Δ_{33} Dynamics in Pion Double Charge Exchange

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The authors calculate effects of direct meson-Δ_{33} interactions on pion double charge exchange to isobaric analog states in ^{18}O. Virtual Δ_{33}'s in the ground state are shown to contribute negligibly, contrary to recent expectations. On-shell Δ_{33} interactions can have a large effect through $\pi+\rho$ exchange, and inclusion of these terms improves agreement with experiment below resonance. Data at the higher energies cannot be explained by a simple combination of sequential pion-nucleon scattering plus direct meson-Δ_{33} terms.

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The expectation that pion double charge exchange (DCX) would be sensitive to Δ_{33} presence in nuclei has focused a great deal of attention on these reactions. Large DCX cross sections observed for $N=Z$ nuclei have, in particular, led to the conjecture[1] that sizable "one-step" contributions to DCX would occur through the Δ_{33} components of the nuclear wave functions [Fig. 1(a)]. This idea provided a realization for a previously proposed two-component phenomenological description[2] of DCX for $N>Z$ nuclei. According to this picture, anomalies observed in

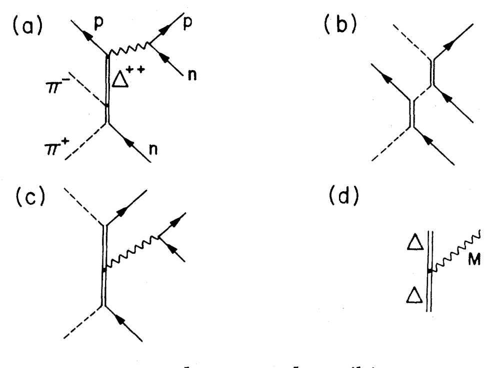

FIG. 1. Feynman diagrams describing Δ_{33} processes that contribute to DCX and that are at issue in this paper. (a) DCX to Δ_{33} components of the final nuclear state. DCX from Δ_{33} in the initial state is also considered. (b) DCX through sequential scattering. (c) DCX through direct Δ_{33}-nucleus interaction. (d) An important interaction contributing to (a) and (c). The meson M may be a π meson or a ρ meson.

^{18}O$(\pi^+,\pi^-)^{18}$Ne to the double isobaric analog state (DIAS) occur as an interference between this "one-step" and the more familiar sequential DCX on two nucleons [Fig. 1(b)]. This model has been used for subsequent explanations[3] and predictions.[4] In this paper we make a detailed microscopic calculation of the terms in Figs. 1(a) and 1(b). We also evaluate the closely related process shown in Fig. 1(c), which we expect to be larger than Fig. 1(a) because in the latter process one of the Δ_{33} is off shell.

The question of how the Δ_{33} interacts with the nucleus is one of quantitative importance in many areas of medium-energy and low-energy nuclear physics. A basic issue is to understand the strength of the coupling shown in Fig. 1(d). We take a simple *Ansatz* for this term suggested by the SU(2)$\otimes$SU(2) quark model. For the pion we take $(f_{\pi\Delta\Delta}/m_\pi)\vec{\Sigma}\cdot\vec{k}\,\vec{\theta}\cdot\vec{\alpha}\,v_{\pi\Delta\Delta}(k)$, where $\vec{\Sigma}$ and $\vec{\theta}$ are the Δ_{33} spin and isospin operators and $v(k) = (1+k^2/\Lambda^2)^{-1}$ is the form factor. The value of the coupling constant[5] is $f_{\pi\Delta\Delta}^2 = \frac{16}{25} f_{\pi NN}^2$, where $f_{\pi NN}^2/4\pi = 0.079$. For the ρ meson we take $(f_{\rho\Delta\Delta}^2/m_\rho)\vec{\Sigma}\cdot\vec{k}\times\vec{\epsilon}\,\vec{\theta}\cdot\vec{\alpha}\,v_{\rho\Delta\Delta}(k)$, where $\vec{\epsilon}$ is the polarization vector of the ρ meson. The value of $f_{\rho\Delta\Delta}$ is obtained from $f_{\rho\Delta\Delta}/f_{\pi\Delta\Delta} = f_{\rho NN}/f_{\pi NN}$ (Ref. 5), where we take the "strong" value[6] for $f_{\rho NN}$: $(f_{\rho NN}^2/f_{\pi NN}^2)(m_\pi^2/m_\rho^2) = 2$. Values of Λ_π and Λ_ρ are controversial and model dependent. Boson-exchange models[7] and helicity-amplitude analyses[6] suggest values of 1.2 GeV/c or greater for $\Lambda_{\pi NN}$ and $\Lambda_{\pi N\Delta}$. Recent calculations have also been made in chiral bag models,[8] which give $\Lambda_{\pi\Delta\Delta}$ to be 0.47–0.74 GeV/c.

