Time Variation of the Fundamental "Constants" and Kaluza-Klein Theories

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Time dependence of the fundamental "constants" is examined within the framework of Kaluza-Klein theories. Relationships among low-energy couplings and masses are derived. It is suggested that detection of a time variation in any of these parameters may provide evidence for extra space dimensions. Experimental bounds are reviewed and new measurements advocated.

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Generalized Kaluza-Klein (KK) models offer the attractive possibility of unifying gravity with the other fundamental forces.[1,2] The basic idea is to enlarge space-time to $4+N$ dimensions in such a way that the N extra spatial dimensions form a very small compact manifold with mean radius R_{KK}. (R_{KK} should probably not be very different from the Planck length[3] $l_P = \sqrt{G} \simeq 1.6 \times 10^{-33}$ cm.) Remarkably, the $(4+N)$-dimensional metric tensor, $g_{\mu\nu}(x)$, then describes general relativity as well as gauge field interactions in our effective four-dimensional world, i.e., for energies $\ll 1/R_{KK}$. Indeed, Witten[2] has shown that for $N \geqslant 7$ such models can accomodate the full $SU(3)_C \otimes SU(2) \otimes U(1)$ gauge symmetry of strong and electroweak interactions.

Although Kaluza-Klein models presently exhibit serious theoretical flaws and are far from being phenomenologically viable,[2] it may not be premature to ask the following questions: Are extra dimensions a physical reality or merely a model-building mathematical tool? If they are real, can we find evidence for their existence?

In this Letter, I would like to suggest that the mean KK radius, R_{KK}, of the extra dimensions might contract, expand, or even oscillate as a function of time. If $\dot{R}_{KK} \neq 0$ (the dot denotes d/dt), it could give rise to observable time variations in the fundamental "constants" of our four-dimensional world and thereby provide a window to the extra dimensions. My suggestion is primarily motivated by the astrophysical observation that the ordinary three spatial dimensions are expanding and the work of Chodos and Detweiler[4] and Freund[5] which showed that incorporating such an expansion in KK cosmologies quite naturally leads to $\dot{R}_{KK} \neq 0$.

Of course, the time variation of fundamental "constants" is not a new issue. In the past, that question was addressed primarily on the basis of Dirac's big numbers hypothesis[6] which led him to conjecture that Newton's gravitational constant G varied as $1/t$ and led others[7] to speculate about the time variation of α, the fine-structure constant. In response to such ideas, tentative bounds have already been given for a variety of parameters[8-12]; some are illustrated in Table I. A word of caution: the quoted bounds are generally obtained under the assumption that the quantity considered is varying alone and that it exhibits a specific monotonic time dependence. Relaxing these assumptions can weaken or in some cases entirely undo the bounds.[13] As we shall now see, KK theories provide a natural theoretical framework for studying time-varying fundamental "constants."

My analysis is based on the following relationships:

$$\alpha_i(m_{KK}) = K_i G/R_{KK}^2 = K_i G m_{KK}^2, \tag{1a}$$

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(m_{KK}) - \pi^{-1}\sum_j C_{ij}[\ln(m_{KK}/m_j) + \theta(\mu - m_j)\ln(m_j/\mu)]. \tag{1b}$$

Equation (1a) is a generic KK quantization formula[1,2,14,15] relating G, R_{KK}, and the short-distance couplings $\alpha_i(m_{KK}) \equiv g_i^2(m_{KK})/4\pi$, $i=1,2,3$, of U(1), SU(2), and $SU(3)_C$ defined at a distance $R_{KK} = 1/m_{KK}$. The K_i are numbers that depend on the N-dimensional topology.[15] In general they can differ; however, if a larger grand unified symmetry such as SU(5) or SO(10) is imposed,[16] one may have $K_1 = K_2 = K_3$. Equation (1b) relates $\alpha_i(m_{KK})$ to the effective long-distance coupling $\alpha_i(\mu)$, $\mu \ll m_{KK}$, measured at laboratory energy μ [for $i=3$, Eq. (1b) is applicable only for $\mu \gtrsim 1$ GeV]. It accounts for leading-logarithmic quantum vacuum-polarization effects of all elementary particles (the sum is over j = leptons, quarks, gluons, $W^\pm$, etc.). The C_{ij} are well-known numbers[17,18] whose sign and magnitude depend on the spin and group-representation quantum numbers of the jth particle. The more familiar fine-structure constant $\alpha(0) \simeq \frac{1}{137}$, QCD mass scale $\Lambda^{(N_F)}$ (N_F is the number of quark flavors), Fermi constant G_F, and weak mixing angle θ_W are obtained via[18]

