Negative Temperature Derivative of Resistivity in Thin Potassium Samples: The Gurzhi Effect?

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The resistivity of free-hanging K wires cooled in He gas varies anomalously with temperature when the diameter d becomes comparable to the mean free path l_{ei} for scattering of electrons by impurities. Wires with $d \lesssim l_{ei}$ display a negative $d\rho/dT$ in the vicinity of 1 K. It is suggested that this is the first experimental evidence for an interaction between surface scattering and normal electron-electron scattering as first proposed by Gurzhi. The new data also show that previous explanations of K data of Rowlands *et al.* are inadequate.

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For a bulk metal with a spherical Fermi surface and isotropic electron-impurity scattering, normal electron-electron scattering (NEES)—i.e., scattering *not* involving a reciprocal-lattice vector—does not contribute to the electrical resistivity ρ, because NEES events conserve total crystal momentum.[1] By itself, electron-electron scattering contributes to ρ only via umklapp electron-electron scattering (UEES)—scattering in which a reciprocal-lattice vector participates. In 1963, however, Gurzhi[2] predicted that NEES should affect the low-temperature resistivity of a *thin*, high-purity wire ($l_{ei} \gg d$) in an unusual way, by causing the temperature-dependent resistivity $\rho(T)$ to *decrease* with increasing temperature ($d\rho/dT < 0$). He argued that when the mean free path (mfp) $l_{ee}{}^N$ between normal electron-electron collisions is much less than d ($l_{ee}{}^N \ll d$), then NEES inhibits electrons near the center of the wire from reaching the surface and scattering from it. Since surface scattering contributes to ρ, this process reduces ρ. According to Gurzhi, the effective mfp for electrons in such a case is not the wire diameter d, but $d^2/l_{ee}{}^N$. As the temperature increases, $l_{ee}{}^N$ decreases, and electrons reach the surface less often, causing the surface contribution to the residual resistivity of the sample to decrease with increasing temperature. This model thus leads to a negative $d\rho/dT$ in thin samples.

In this Letter we report the appearance of a negative $d\rho/dT$ in thin samples of potassium (K) below 1.3 K. Since K is thought to have a nearly spherical Fermi surface, and below 1.3 K electron-phonon scattering is negligible in K,[3] the most likely explanation for this behavior involves the increasing importance of the Gurzhi effect relative to UEES as the sample becomes thinner. This apparently occurs despite the fact that we are not in the regime of experimental parameters where the Gurzhi effect should be most pronounced.

We begin with a review of recent measurements of ρ for K for three reasons. Firstly, to place our data in context. Secondly, to describe the data of Rowlands, Duvvury, and Woods[4] for which our new results provide a better understanding than was previously available. Thirdly, to establish that there is no published alternative to the Gurzhi effect to explain our data.

As we have indicated, for bulk K below 1.3 K, electron-phonon scattering is negligible and NEES cannot contribute to $\rho(T)$. According to the standard model, $\rho(T)$ should then be dominated by UEES which yields $\rho(T) = AT^2$.[1] In the standard model, the coefficient A is expected to be insensitive to small changes in sample properties.

In 1978, Rowlands, Duvvury, and Woods[4] published measurements of $\rho(T)$ for high-purity K from 4.2 to 0.5 K which were at variance with both expectations. For wires with $d = 0.8$ mm suspended freely in He gas, they found that below 1.3 K, $\rho(T)$ was better fitted by $\rho \propto T^{3/2}$ than by $\rho \propto T^2$. Furthermore, the magnitude of $\rho(T)$ decreased with sample aging at room temperature. They suggested that their data might be due primarily to an interaction between NEES and surface scattering in the analog of a Knudsen flow regime ($l_{ee}{}^N \gg d$). In this regime this interaction leads to a positive $d\rho/dT$. Their model assumed little or no contribution to $\rho(T)$ from UEES. Subsequently, their data were interpreted as evidence for electron-phason (i.e., charge-density-wave induced) scattering,[5] and, alternatively, in terms of electron-electron scattering in the presence of anisotropic scatterers such as dislocations.[6] This latter model, which was developed primarily to explain low-temperature data on K wires encased in polyethylene,[3,7] required the data of Rowlands, Duvvury, and Woods to vary as T^2. Within the uncertainties of the data, such a variation could not be completely ruled out. Some support for the Rowlands-Duvvury-Woods model was provided by Black,[8] who made Monte Carlo calculations of the interaction between NEES and electron-surface scattering.

