Forward Glory Effects in the Elastic Scattering of $^{12}C+^{12}C$

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It is shown that the elastic scattering of $^{12}C + ^{12}C$ in the center-of-mass energy range $6 < E < 31$ MeV exhibits forward glory enhancement. Semiclassical analysis of the quantity

$$\Delta\sigma_T \equiv \sigma_R - \int \left[\frac{d\sigma_{\rm Ruth}}{d\Omega} - \frac{d\sigma_{\rm el}}{d\Omega}\right] d\Omega$$

indicates that the best candidate for a $^{12}C + ^{12}C$ interaction potential is a small-radius, deep Woods-Saxon potential, in qualitative agreement with the results obtained from recent analyses of $d\sigma_{\rm el}/d\Omega$ done at higher energies.

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Recently several analyses of elastic and inelastic scattering data of light-heavy-ion systems at intermediate energies have been reported.[1] The main conclusion reached has been the removal of some of the major ambiguities attached to the ion-ion optical potential usually extracted from data taken at lower energies. This is accomplished at intermediate energies because of the incipient dominance of the far-side amplitude over the near-side one,[2] thus leading to a greater degree of sensitivity to the details of the ion-ion interaction at shorter separation distances.

In a parallel theoretical development,[3] the observation has been made that the extraction and analysis of the forward glory contribution to the elastic scattering would also furnish further constraints on the interaction potential. It has been suggested in Ref. 3 that a careful study of the quantity

$$\Delta\sigma_T = \sigma_R - \int d\Omega \left[\frac{d\sigma_{\rm Ruth}}{d\Omega} - \frac{d\sigma}{d\Omega}\right], \qquad (1)$$

where σ_R is the total reaction cross section, $d\sigma/d\Omega$ the elastic differential cross section, and $d\sigma_{\rm Ruth}/d\Omega$ the differential Rutherford cross section, can supply the above mentioned constraint. This comes about as a consequence of the optical theorem which relates $\Delta\sigma_T$, which is the difference between the total cross section and the total Rutherford cross section, to the imaginary part of the forward "nuclear" scattering amplitude, viz.,[4]

$$\Delta\sigma_T = (4\pi/k)\,{\rm Im}[f(0) - f_{\rm Ruth}(0)], \qquad (2)$$

where f and $f_{\rm Ruth}$ are the total and Rutherford scattering amplitudes, respectively, and k is the asymptotic wave number of relative motion. The occurrence of forward glory, a refractive effect, leads to a major enhancement in $\Delta\sigma_T$.

In this Letter, we present evidence for the forward glory enhancement in $\Delta\sigma_T$ for the scattering of ^{12}C on ^{12}C in the energy range $6 < E_{\rm c.m.} < 31$ MeV.

We have determined the quantity $\Delta\sigma_T$ from existing experimental data on the total reaction cross section, σ_R, and published values of the sum-of-differences cross section,

$$\sigma_{\rm SOD} \equiv \int d\Omega \left[\frac{d\sigma_{\rm M}}{d\Omega} - \frac{d\sigma_{\rm el}}{d\Omega}\right]$$

where $d\sigma_{\rm M}/d\Omega$ is the Mott cross section. The total reaction cross section σ_R is obtained from the summed contribution of the complete fusion cross section, σ_F, and the total, angle-integrated, cross section of direct processes σ_D.

Recently Kolata *et al.*[5] measured the total α yield in the $^{12}C + ^{12}C$ fusion. The contribution to σ_F arising from the 3α evaporation, not taken into account in previous fusion measurements, was determined. An anomalous α yield, which seems to be a direct process, was included in σ_D. We used σ_R from Ref. 5, and calculated the σ_R from other fusion measurements,[6-8] summing to them the 3α evaporation, and considered σ_D as composed of the total angle-integrated inelastic cross section[5,9] and the anomalous α yield.[5]

The quantity $\sigma_{\rm SOD}$ was constructed by Treu *et al.*[10] and more recently by Ledoux *et al.*[11] from the measured elastic scattering angular distributions. The quantity of interest in this Letter, $\Delta\sigma_T$, was then evaluated, as indicated in Eq. (1), namely $\Delta\sigma_T = \sigma_R - \sigma_{\rm SOD}$.