$$\alpha^{-1}(\mu) = \tfrac{5}{3}\alpha_1^{-1}(\mu) + \alpha_2^{-1}(\mu), \tag{2a}$$

$$\Lambda^{(N_F)} \simeq \mu \exp[-6\pi/(33-2N_F)\alpha_3(\mu)], \tag{2b}$$

$$G_F \simeq \pi\alpha_2(m_W)/\sqrt{2}m_W^2, \tag{2c}$$

$$\tan^2\theta_W(m_W) = \tfrac{3}{5}\alpha_1(m_W)/\alpha_2(m_W), \tag{2d}$$

with $m_W = W^\pm$ boson mass $\simeq 81$ GeV.

Differentiating Eqs. (1) with respect to t gives

$$\frac{\dot{\alpha}_i(m_{KK})}{\alpha_i(m_{KK})} = \frac{\dot{K}_i}{K_i} + \frac{\dot{G}}{G} + \frac{2\dot{m}_{KK}}{m_{KK}}, \tag{3a}$$

$$\frac{\dot{\alpha}_i(\mu)}{\alpha_i^2(\mu)} = \frac{\dot{\alpha}_i(m_{KK})}{\alpha_i^2(m_{KK})} + \frac{1}{\pi}\sum_j C_{ij}\left[\frac{\dot{m}_{KK}}{m_{KK}} - \frac{\dot{m}_j}{m_j} + \theta(\mu - m_j)\left(\frac{\dot{m}_j}{m_j} - \frac{\dot{\mu}}{\mu}\right)\right]. \tag{3b}$$

These simple formulas relate the time dependence of fundamental masses and couplings.

So far, the basic mass (length, time) unit has been left arbitrary. In order to interpret astrophysical red shifts (Hubble's constant) as resulting from expansion of our three spatial dimensions, rather than a changing length scale, I now adopt atomic mass units such that $\dot{m}_e = \dot{\mu} = 0$. To further simplify matters, the remaining discussion will be restricted to scenarios in which $\dot{K}_i = \dot{m}_j = 0$. A more complete analysis will be presented elsewhere.

(1) $\dot{\alpha}_i(m_{KK}) = 0$.—This is the constraint employed by Chodos and Detweiler.[4] It could arise naturally if $\sqrt{G}$ and m_{KK} are simply related by quantum loop effects.[15,19] In that case, one finds from Eq. (3) (with $\dot{\mu} = \dot{m}_j = \dot{K}_i = 0$)

$$\dot{G}/G = -2\dot{m}_{KK}/m_{KK}, \tag{4a}$$

$$\frac{\dot{\alpha}_i(\mu)}{\alpha_i^2(\mu)} = -\frac{1}{2\pi}\sum_j C_{ij}\frac{\dot{G}}{G}, \tag{4b}$$

$$\frac{\dot{\alpha}}{\alpha} = -\frac{\alpha}{2\pi}\sum_j\left(\frac{5}{3}C_{1j} + C_{2j}\right)\frac{\dot{G}}{G}. \tag{4c}$$

For $m_{KK} \sim t^a$, Eq. (4a) implies $\dot{G}/G = -2a/t$, which realizes Dirac's conjecture[6] if $a = \frac{1}{2}$.[4] Employing the universe lifetime $\tau_U \simeq 2\times 10^{10}$ yr, one expects in such cosmologies

$$|\dot{G}/G| = |a| \times 10^{-10}\ \text{yr}^{-1}. \tag{5}$$

Comparison with Table I indicates that $|a|$ must be quite small if the experimental bound is to be respected. However, given the assumptions that go into such bounds and the uncertainty in τ_U, $|a| \simeq \frac{1}{2}$ may still be viable. Continued attempts to find a variation in G are clearly interesting.

TABLE I. Bounds on some fundamental quantities (g_p = proton gyromagnetic ratio).

Quantity Q	Bound on $\lvert\dot{Q}/Q\rvert$ (yr^{-1})	Method	Ref.
G	$< 1\times 10^{-11}$	Astrophysics	8
α	$< 1\times 10^{-17}$	Geochemical	9
α	$< 5\times 10^{-15}$	Geochemical	10
α	$< 4\times 10^{-12}$	Astrophysics	11
α	$< 4\times 10^{-12}$	Laboratory	12
$g_p m_e/m_p$	$< 8\times 10^{-12}$	Astrophysics	11
$\alpha^2 g_p m_e/m_p$	$< 2\times 10^{-14}$	Astrophysics	11

The relationship between α and G in Eq. (4c) realizes a suggestion of Peebles and Dicke.[7] If it holds, then one expects $|\dot{\alpha}/\alpha|$ to be considerably smaller than $|\dot{G}/G|$. However, the stringent bound on $|\dot{\alpha}/\alpha|$ in Table I, if applicable, seems to suggest that $\sum_j C_{ij}$ is very small, perhaps zero. Note that in this scenario, nonfundamental masses such as m_p, the proton mass, may still be time dependent,

$$\frac{\dot{m}_p}{m_p} \simeq \frac{-(33-2N_F)}{54}\frac{\dot{G}}{G};$$

and so one might look for a time variation in m_p/m_e (see Table I).