In 1982, we published[9] measurements from 4.2 to 0.08 K of $\rho(T)$ for high-purity, freely hanging K wires, with d ranging from 3 to 0.9 mm, prepared and cooled in an Ar atmosphere. We found $\rho(T)$ to vary closely as AT^2 from 1.3 K down to 0.3 K, below which there occurred a deviation from T^2 behavior which we believe is not relevant to the issues addressed in this Letter.[10] For a variety of reasons,[9] including the fact that the coefficient A was insensitive to modest changes in sample properties, we concluded that, standing alone, our data were consistent with simple UEES in K and did not require any of the alternative models mentioned above. Subsequent measurements in our laboratory on $d = 1.5$-mm samples prepared in He gas, and independent measurements[11] on $d = 3$-mm samples also prepared in He gas, provided support for our results. None of these studies engendered any understanding of what Rowlands, Duvvury, and Woods had observed.

In this Letter we report studies of $\rho(T)$ from 1.8 to 0.08 K for free-hanging, high-purity K wires with 0.09 mm $\leq d \leq$ 1.5 mm prepared and cooled in He gas. We find that data for these wires display a clear pattern of unusual behavior which is consistent with that reported by Rowlands, Duvvury, and Woods in the region of overlap, but more complex in form. Data for wires cooled in Ar, or in partial vacuum, are similar in form to those for wires cooled in He; but, for reasons which are not yet understood, they tend to behave like data for samples in He of larger d and to show greater variability for fixed d.[12]

Details of our experimental apparatus and techniques have already been published.[9,13] We note here only that the samples were in the form of extruded, freely hanging wires suspended by their ends and by potential leads of the same K as the samples. For samples with $d \geq 0.25$ mm, the current and potential leads were of the same thickness as the samples. For samples with $d \leq 0.25$ mm, the current and potential leads had $d = 1.5$ mm. Comparison measurements on $d = 0.25$-mm samples with both types of leads yielded similar results.

Figure 1 shows a normalized $d\rho/dT$ [13] plotted as a function of T from 0.08 to 1.8 K for selected K samples with $0.09 \leq d \leq 1.5$ mm, prepared in He gas. The solid symbols indicate samples

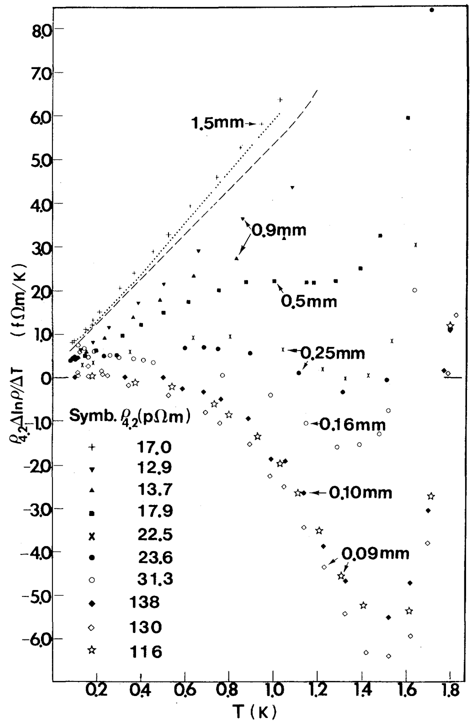

FIG. 1. $\rho_{4.2\,\mathrm{K}}(\Delta\ln\rho/\Delta T)$ vs T for thin wires of K cooled in a He atomsphere. The diameters of the wires are indicated. The dotted and dashed lines indicate the behavior of data for wires of $d = 3.0$ mm and $d = 0.9$ mm, respectively, cooled in an Ar atmosphere (see Ref. 9). The filled circles and the exx's represent $d = 0.25$-mm samples with $d = 0.25$-mm and $d = 1.5$-mm leads, respectively.

whose diameters were determined from the diameters of the stainless-steel dies through which they were extruded. The open symbols indicate samples whose surfaces became corroded while inside the sample can, so that their effective diameters became smaller. Their reduced diameters were inferred from their increased room-temperature resistances. Two samples of each diameter were measured concurrently. The data for such sample pairs always agreed well with each other; one of the largest disparities is indicated by the different symbols for the two thinnest ($d = 0.09$ mm) samples.

