Fission-Fragment Angular Distributions

P. D. Bond[(a)]
Kernfysisch Versneller Instituut, 9747 AA Groningen, The Netherlands
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The universally used "exact" formula for fission-fragment angular distributions is shown to be valid only under restrictive assumptions. The more general expression, which depends crucially on the final fragment spin distributions, predicts dramatically more anisotropic angular distributions for fission from nuclei at high spin. Recent "anomalous" results are analyzed.

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There has recently been a great deal of discussion of the apparent failure of the rotating-liquid-drop model (RLDM) to reproduce fission-fragment angular distributions from nuclei formed at high angular momentum.[1-5] These conclusions have been based upon analysis with an expression for the angular distribution which is often referred to as being exact.[6] It is shown in this Letter that, in fact, it is "exact" only in a certain approximation and that this approximation fails most dramatically for systems with large angular momentum. The more general angular-distribution formula is presented below and used to analyze angular distributions of fission fragments from nuclei of high spin. The observed large anisotropies are reproduced and are a consequence of the limited spins in the final fragments.

For compound nuclei formed with spin projection $M=0$ along the beam direction the angular distribution formula which has been universally used and found to work well for many cases of fission is[6]

$$W(\theta)\propto \sum_{I}^{I_{\max}} \left\{(2I+1)T_I \sum_{K=-I}^{I} \left[\tfrac{1}{2}(2I+1)\right] |D_{K0}{}^{I}(\theta)|^2 \exp(-K^2/2K_0^2)\left[\sum_{K=-I}^{I}\exp(-K^2/2K_0^2)\right]^{-1}\right\}. \quad (1)$$

The term $(2I+1)T_I$ reflects the formation cross section for a specific compound nucleus of spin I (T_I is the transmission coefficient). The factor $[\frac{1}{2}(2I+1)]|D_{K0}{}^{I}(\theta)|^2$ is the properly normalized angular distribution function[7] for a state of spin I to decay at an angle θ to the beam direction if the I projection along the direction of emission is K. It is assumed that a deformed nucleus fissions along its symmetry axis and the exponential term is the density of states (prior to scission) of the I projection, K, along this symmetry axis. The parameter K_0^2 is defined as $K_0^2=\mathcal{I}_{\rm eff}T/\hbar^2$, where T is the nuclear temperature and $\mathcal{I}_{\rm eff}$ is given by the RLDM. At high angular momentum the RLDM predicts that the fission barrier vanishes even for a spherical nucleus, which leads to $K_0^2\to\infty$. This produces a uniform K distribution and hence from Eq. (1) an isotropic angular distribution of fission fragments. This predicted isotropy for fission fragments from nuclei at high spin is not seen in experiments[1-4] (see Fig. 1) and has led to the suggested failure of the RLDM.

While Eq. (1) has been successfully applied to many systems, it is valid only when the density of final states can be taken as uniform. A derivation of the general angular-distribution formula and discussion of other cases of the M projection will be presented elsewhere.[8] In summary, angular momentum conservation, the density of final states, and the transmission coefficients of the final fragments are considered. As in previous work[9] the assumption of compound-nucleus formation is made so that all interferences of different partial waves are neglected. The resulting expression for the fission-fragment angular distribution is

$$W(\theta)\propto \sum_{I,l,S,K} (2I+1)T_I(2l+1)T_l \frac{|\langle IKl0|SK\rangle|^2}{2S+1}\rho(S)\rho(K)\left[\tfrac{1}{2}(2I+1)\right]|D_{K0}{}^{I}(\theta)|^2. \quad (2)$$

In Eq. (2) $(2I+1)T_I$ is the fusion cross section for spin I; $(2l+1)T_l\,|\langle IKl0|SK\rangle|^2$ reflects the probability of state I to fission into two fragments with relative angular momentum l, combined channel spin S (the vector sum of the two fragment spins), and projection K along the emission axis; $\rho(S)$ and $\rho(K)$ are densities of states; and $[\frac{1}{2}(2I+1)]|D_{K0}{}^{I}(\theta)|^2$ is the angular distribution function. In the following the density of states $\rho(S)$ is chosen as[9] $\rho(S)\propto(2S+1)\exp[-(S+\frac{1}{2})^2/2S_0^2]$ and $\rho(K)$ is chosen as in Eq. (1).