We construct DCX cross sections from the solution of the Klein-Gordon equation for the pion. The interaction with the nucleus is described by an optical potential[9] U. The interactions in Figs. 1(a) and 1(c) are represented by a second-order term in U, which is built out of the pion ($\vec{\phi}$) and nuclear ($\vec{T}_N$) isospin operators, $\Delta U = (\vec{\phi}\cdot\vec{T}_N)^2 U_2$, and which we calculate according to well-defined diagrammatic rules.[10] Because we are interested in the qualitative effects of the terms in Fig. 1, we make the following simplifications: (1) $U - \Delta U$ is taken to be the free pion-nucleon amplitude averaged over the nuclear ground state; and (2) closure is assumed for intermediate nuclear states, i.e., nucleon and Δ_{33} recoil and interactions are neglected except to the extent described by ΔU.

The isotensor potential can be constructed[11] in terms of ground-state expectation values of Figs. 1(a) and 1(c), $U_2 = \sum\langle \mathrm{gs}|\theta_\mu(i,j)|\mathrm{gs}\rangle$, where $|\mathrm{gs}\rangle$ is the ground state of ^{18}O, where the sum runs over all pairs of nucleons and diagrams μ, and where $\theta_\mu(i,j)$ is the piece of diagram μ proportional to the isotensor $\tau_{ij} = (\vec{\phi}\cdot\vec{\tau}_i\,\vec{\phi}\cdot\vec{\tau}_j + \vec{\phi}\cdot\vec{\tau}_j\,\vec{\phi}\cdot\vec{\tau}_i)/2$, with $\vec{\tau}$ the Pauli isospin matrix of the nucleon. We are able to express the operators $\theta_\mu(i,j) + \theta_\mu(j,i)$ in the form

$$2kk'C_\mu \exp[-i\vec{q}\cdot(\vec{R}+\vec{r}/2)]\textstyle\sum_\nu \alpha_{\mu\nu}\bar{\theta}_\nu(i,j), \quad (1)$$

where $\vec{q} = \vec{k}' - \vec{k}$, $\vec{R} = (\vec{r}_i + \vec{r}_j)/2$, $\vec{r} = (\vec{r}_i - \vec{r}_j)$, and $\bar{\theta}_\nu(1,2)$ is

$$\bar{\theta}_\nu(1,2) = \int \frac{d^3k''}{(2\pi)^3}\,\frac{k''^2 M_\nu e^{i\vec{k}''\cdot\vec{r}}}{-k''^2 + m^2}. \quad (2)$$

The vectors $\vec{k}'$ and $\vec{k}$ are respectively the final and initial pion momenta. In Eq. (1) the term C_μ is the product of standard propagator factors for the Δ_{33}, i.e., Fig. 1(a) for the π meson gives

$$C_\mu = \frac{f_{\pi NN} f_{\pi N\Delta}{}^2 f_{\pi\Delta\Delta} m_\pi{}^{-4}}{S^{1/2} - m_\Delta + i\Gamma(S)/2}\,\frac{1}{m_n - m_\Delta},$$

and Fig. 1(c) gives for the π meson

$$C_\mu = \frac{f_{\pi NN} f_{\pi N\Delta}{}^2 f_{\pi\Delta\Delta} m_\pi{}^{-4}}{[S^{1/2} - m_\Delta + i\Gamma(S)/2]^2}.$$