Because of the dispersion inherent in both σ_R and $\sigma_{\rm SOD}$, we present our $\Delta\sigma_T$ as a band, whose width is much smaller than its mean value. This band of points representing $\Delta\sigma_T$ is plotted in Fig. 1 versus the center-of-mass energy. One sees clearly the beginning of the large-period oscilla-

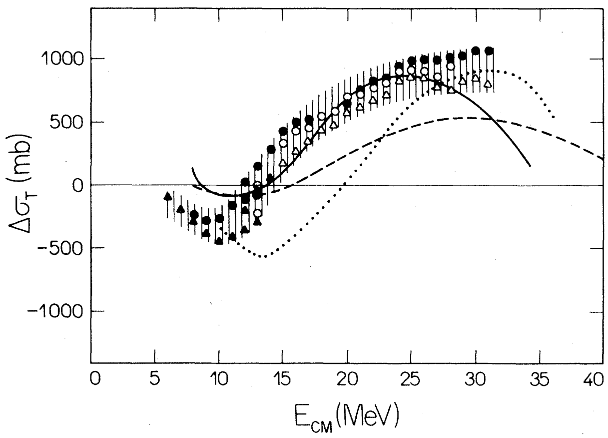

FIG. 1. The quantity $\Delta\sigma_T$ for the $^{12}C + {}^{12}C$ system, calculated with the Yale potential V=14 MeV, r_v =1.35 fm, a_v =0.35 fm (dotted curve); with the small-radius, deep interaction, V=250 MeV, r_v =0.66 fm, a_v =0.63 fm (full curve); and with the sharp-cutoff limit, Eq. (3) (dashed curve). The data points were extracted from Ref. 5 (open triangles), Ref. 6 (open circles), Ref. 7 (full circles), and Ref. 8 (full triangles), with use of σ_{SOD} from Refs. 10 and 11. The sizes of the vertical bars indicate the overall error in the derived data points.

tory behavior expected from the theoretical study of Ref. 3. To ascertain the refractive nature of $\Delta\sigma_T$, we show in the figure $\Delta\sigma_T$ calculated in the sharp-cutoff approximation[12]:

$$\Delta\sigma_T{}^{\text{s.c.}} = \frac{2\pi}{k^2}\sum_{l=0}^{l_c}(2l+1)\cos 2\sigma_l\,, \tag{3}$$

where σ_l is the Coulomb phase shift of the lth partial wave and l_c is the sharp-cutoff angular momentum that specifies the value of the total reaction cross section through $\sigma_R = (\pi/k^2)(l_c+1)^2$. We consider as a criterion for the refractive enhancement in $\Delta\sigma_T$ due to forward glory scattering the following:

$$\Delta\sigma_T/\Delta\sigma_T{}^{\text{s.c.}} > 1, \tag{4}$$

which is clearly satisfied by the $^{12}C + {}^{12}C$ $\Delta\sigma_T$ data shown in Fig. 1.

It has been pointed out in Ref. 3 that different optical potentials that give similar reasonable fits to the ratio to Rutherford scattering at small angles may give quite different $\Delta\sigma_T$'s. Thus through the confrontation of the calculated $\Delta\sigma_T$ with the experimental one, a less ambiguous optical potential may be deduced.[13] We have tested this idea on our $^{12}C + {}^{12}C$ case. We have considered two optical potentials that both generate forward glory scattering, that is, the corresponding classical deflection function $\theta(l)$ passes through zero at a finite value of l, l_{g1}.