It is easily seen that Eq. (1) is the limiting case of Eq. (2) when the density of final states is uniform ($S_0^2 \to \infty$). In that case the S sum of the Clebsch-Gordan coefficient can be made and one obtains Eq. (1). In practice, if the maximum K value is limited by I or K_0 and not S_0, Eq. (1) may be a reasonable approximation to Eq. (2). This limit could be approached when the compound nucleus is formed with low spin.

However, generally the sum over S cannot be made as the values of S which contribute are limited by the parameter S_0^2 in the density of states. In the case of large I, $K_0^2 \to \infty$ and $S<I$ so that it is the value of S (via the Clebsch-Gordan coefficient) that severely restricts the values of K which contribute to fission. Thus, small K values will be favored and anisotropic angular distributions will result.

As an instructive, but approximate, limit of Eq. (2) let us consider the case of I very large and hence S smaller than I. The Clebsch-Gordan coefficient can be approximated by its asymptotic value[7,10] and the l and S sums can be converted to integrals. The result is (with the assumption $T_l = 1$ for $l \leq I_{max}$)

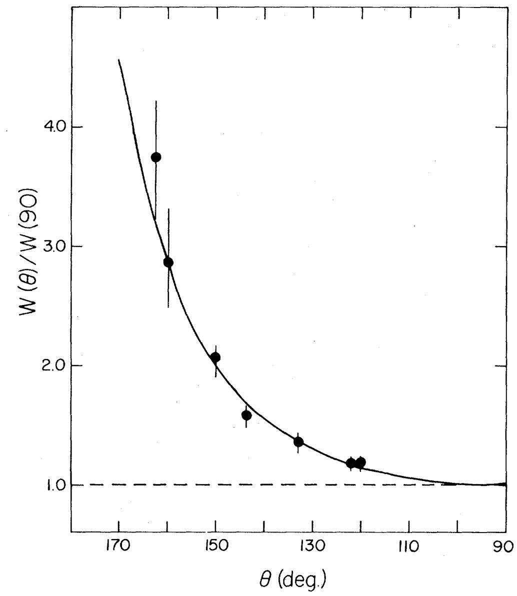

FIG. 1. Fission-fragment angular distributions for $^{40}Ar + ^{238}U$. The data are from Ref. 3 and the solid line is calculated with use of Eq. (3). The dashed line is calculated with Eq. (1).

$$W(\theta) = \sum_I (2I+1) T_I \left[\mathrm{erf}\left(\frac{I_{max}-I}{\sqrt{2}\,S_0}\right) + \mathrm{erf}\left(\frac{I}{\sqrt{2}\,S_0}\right)\right] \sum_K \exp\left(-\frac{K^2}{2\sigma^2}\right)\left(\frac{2I+1}{2}\right)|D_{K0}{}^I(\theta)|^2 \left[\sum_{K=-I}^{I} \exp\left(-\frac{K^2}{2K_0{}^2}\right)\right]^{-1}, \quad (3)$$

where $1/\sigma^2 = 1/S_0^2 + 1/K_0^2$ and erf is the error function. This approximation to Eq. (2), valid when $I_{max} > S_0$, has a nearly identical form as Eq. (1); however, the meaning of σ^2 is vitally different from K_0^2 and the predicted angular distributions are very different.

We first consider a case where Eq. (1) predicts an isotropic distribution, i.e., $K_0^2 \to \infty$ for nearly all partial waves. Such a case, $^{40}Ar + ^{238}U$, was studied in Ref. 3. The cross section for fusion-fission indicated that partial waves up to 131 contribute, whereas by partial wave 31 the RLDM predicts that $K_0^2 = \infty$. Shown in Fig. 1 is the measured angular distribution together with calculations using Eqs. (1) and (3). The RLDM dependence of K_0^2 on I was simulated as $K_0^2 = K_0^2(I=0)/[1-(I/I_\infty)^3]$ for $I \leq I_\infty = 31$ and $K_0^2 = \infty$ for $I > I_\infty$. The value of $S_0^2 = 183$ was estimated from statistical theory as $S_0^2 = \frac{4}{5} mR^2T/\hbar^2$, where m is the mass of the two equal-mass fission fragments, $T \approx [(8/2m)(E_{c.m.} + Q - E_{CB} - \hbar^2\langle l^2\rangle/2\mu R^2)]^{1/2} \approx 1.7$ MeV, and we choose $r_0 = 1.22$ fm. The angular distribution calculated with Eq. (3) is in excellent agreement with the data (Fig. 1). The only arbitrary parameter in the calculation is the normalization to the data at 120°. It should be noted that a range of S_0^2 values would fit this data because the strong sensitivity to S_0^2 occurs at angles closer to 180°.