Similar expressions exist for the ρ meson. For $f_{\pi N\Delta}$ we take the value determined from the width of the Δ_{33}, $f_{\pi N\Delta}{}^2/4\pi = 0.37$. In Eq. (2) m is the mass of the exchanged meson and M_ν are scalars formed from contractions of tensors in the momenta $\vec{k}'$, $\vec{k}$, $\vec{k}''$, and the Pauli spin matrices $\vec{\sigma}_1$ and $\vec{\sigma}_2$. We take $m_\rho = 650$ MeV to adjust for the 2π continuum in the projection of the $N\bar{N}\to\pi\pi$ helicity amplitude onto the state with quantum numbers of the ρ meson. Only five of the scalars formed in this way contribute to DCX to the DIAS, according to the approximations we have stated, and these are given in Table I. The relative strengths $\alpha_{\mu\nu}$ are given in Table II. Reductions of the diagrams in Figs. 1(a) and 1(c) to the form in Eqs. (1) and (2) are tedious and aided by use of the identity[13]

$$\vec{S}^+\cdot\vec{A}\,\vec{\Sigma}\cdot\vec{B}\,\vec{S}\cdot\vec{C} = \tfrac{5}{6} i\vec{A}\cdot\vec{B}\times\vec{C} - \tfrac{1}{6}(\vec{\sigma}\cdot\vec{A})(\vec{B}\cdot\vec{C}) - \tfrac{1}{6}(\vec{A}\cdot\vec{B})(\vec{C}\cdot\sigma) + \tfrac{2}{3}\vec{A}\cdot(\vec{B}\cdot\vec{\sigma})\vec{C}. \quad (3)$$

To take the ground-state expectation value of Eq. (1) we assume $|\mathrm{gs}\rangle$ to be a Slater determinant of single-particle orbits. Note that only valence neutrons contribute to ΔU. For these we use $d_{5/2}$ harmonic-oscillator wave functions coupled to $J = 0$ and make a Moshinsky transformation to express them in terms of $\vec{r}$ and $\vec{R}$. Because this wave function is antisymmetric, we do not have to consider direct and exchange diagrams separately. Additionally, we assume that the wave function consists of a short-range correlation function acting between the active nucleons. We are then eventually able to write $\langle\vec{k}'|U_2|\vec{k}\rangle$ as

$$\tfrac{1}{2} v_\pi(k')v_\pi(k)\int d^3R\,\exp(i\vec{q}\cdot\vec{R})[\vec{k}'\cdot\vec{k}\,\rho^{(S)}(R) + \rho^{(T)}(R)C_2(\hat{R})\cdot T_2(\vec{k}',\vec{k})], \quad (4)$$

where $\rho^{(S)}$ involves expectation values of M_ν for $\nu = 2$, 6, and $\rho^{(T)}$ for $\nu = 4$, 5, 7 in Eq. (2) (see Table I). Evaluation of $\rho^{(S)}$ and $\rho^{(T)}$ involves an integration over the coordinate $\vec{r}$. We find that in the surface region of the nucleus, which is the most important for pion scattering near resonance, the scalar $\rho^{(S)}$ and tensor $\rho^{(T)}$ densities are, to a reasonable approximation, proportional to the square of the valence neutron density $\Delta\rho$. We therefore write $\rho^{(S)} \equiv \lambda^{(S)}\Delta\rho^2(R)/\rho_0$ and $\rho^{(T)} \equiv \lambda^{(T)}\Delta\rho^2(R)/\rho_0$, where $\rho_0 = 0.16$ fm^{-3} is the central density of nuclei.

TABLE I. Tensors contributing to the sum in Eq. (2). Notation is the same as Ref. 12.

ν	Tensors M_ν
2	$3T_0(\hat{k}',\hat{k})\cdot T_0(\vec{\sigma}_1,\vec{\sigma}_2)$
4	$-\sqrt{2}\,T_2(\hat{k}',\hat{k})\cdot C_2(\hat{k}'')T_0(\vec{\sigma}_1,\vec{\sigma}_2)$
5	$T_2(\hat{k}',\hat{k})\cdot T_2(\vec{\sigma}_1,\vec{\sigma}_2)$
6	$-\sqrt{2}\,T_0(\hat{k}',\hat{k})\cdot T_0[C_2(\hat{k}''),T_2(\vec{\sigma}_1,\vec{\sigma}_2)]$
7	$(2/3)^{1/2}T_2(\hat{k}',\hat{k})\cdot T_2[C_2(\hat{k}''),T_2(\vec{\sigma}_1,\vec{\sigma}_2)]$

TABLE II. Coefficients $\alpha_{\mu\nu}$ in Eq. (2). The coefficients $\alpha_{\mu\nu}$ for the term similar to Fig. 1(a) but with the Δ_{33} in the initial state are the same as for Fig. 1(a).