The first optical potential we considered in our analysis is the one suggested by Reilly *et al.*[14] This Woods-Saxon potential, whose parameters are V=14 MeV, a_v=0.35 fm, r_v=1.35 fm, W = 0.4 MeV +0.1$E_{\text{c.m.}}$, a_w=0.35 fm, r_w=1.40 fm, reproduces reasonably well the elastic scattering angular distributions of $^{12}C + {}^{12}C$ and accounts for the structure seen in the excitation function at 90°. We calculated $\Delta\sigma_T$ using the Ford-Wheeler,[15] stationary-phase approximation, which gives

$$\Delta\sigma_T = \frac{4\pi}{k^2}(l_{g1}+\tfrac{1}{2})\left[\frac{2\pi}{(d\theta/dl)_{l_{g1}}}\right]^{1/2}|S_{l_{g1}}{}^n|\sin\left[2(\sigma_{l_{g1}}+\delta_{l_{g1}}{}^n)-\frac{\pi}{4}\right], \tag{5}$$

where δ_{g1} is the nuclear phase shift evaluated at the forward glory angular momentum l_{g1}, and $S_{l_{g1}}{}^n$ is the reflection coefficient at l_{g1}. We have found that the reflection coefficient $|S_{l_{g1}}{}^n|$ for the Yale potential is very close to unity in the energy range of interest. Further, the effect of absorption on l_{g1} was found to be small too. This convinced us that the use of a classical deflection function is more than adequate. The nuclear phase shift $\delta_l{}^n$ is a rapidly varying function of l in the forward glory region. However, its value at l_{g1}, $\delta_{l_{g1}}{}^n$, is quite small as our optical-model calculation has shown us.

The result of our calculation of $\Delta\sigma_T$, Eq. (5), for the Yale potential, using $\theta(l)$ and l_{g1} generated classically from the real part alone and setting $|S_{l_{g1}}{}^n|$=1 and $\delta_{l_{g1}}{}^n$=0, is shown in Fig. 1 as the dotted curve. We see clearly that the magnitude of $\Delta\sigma_T$ comes out right; however, there is a major discrepancy in the phase. We have also calculated $\Delta\sigma_T$ for a potential tailored according to that obtained from the analysis of the intermediate-energy data; a rather small-radius deep Woods-Saxon interaction. We have taken V=250 MeV, r_v=0.66 fm, and a_v=0.63 fm.[16] The result is presented in Fig. 1 as the full curve. The agreement with the general trend of the data is striking. This agreement with the behavior of the data is made meaningful by the fact that the elastic scattering angular distribution with this real potential and with W=0.4 MeV+ 0.3E, r_w =0.93 fm, a_w=0.35 fm, is coming out as reasonable as the one obtained with the Yale potential, as clearly seen in Fig. 2.

The fact that $\Delta\sigma_T$ is acting as a filter to the appropriate optical potential that best represents the interacting system would be understood easily by the fact that $d\sigma(\theta)/d\Omega$ probes a certain combination of the optical-potential parameters, whereas $\Delta\sigma_T$ tests a different combination. This situation becomes quite clear at higher energies where the forward glory impact parameter, $b_{g1} = l_{g1}/k$, becomes independent of energy[17]:

$$b_{g1} = R_v\left\{1 + \frac{a_v}{2R_v - 3a_v}\left\{3\ln R_v + \ln\left[\frac{\pi}{2a_v}\left(\frac{V}{z_1 z_2 e^2}\right)^2\right]\right\}\right\} \tag{6}$$

and the slope of $d\sigma(\theta)/d\Omega$ in the drop-off region (the region of the quarter-point angle) is determined basically by[2,18]

$$l_0 = 2ka[\pi - \arctan(W/V)] \tag{7}$$

if an equal geometry for $W(r)$ and $V(r)$ is assumed. Therefore the two equations above furnish two invariants for $d\sigma(\theta)/d\Omega$ and $\Delta\sigma_T$, supplying, thus, important constraints on the parameters of the ion-ion interaction, as our calculations clearly indicate (see Figs. 1 and 2). Though not quite applicable at the low energies we are considering, Eqs. (6) and (7) do supply two reasonable qualitative constraints.