For $\dot{\alpha}_i(m_{KK}) = 0$ and $\dot{m}_{KK} \neq 0$, low-energy couplings and masses can still be time independent, i.e., $\dot{\alpha}_i(\mu) = \dot{m}_p = 0$, but only if $\sum_j C_{ij} = 0$. {In the standard model,[18] with neglect of scalars, $\sum_j C_{ij} = -2, \frac{5}{3}, \frac{7}{2}$ for $i = 1,2,3$, while $\sum_j C_{ij} = \frac{1}{6}(55 - 4n_g)$ [$\frac{1}{6}(88 - 4n_g)$] in the n_g-generation SU(5) [SO(10)] model.[16]} Some supersymmetrical models naturally give $\sum_j C_{ij} = 0$, which is quite interesting since supersymmetry may also be required to tame the ultraviolet divergencies of quantum gravity.

Finally, there is the possibility that $\dot{m}_j/m_j = \dot{m}_{KK}/m_{KK} = \dot{\mu}/\mu = 0$. In that case no relative time variation is detectable. Such a situation might arise if all masses have a dynamical origin (dimensional transmutation) which implies that m_i/m_j is a time-independent calculable number.

(2) $\dot{\alpha}_i(m_{KK}) \neq 0$.—KK cosmologies with this property have been studied by Freund[5] at the classical level. One possibility,

$$\frac{\dot{\mu}}{\mu} = \frac{\dot{m}_j}{m_j} = \frac{\dot{m}_{KK}}{m_{KK}} = \frac{\dot{K}_i}{K_i} = 0,$$

leads to

$$\frac{\dot{\alpha}_i(\mu)}{\alpha_i^{\,2}(\mu)} = \frac{\dot{\alpha}_i(m_{KK})}{\alpha_i^{\,2}(m_{KK})} = \frac{1}{\alpha_i(m_{KK})}\frac{\dot{G}}{G}. \tag{6}$$

If asymptotic freedom is effective up to m_{KK}, then one expects for the QCD coupling $\alpha_3(\mu)/\alpha_3(m_{KK}) \sim \ln(m_{KK}/\mu) \gg 1$. Hence, in this scenario the time variation of the QCD coupling is significantly enhanced at low energies. The mass ratio m_p/m_e provides a sensitive test of this possibility. If in addition, grand unification holds,

$$\frac{\dot{\alpha}_i(\mu)}{\alpha_i^{\,2}(\mu)} = \frac{3}{8}\frac{\dot{\alpha}}{\alpha^2}. \tag{7}$$

The quantity $\dot{\alpha}/\alpha$ is suppressed by $8\alpha/3\alpha_3(\mu)$ relative to $\dot{\alpha}_3(\mu)/\alpha_3(\mu)$; but one must now contend with the stringent bound on $|\dot{\alpha}/\alpha|$ in Table I.

(3) m_{KK} *oscillates*.—As a far-out speculation, let me consider a cosmology in which R_{KK} was contracting until at very short distances $\simeq\sqrt{G}$, quantum gravity began exerting an opposing pressure. In that case, m_{KK} might now be oscillating about an equilibrium position. The amplitude of such oscillations would be damped as power is radiated into our three spatial dimensions by oscillating charges. If the present amplitude and frequency are sufficiently small (and have been for the last 20 billion years), then all experimental bounds can be circumvented. Of course in the early oscillation stage, the power radiated should have been enormous. Perhaps it was the source of the 3-K cosmic background radiation. This speculative cosmology is under investigation.

In conclusion, I would like to advocate new diverse experiments designed to seek out time variations in the fundamental "constants." (Perhaps the existence of a theoretical framework in KK models will now make such measurements more appealing to experimentalists.) It would be nice if, in addition to astrophysical and geochemical bounds, new precise short-time laboratory experiments (such as in Ref. 12) could be undertaken. All imaginable quantities G, α_i, α, m_p/m_e, etc. should be closely scrutinized. New clever experimental ideas would be most welcome. If a time variation is detected, it could be our window to the extra dimensions, an exciting possibility.

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