μ/ν	2	4	5	6	7
Fig. 1(a)	0	$-\frac{1}{3}$	$-\frac{1}{3}$	0	$\frac{1}{6}\sqrt{21}$
Fig. 1(c)	$\frac{20}{81}$	$-\frac{4}{27}$	$-\frac{4}{27}$	$\frac{20}{27}\sqrt{5}$	$\frac{2}{27}\sqrt{21}$

We show some numerical results for the $\lambda^{(S)}$ and $\lambda^{(T)}$ coefficients in Table III. Note that for Fig. 1(a) $\lambda^{(S)}$ vanishes. This occurs because $\vec{\Sigma}\cdot\vec{S}=0$. The calculation corresponds to $\Lambda_\pi=1.2$ GeV/c, and the results for several different values of Λ_ρ are shown. We took the short-ranged correlation function to be a step at $\vec{r}=0.5$ fm. Note that $\lambda^{(S)}$ is strongly sensitive to Λ_ρ. This arises from the fact that $\lambda^{(S)}$ is dominated by the ρ-meson spin-spin force in the Δ_{33}-nucleon interaction, which adds constructively to the spin-spin component of the π-meson exchange. The effect is analogous to that of the ρ meson in the Lorentz-Lorenz effect in pion-nucleus scattering.[14] The sensitivity to Λ_ρ arises from a cancellation between the long-range attraction in the meson exchange and the short-range delta-function repulsion, which is spread out beyond the range of the short-range correlation function to an extent dependent on Λ. Because relative S waves do not contribute to $\lambda^{(T)}$, this coefficient is dominated by the tensor component of the Δ_{33}-nucleon interaction. The ρ meson adds destructively to the π exchange in the tensor force and with a smaller coefficient than in the spin-spin channel. For these reasons we see that $\lambda^{(T)}$ is smaller than $\lambda^{(S)}$ and less sensitive to Λ_ρ. Also, $\lambda^{(T)}$ increases as the ρ meson is damped so that $\lambda^{(T)}>\lambda^{(S)}$ for very small Λ_ρ.

Also shown in Table III is the probability P_Δ that the Δ_{33} is admixed into the nuclear wave function. If we write the wave function for the valence neutrons as

$$|\psi\rangle=|NN\rangle+\textstyle\sum_i\beta_i|N\Delta;i\rangle, \tag{5}$$

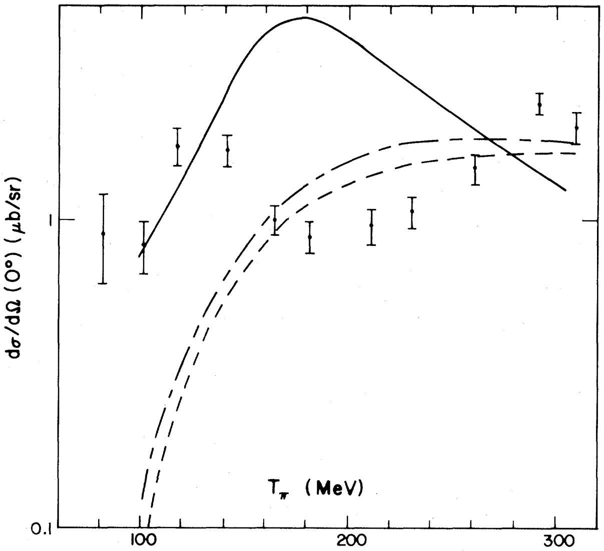

FIG. 2. Cross section for $^{18}O(\pi^+,\pi^-)^{18}Ne$ to the double isobaric analog state as a function of energy compared to the experiment of Ref. 15. The long/short-dashed curve is the sequential process. The short-dashed curve is the sequential plus terms of type in Fig. 1(a). The solid curve is the sequential plus Fig. 1(c).

where $|N\Delta;i\rangle$ represents the excited Δ-N states which have the same spin and parity as $|NN\rangle$, then

$$P_\Delta=\textstyle\sum\beta_i^2/(1+\sum\beta_i^2). \tag{6}$$

We evaluated the β_i in perturbation theory assuming the same model used to calculate Fig. 1. We find that P_Δ is rather insensitive to Λ_ρ because it is dominated by the tensor component of the interaction.