In connection with the deep potential that gave the best account of $\Delta\sigma_T$, it is important to stress that exactly this type of potenital is seen to emerge from the analysis of intermediate-energy data. At these higher energies a remnant of a nuclear rainbow scattering (scattering to negative angles) is seen to occur. At the low energies considered in this Letter, our deep potential generates a strong orbiting situation, which would persist up to a critical energy given approximately by[17]

$$E_{cr} = \frac{V_c(R_v)}{2} + \frac{V}{8a_vR_v}[(R_v - 2a_v)^2 + 2a_v^2], \tag{8}$$

where $V_c(R_v)$ is the Coulomb interaction at the nuclear potential radius R_v. Using the parameters of our potential, V=250 MeV, r_v=0.66 fm, a_v=0.63 fm, we obtain E_{cr}=72 MeV, well above the energy at which our collected data points end. It would be quite interesting to extend the present study to energies higher than E_{cr}, where both nuclear rainbow and forward glory would be acting.

In conclusion, we have presented in this Letter strong evidence for the forward glory scattering phenomenon in the $^{12}C + {}^{12}C$ system, as exemplified by the enhancement and oscillation in $\Delta\sigma_T$. To our knowledge, this is the first time that such a phenomenon has been "seen" in nuclear heavy-ion scattering.[19] We have clearly shown that a joint analysis of both $d\sigma/d\Omega$ and $\Delta\sigma_T$ reveals a less ambiguous interaction potential. The $^{12}C + {}^{12}C$ potential we obtained from our analysis is quite deep and resembles closely the interaction potential deduced from analysis done on the elastic scattering of $^{12}C + {}^{12}C$ at intermediate energies ($E \sim 15$ MeV/nucleon) and that calculated from the double folding model.[20]

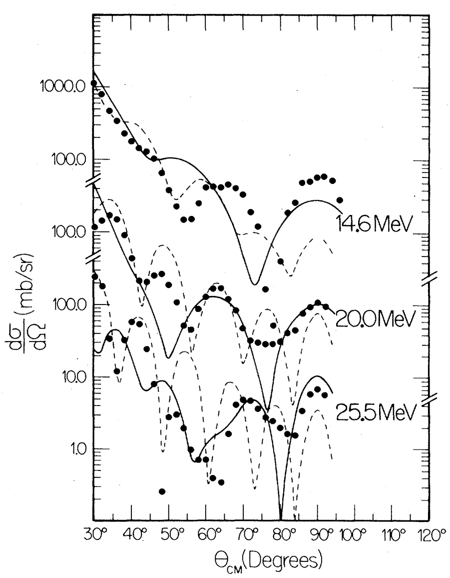

FIG. 2. The elastic scattering angular distribution for $^{12}C + {}^{12}C$ at three center-of-mass energies (Ref. 11). The theoretical curves were obtained with the Yale potential (see caption to Fig. 1) with W=0.4 MeV + 0.1 $E_{c.m.}$, r_w=1.40 fm, a_w=0.35 fm (dashed curve), and with the small-radius, deep potential of Fig. 1 with W=0.4 MeV + 0.3 $E_{c.m.}$, r_w=0.93 fm, and a_w=0.35 fm (full curve). The total reaction cross section obtained with the latter potential is slightly smaller than the experimental value. The values of $|S_{l_{g1}}|$ in both cases come out close to unity.

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Note added.—Because of the difficulty inherent in evaluating $\int d\Omega(d\sigma/d\Omega - d\sigma_{\mathrm{Ruth}}/d\Omega)$ [C. Marty, Z. Phys. A 309, 261 (1983)], we suggest that a more precise analysis of the forward glory effects should follow the method used by Jappesen *et al.* [Phys. Rev. C 27, 697 (1982)] in their effort to extract the forward pion-nucleus amplitude.

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