Degradation of Proton Momentum through Nuclei

Rudolph C. Hwa
Institute of Theoretical Science and Department of Physics, University of Oregon, Eugene, Oregon 97403
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The inclusive reactions $p + A \to p + X$ are studied in the framework of an evolution process in momentum degradation. All measured inclusive cross sections between C and Pb for all x values can be understood in terms of one basic degradation function which involves only one adjustable parameter. The inferred degradation length in nuclear matter is 17 fm.

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The recently published data of Barton *et al.*[1] on the A dependence of inclusive hadron fragmentation provides a wealth of experimental information on the effects of nuclear media on the propagation of hadrons through nuclei. A presentation of that subject by Busza[2] at the Quark Matter 1983 conference generated considerable interest in the implication of the data on the extent of the fragmentation region in ultrarelativistic heavy-ion collisions. Theoretical understanding of the data has been virtually nonexistent; in Ref. 1 arguments are given to rule out all mechanisms of particle production discussed by the authors. I present here a simple picture to describe all proton-nucleus inclusive reactions of the type $p + A \to p + X$ for $A \geq 12$. With only one free parameter one can fit all those inclusive cross sections apart, of course, from the scale of the cross sections. From that parameter, one can learn about the stopping power of the nuclear medium on proton beams.

The data for the inclusive cross sections have been summarized by an empirical formula,[1]

$$E\, d^3\sigma/dp^3 = \sigma_0 A^{\alpha(x)}. \tag{1}$$

It was found that the dependence of α on the momentum fraction x roughly follows some universal trend. No theoretical significance in the form of (1) is claimed. The result of the present study can be taken to suggest that (1) is a poor way of describing the data, which partly explains why the error bars on α are large. Furthermore, there are reasons to believe that α should not be universal; there should be differences between the cases where mesons as opposed to baryons are detected, even if (1) is roughly right. Careful examination of the data on α reveals such differences. In this note, I suggest a physically more sensible way of describing the data.

Although the proper basis for discussing hadron-nucleus collisions should be in terms of quarks, I shall, insofar as possible, stay at the hadronic level in order to be as model independent as possible. Consider a proton traversing a nucleus A at high energy. Let there be N nucleons in the tube that the proton tunnels through, a number which is defined as the product of the tube volume and the nucleon-number density of the nucleus. Let $H(x,N)$ be the probability (more precisely, the invariant distribution function) that the detected proton, after traversing N nucleons, has a momentum fraction x relative to the incident momentum. For a larger nucleus for which the detected proton traverses one more nucleon, the corresponding distribution should satisfy the convolution equation

$$H(x, N+1) = \int_x^1 dx' x'^{-1} H(x', N) Q(x/x'), \tag{2}$$

where $Q(z)$ is the probability in invariant phase space that a proton has momentum fraction z after the collision with one more nucleon. Equation (2) expresses the notion of successive degradation of the proton momentum as it goes through the nucleus.[3] In (2), we focus only on the longitudinal-momentum changes, assuming that the transverse momentum has been integrated over for each distribution function. Since the inclusive cross sections are roughly factorizable in p_L and p_T, we could also regard (2) as a statement of p_L distributions at fixed p_T.

If the $(N+1)$st nucleon is absent, $Q(z)$ would be $\delta(z-1)$. Its presence entails nonzero probability for the emerging proton to be at $z < 1$. The z dependence of that probability can be inferred from the data on pp collision,[1] the inclusive cross section for which increases linearly with z except for the elastic peak at $z = 1$. Thus we may write

$$Q(z) = \lambda z + \lambda' \delta(z-1). \tag{3}$$

The constraint on λ and λ' depends upon the attenuation of the proton flux as it passes through the last nucleon. On that point, I appeal to the quark picture of hadron collisions.[4] The valence and sea quarks in a proton interact with varying effectiveness with the target nucleus in accordance with the empirical rule of short-range correlation in rapidity. The valence quarks, having more momenta on the average, pass through the nucleus with more ease than sea quarks and gluons. The "proton" that has been identified as having momentum fraction x' in (2) after traversing N nucleons should strictly be regarded as three valence quarks which are to recombine outside the nucleus. The last step of traversing the $(N+1)$st nucleon is in reality three valence quarks passing through that nucleon with different rapidities, all of which are far separated from the rapidities of the partons in that nucleon. Since the flavors of those quarks do not change, the proton flux should not be attenuated in any significant measure.

The conclusion of the foregoing discussion is that when N is large, the total probability for a proton to remain a proton after the $(N+1)$st collision is very nearly one; for definiteness, let the constraint be precisely

$$\int_0^1 dz\, z^{-1} Q(z) = 1. \tag{4}$$

From here on I shall not make reference to the quark basis again in the consideration below except in the conclusion. A more fundamental treatment would have to be based on quarks and gluons, but it would involve additional assumptions which I attempt to avoid in this paper.

