to a temperature[16]

$$T_r \sim (\Gamma M_p)^{1/2}. \tag{20}$$

For $|\beta| \sim 10^{-6}$, we have $m_3 \sim 10^{12}$ GeV and $T_r \sim 3 \times 10^9$ GeV.

Baryon asymmetry can be generated in this model through decays of the SU(3) triplet scalar bosons. If ϵ is the baryon number produced per one triplet decay, the resulting baryon asymmetry is $\eta = n_B/n_\gamma \sim \epsilon T_r/m_3$.[17] In the minimal SU(5) model ϵ is $< 10^{-15}$, and so η is far too small.[18] We note that this problem is not peculiar to our scenario, but is present in all cosmological models based on SU(5) with one Higgs 5-plet. The disease can be cured either by extending the Higgs sector of the model or by working with a larger gauge group.[18] The inflationary scenario described here is easily extended to gauge groups larger than SU(5).

To conclude, we have shown that a straightforward extension of the minimal SU(5) Higgs system yields a satisfactory inflationary scenario. A scalar field that transforms as a singlet under the unifying gauge symmetry is all that needs to be added. Such scalars automatically appear in Kaluza-Klein theories.[19]

The authors are grateful to Tanmay Vachaspati for his help in the derivation of Eq. (18) and to A. Guth for a useful discussion. This work was supported in part by the U.S. Department of Energy under Contract No. DE-AC02-78ER05007 and in part by National Science Foundation through Grant No. PHY-8206202.

[1]A. H. Guth, Phys. Rev. D 23, 347 (1981); A. D. Linde, Phys. Lett. 108B, 389 (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982).

[2]See Proceedings of Monopole '83, Ann Arbor, Michigan, October 1983 (to be published).

[3]S. W. Hawking, Phys. Lett. 115B, 295 (1982); A. H. Guth and S. Y. Pi, Phys. Rev. Lett. 49, 1110 (1982); A. Starobinsky, Phys. Lett. 117B, 175 (1982); J. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. D 28, 679 (1983).

[4]A. Albrecht *et al.*, Nucl. Phys. B229, 528 (1983); G. Lazarides and Q. Shafi, in *Proceedings of the Nuffield Workshop on the Very Early Universe*, edited by G. W. Gibbons, S. W. Hawking, and S. Siklos (Cambridge Univ. Press, Cambridge, England, 1983); J. Ellis *et al.*, Phys. Lett. 120B, 331 (1983); D. V. Nanopoulos *et al.*, Phys. Lett. 123B, 41 (1982), and 124B, 171 (1983); A. D. Linde, to be published.

[5]B. A. Ovrut and P. J. Steinhardt, Rockefeller University Report No. RU83/B/65, 1983 (to be published).

[6]M. Magg and Q. Shafi, Z. Phys. C 4, 63 (1980).

[7]In particular, it is shown in Ref. 6 that for $15a + 7b > 0$ and $\beta < 0$, the $SU(3) \otimes SU(2) \otimes U(1)$ phase is the absolute minimum of the effective SU(5) potential. The $SU(4) \otimes U(1)$ phase is not even a local minimum of this potential for the above range of the parameters, so that problems of the kind discussed by J. Breit, S. Gupta, and A. Zaks [Phys. Rev. Lett. 51, 1007 (1983)] are avoided.

[8]S. Weinberg, Phys. Rev. D 9, 3357 (1974); L. Dolan and R. Jackiw, Phys. Rev. D 9, 3320 (1974).

[9]A. Vilenkin, Phys. Lett. 117B, 25 (1982), and Phys. Rev. D 27, 2848 (1983).

[10]G. Shore, Ann. Phys. (N.Y.) 128, 376 (1980); B. Allen, to be published; A. Vilenkin, Nucl. Phys. B226, 504 (1983).

[11]G. W. Gibbons and S. W. Hawking, Phys. Rev. D 15, 2738 (1977).

[12]A. D. Linde, Phys. Lett. 116B, 335 (1982); A. Vilenkin, Nucl. Phys. B226, 527 (1983).

[13]A. Vilenkin and L. H. Ford, Phys. Rev. D 26, 1231 (1982); A. D. Linde, in Ref. 12.

[14]M. S. Turner, Phys. Rev. D 28, 1243 (1983).

[15]The doublet mass m_2 is fine tuned to be near zero in flat space-time with φ and Φ at the minima of the effective potential. Expansion of the universe and oscillations of the scalar fields may spoil this fine tuning (by affecting the coupling constants) and lead to a more complicated reheating picture. We hope to return to this question in a future publication.

[16]A. Albrecht and P. J. Steinhardt, to be published.

[17]L. Abbot, E. Farhi, and M. Wise, Phys. Lett. 117B, 29 (1982).

[18]E. W. Kolb and M. S. Turner, Enrico Fermi Institute Report No. 83-14, 1983 (to be published), and references therein.

[19]Q. Shafi and C. Wetterich, Phys. Lett. 129B, 387 (1983).