Finally, we show the 5° cross section for $^{18}O(\pi^+,\pi^-)^{18}Ne$ to the DIAS in Fig. 2. The experimental points are taken from Ref. 15. The long-dashed curve corresponds to sequential scattering through the iterated optical potential $U-\Delta U$. The short-dashed curve is sequential scattering plus the result in Fig. 1(a); in this case the net cross section is slightly less than the sequential. This result shows that the Δ_{33} in the wave function contributes negligibly to the analog transitions at

TABLE III. Parameters λ characterizing the isotensor potential for 160-MeV pions. Λ_ρ is given in GeV/c. P_Δ is the Δ_{33} probability of the valence neutron in ^{18}O. Units of λ are fm^3. $\Lambda_\pi=1.2$ GeV/c.

		Fig. 1(a)	Fig. 1(c)	
Λ_ρ	P_Δ (%)	$\lambda^{(T)}$	$\lambda^{(S)}$	$\lambda^{(T)}$
1.2	3.7	$-0.34-0.66i$	$-7.0+9.9i$	$0.58-0.82i$
1.1	5.2	$-0.34-0.66i$	$-2.8+4.1i$	$0.58-0.83i$
No ρ	25	$-0.85-1.65i$	$-0.6+0.8i$	$1.45-2.04i$

these energies. The effect of Fig. 1(a) is smaller than one would naively estimate from the magnitude of P_Δ in Table III because (1) the coefficient $\lambda^{(S)} \equiv 0$ and (2) the cross section for the transition operator[12] $T_2(\vec{k}'\vec{k}) \cdot C_2(R)$ in Eq. (4) is intrinsically small (it would vanish in the Born approximation in the forward direction). We conclude that the deltas in the wave function cannot be the terms responsible for the interfering amplitude in Refs. 1–4, contrary to the hopes expressed therein.

Also shown in Fig. 2 is the cross section corresponding to the term in Fig. 1(c). The solid curve shows the sum of this term evaluated for $\Lambda_\rho = 1.2$ GeV/c plus the sequential term, Fig. 1(b). Note that the result is quite large and that the enhancement improves agreement with experiment at low energies. Such large relative contributions of the ρ meson are already familiar in the nucleon-nucleon interaction, where one finds that the ρ-meson contribution to the spin-spin force is several times larger than that of the π meson. However, at resonance the result is a factor of 6 larger than experiment. The effect on the angular distribution is such that the position of the first minimum is moved to smaller angles but not by enough to agree with the experimental data at 164 MeV.[16] Phenomenological analysis[17] of 164-MeV elastic and single- and double-charge-exchange data throughout the periodic table shows that in order to understand all the data a large isovector optical potential and, in addition, a large isotensor potential whose imaginary part is *opposite* to that evaluated from Fig. 1(c) are required. We thus conclude that if Λ_ρ can be as large as 1.2 GeV/c, there must be other terms which give a substantial contribution and which are out of phase with Fig. 1(c) near and above resonance.

The results of our calculation are conclusive regarding the Δ_{33} in the nuclear wave function: This term has a negligible effect on double charge exchange to the DIAS in the resonance region. We have also shown that Fig. 1(c) is an important process for DCX, although we can draw no firm conclusion about the size of the Δ_{33}-nucleus interaction based on theory at this time. For large Λ_ρ ($\gtrsim 1.2$ GeV/c) the enhancement at low energy constitutes an improvement, but some yet to be determined mechanism would then be needed to cancel its effect near and above resonance. For smaller Λ_ρ, e.g., values calculated in Ref. 8, the effect of Fig. 1(c) is quite small. A firmer conclusion about the dynamics underlying the process in Fig. 1(c) must await a more complete assessment of the other terms that may contribute to the experiment.

Nonanalog double charge exchange represents another reaction to which the terms in Fig. 1 contribute. Because the sequential process is suppressed at least in the simpler nuclear models, we are hopeful that a combined analysis of DCX involving analog and nonanalog transitions will help resolve some of the ambiguities that remain.[18]

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