From (3) and (4), we have $\lambda + \lambda' = 1$. Let us define $Q(z) = \delta(z-1) + D(z)$ so that the degradation function $D(z)$ is $D(z) = \lambda[z - \delta(z-1)]$, which satisfies $\int_0^1 (dz/z) D(z) = 0$. It then follows in the large-N (continuum) limit that

$$dH(x,N)/dN = \int_x^1 dx'\, x'^{-1} H(x',N) D(x/x'). \tag{5}$$

Converting this to the moment equation where

$$\tilde{H}(n,N) = \int_0^1 dx\, x^{n-2} H(x,N), \tag{6}$$

and similarly for $\tilde{D}(n)$, we have $d\tilde{H}(n,N)/dN = \tilde{H}(n,N)\tilde{D}(n)$ and the obvious solution

$$\tilde{H}(n,N) = \tilde{H}(n,N_0) \exp[(N-N_0)\tilde{D}(n)], \tag{7}$$

where $\tilde{D}(n) = \lambda(n^{-1} - 1)$. Equation (7) completely determines the inclusive distributions for all (large) nuclei, given some reference at A_0 for scale.

Let us now relate N to A by geometry. If R_A denotes the radius of the target nucleus, the average length of a tube through the nucleus, averaged over all impact parameters, is $4R_A/3$. The average number of nucleons, N, in the tube of radius r_p, the proton radius, is then $(r_p/R_A)^2 A$. Using the empirical formula $R_A = 1.2A^{1/3}$ fm, and with $r_p = 0.8$ fm, we get $N = cA^{1/3}$ where $c = 0.44$. Expressing (7) in terms of A, we therefore have

$$\tilde{H}(n,A) = \tilde{H}(n,A_0) \exp[\Delta(1-n)/n], \tag{8}$$

where $\Delta = \lambda c (A^{1/3} - A_0^{1/3})$.

The data of Ref. 1 on $p + A \to p + X$ at $P_{\text{beam}} = 100$ GeV/c are reproduced in Fig. 1, the curves in which should be ignored for now. Those inclusive cross sections are for $p_T = 0.3$ GeV/c. Assuming factorization in p_L and p_T, we adopt the relationship

$$E\, d^3\sigma/dp^3 = \sigma_1 A^{2/3} H(x,A), \tag{9}$$

where $A^{2/3}$ is the explicit geometrical factor to which we assume the inelastic pA cross section is proportional, and σ_1 contains a universal scale and a p_T dependence common for all A. There is evidence that (9) is a sensible form to exhibit the dependences on A, p_T, and x, at least in neutral-strange-particle production.[5] From the data in Fig. 1, we notice that $E\, d^3\sigma/dp^3$ for an Ag target is nearly independent of x, although it is almost also true for Cu. I shall, for definiteness, choose Ag to be the reference nucleus ($A_0 = 108$), which possesses the required property that $H(x, A_0) = H_0$, a constant. Using (9), we can then extract from the data in Fig. 1 the ratio

$$R(x,A) = H(x,A)/H(x,A_0). \tag{10}$$

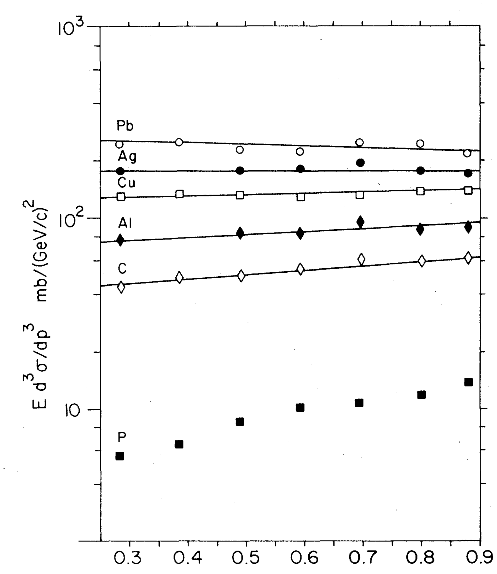

FIG. 1. Invariant differential cross sections for $p + A \to p + X$ at an incident momentum of 100 GeV/c and transverse momentum 0.3 GeV/c. Data are from Ref.1.

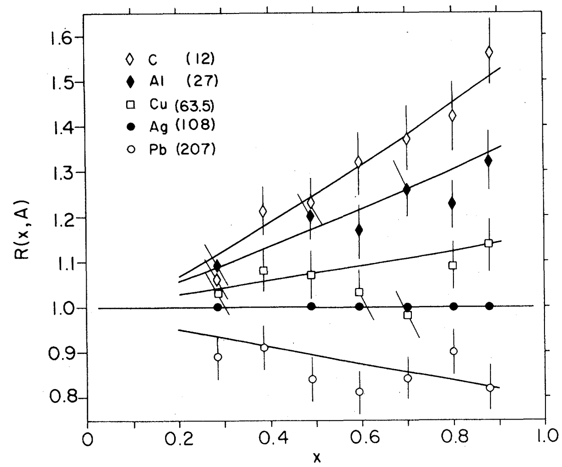

FIG. 2. Ratios of invariant x distributions of A to that of Ag. Data are from Ref. 1. An average of 5% error is assigned to all points.

The data[6] for $R(x,A)$ are shown in Fig. 2. The essence of this work is to provide a one-parameter fit of all the data points in Fig. 2, which is evidently a more amplified version of Fig. 1.