Next we turn to the case of $^{20}Ne + ^{209}Bi$.[2] In this case $I_{max} \approx 102\hbar$, the RLDM dependence of K_0^2 is simulated as above but with $I_\infty = 71$ (Ref. 2), and we calculate $S_0^2 = 110$. The measured angular distribution[2] and the calculated one (solid line) are shown in Fig. 2. Again the agreement is excellent above 20° but the calculation produces a stronger asymmetry at more forward angles than what is observed. It is the angular region near 0 and π which is most sensitive to K_0^2 and S_0^2. Because of the form of Eq. (3) it is not easy to determine from the angular distribution alone which of the parameters, K_0^2 or S_0^2, might be causing the lack of agreement with data at the most forward angles.

An estimate of S_0^2 can be obtained from independent data. Fission-fragment gamma-ray multiplicities in a nearly identical system, $^{26}Mg + ^{208}Pb$, at about the same input angular momentum have been measured.[11] In that experiment a

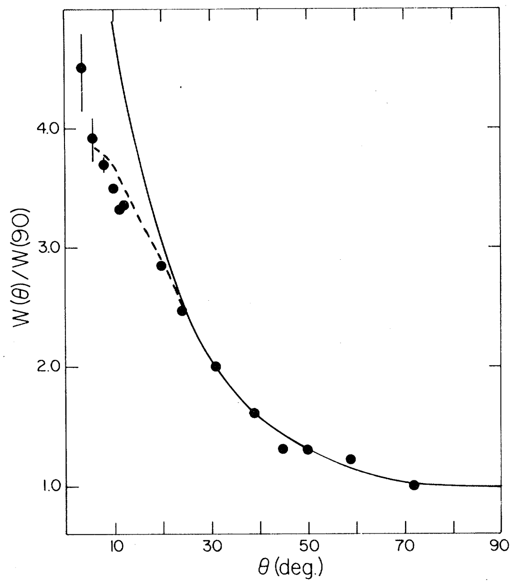

FIG. 2. Fission-fragment angular distribution for $^{20}Ne + ^{209}Bi$. Data are from Ref. 2 and the two calculated curves are made with Eq. (3) (see text).

determination of $J = |j_1| + |j_2|$ is made (j_1 and j_2 are the final-fragment spins *after* particle emission). The fission-fragment angular distribution depends upon S_0^2 which is related to the fragment spins (S_i) *prior* to post-fission particle emission. First consider the case of $\langle |S_i| \rangle = \langle |j_i| \rangle$. If the two fission fragments are assumed to have the same mass and hence the same average spin, then $S_0^2 = (16/9\pi)\langle |j| \rangle^2$. From Ref. 11 we find $\langle |j| \rangle \approx \frac{1}{3}J \approx 13.5$; hence $S_0^2 \approx 100$, which is extremely close to the naive estimate used in the calculation. However, one should expect $\langle |S_i| \rangle \geq \langle |j_i| \rangle$ because of particle emission. It is possible to fit the data by varying S_0^2 (see dashed line in Fig. 2). The value of S_0^2 required corresponds to $\langle |S_i| \rangle \approx 26$ which means that particle emission would have to remove an unreasonably large amount of angular momentum to account for the data. Thus it is unlikely that this process is the sole contributor to the lack of agreement at forward angles. There are other possible causes for the angular distribution to be flatter than the basic prediction for the $^{20}Ne + ^{209}Bi$ case, e.g., the effect of $M \neq 0$ due either to the $\frac{9}{2}$ spin of the target or to mechanisms other than compound-nucleus formation. Of course, the approximations made in obtaining Eq. (3) will have to be tested and, finally, a modification of the RLDM might be required.[1-5]

In conclusion, the suggestions of recent publications that the standard theory of fission fails at high angular momentum have been based upon analysis with an inappropriate theoretical angular-distribution formula. The general expression [Eq. (2)] presented above is crucially dependent upon the spins of the fragment nuclei and ties the parameters of the RLDM and the spin distribution in the final nuclei together. In agreement with data, strong asymmetries are predicted for fission from nuclei at high spin. The cause of remaining discrepancies at the most forward angles in some systems is not yet clear.

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[(a)]Permanent address: Brookhaven National Laboratory, Upton, N.Y. 11973.

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