To achieve that, I first note that $|\Delta|$ in (8) is less than 0.5 for $A \geq 12$ and $\lambda < 0.5$, an upper bound which will prove realistic *a posteriori*. We may therefore expand the exponential in (8) and keep the leading terms in

$$\frac{\tilde{H}(n,A)}{H_0} = \frac{1}{n-1} + \sum_{j=1}^{\infty} a_j n^{-j}, \tag{11}$$

where $a_1 = -\Delta + \Delta^2/2 - \Delta^3/6$, $a_2 = -\Delta^2/2 + \Delta^3/3$, and $a_3 = -\Delta^3/6$ to order Δ^3. Inverting the moments by Mellin transform is now simple, yielding

$$R(x,A) = 1 + \sum_{j=1}^{\infty} \frac{a_j x(-\ln x)^{j-1}}{(j-1)!}. \tag{12}$$

On the basis of this equation, I have fitted the data for all A in Fig. 2 by adjusting λ, the only free parameter, and obtained the best value

$$\lambda = 0.43. \tag{13}$$

The corresponding curves are shown in Fig. 2. Evidently, the fit is very good, given the inaccuracy of the data. It is far better than the fit based on the *ad hoc* formula in (1).

Setting $E\,d^3\sigma/dp^3$ for ^{108}Ag at an average value of 177 mb/(GeV/c)^2 as input, we can use (9) and (10) to calculate the inclusive cross sections for all other A values. The results are shown as solid lines in Fig. 1. In that logarithmic plot the fit appears excellent. I have not extended the calculation to the proton case since both the physics and mathematics of this approach would be invalid.

The success of this description lends support to the basic soundness of my approach to the $p \to p$ inclusive processes. If we had allowed some absorptivity in (4), i.e., letting its right-hand side be $1-\eta$ for some positive η, then it can be shown that the data would rule out $\eta > 0.1$.

The conclusion that can be drawn from this analysis is that the convolution equation (2) is physically sensible. More specifically, we learn from (13) that when one more nucleon is added to the end of a tube in a large nucleus, 43% of the time the "proton beam" suffers momentum degradation without significant attenuation in the total number of protons. This property can be quantified by defining and determining a degradation length Λ_p for a proton going through a nucleus as follows: Let Λ_p^{-1} be the fraction of momentum loss per unit length of propagation through a nucleus, i.e., $\Lambda_p^{-1} = p_L^{-1}\,dp_L/dL$, where L is the distance traversed. The average momentum fraction after traversing a nucleon is $\langle z\rangle = \int_0^1 dz\, Q(z) = 1 - \lambda/2$. The fraction of momentum loss is, therefore, $1 - \langle z\rangle = \lambda/2$ in thickness $\Delta L = L/N = 3.6$ fm in nuclear matter where we have used $L = 4R_A/3$ for the length of the tube containing N nucleons. Consequently, we obtain

$$\Lambda_p = 17 \text{ fm}, \tag{14}$$

which is far greater than what one might naively have expected.

The above result is to be contrasted with the case of pp collisions in which inelastic scattering accounts for 80% of the total cross section and conversion from proton to neutron is not uncommon. The difference can be understood qualitatively in the quark picture by recognizing that in pp collisions, the sea quarks and gluons of the projectile can readily get past the target, whereas in pA collisions, they cannot. Thus, in the latter case the notion of a "proton" going through a nucleus should be replaced by the three uud valence quarks without accompanying sea quarks and gluons as discussed immediately following (3). In the same picture, a detected pion, say π^+, should be regarded as $u\bar{d}$ quarks going through the nucleus; the degradation length would be different from that of uud. A more thorough understanding of these processes must await the completion of a more in-depth investigation of the problem in the framework of parton interaction and recombination. What can be stated at this point is that information gained from hadron-hadron collisions cannot be directly applied to the fragmentation regions of relativistic nucleus-nucleus collisions, a comment that is consistent with other cautionary remarks already made with regard to heavy-ion collisions.[7]

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[1]D. S. Barton *et al.*, Phys. Rev. D 27, 2580 (1983).

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[4]R. C. Hwa, in *Partons in Soft-Hadronic Processes*, edited by R. T. Van de Walle (World Scientific, Singapore, 1981), and Phys. Rev. D 22, 1593 (1980).

[5]P. Skubic *et al.*, Phys. Rev. D 18, 3115 (1983).

[6]The actual cross sections for Ag (rather than the average H_0) have been used in the denominator so that the systematic errors are minimized in the ratios. For the $x = 0.387$ point, I have used the same section as at $x = 0.284$.

[7]R. C. Hwa, in Proceedings of the Fourteenth International Symposium on Multiparticle Dynamics, Tahoe City, 1983, edited by P. M. Yager and J. F. Gunion (World Scientific, Singapore, to be published), and in "Quark Matter 1983," Proceedings of the Third International Conference on Ultra-Relativistic Nucleus-Nucleus Collisions, Brookhaven National Laboratory, edited by T. W. Ludlam and H. Wegner (Nucl. Phys. A, to be published).