The crosses indicate data for a $d = 1.5$-mm sample. For comparison, the dotted and dashed lines indicate data for 3.0-mm and 0.9-mm samples, respectively, in Ar gas.[9] Together, these data indicate the range of variation for all of our "thick" samples in both Ar ($d = 0.9$–3 mm) and He ($d = 1.5$ mm). The erect and inverted filled triangles represent data for two independent d

$=0.9$-mm samples in He. Data for our other $d = 0.9$-mm samples spanned the range bounded approximately by the dashed line and the upright triangles (lower set of data). The filled circles indicate data for a $d=0.25$-mm sample with $d=0.25$-mm leads, and the exx's indicate data for a $d=0.25$-mm sample with $d=1.5$-mm leads. Data for our other $d=0.25$-mm samples approximately spanned the range indicated by the symbols for these two samples.

Figure 2 compares the data of Fig. 1 (solid lines) with data for the $d=0.8$-mm samples of comparable purity [residual resistivity ratio $(\mathrm{RRR}) = R_{293\,\mathrm{K}}/R_{4.2\,\mathrm{K}} \approx 6000$] from Rowlands, Duvvury, and Woods.[4] Both the form and magnitude of their data are consistent with what we expect for samples of $d=0.8$ mm in He gas. However, the more complex behavior of our data rules out the simple $T^{3/2}$ form for $\rho(T)$ that they originally proposed.

Our thickest samples had RRR's of about 6000, which corresponds to a bulk electron mean free path l_{ei} for electron-impurity scattering of about 0.2 mm. Samples comparable to or thinner than this value of 0.2 mm displayed negative values of $d\rho/dT$ in the vicinity of 1 K, followed by a rapidly increasing $d\rho/dT$ above 1.3 K due to electron-phonon scattering. The changes in sign of $d\rho/dT$ from negative to positive shown in Fig. 1 indicate resistivity minima in these very thin samples.

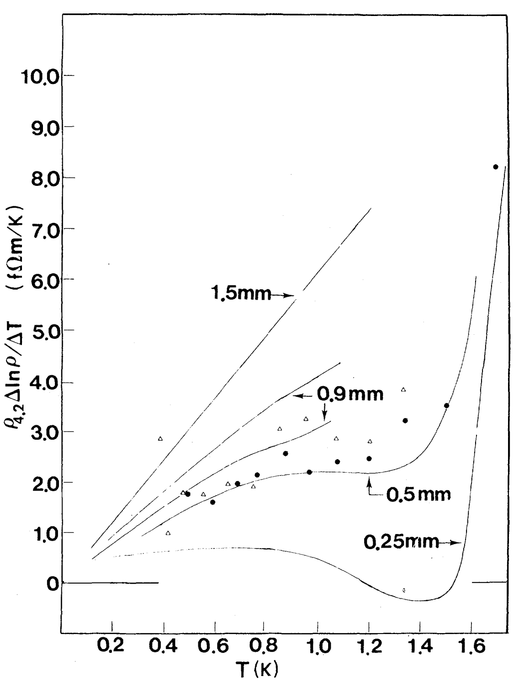

FIG. 2. $\rho_{4.2\,\mathrm{K}}(\Delta \ln \rho/\Delta T)$ vs T for two samples from Ref. 4 of $d=0.8$-mm K wires cooled in a He atmosphere. For comparison, the solid lines represent data from Fig. 1 for K samples having the diameters indicated.

The variations with sample diameter shown in Fig. 1 cannot be due to plastic deformation during either fabrication or cooling of the thinner samples, since plastic deformation makes $d\rho/dT$ become more positive,[14] exactly the opposite of what is seen here.

To test for effects of surface corrosion, two different thicknesses of samples were allowed to thin by means of such corrosion, which occurs naturally inside the sample can. In Fig. 1, the two samples with $d=0.16$ mm and $d=0.09$ mm, labeled by open circles and diamonds, respectively, are corroded versions of the two samples with $d=0.25$ mm and $d=0.1$ mm, labeled by filled circles and diamonds. The changes seen with decreasing thickness due to corrosion are similar to those seen with decreasing thickness in freshly extruded samples. The important quantity must thus be the size of the uncorroded portion of the sample.

To test whether the electron mean free path, l_{ei}, is also an important parameter, measurements of $d\rho/dT$ were made on $d=0.25$-mm samples of K-0.08%-Rb alloys in He gas. These alloys have $l_{ei} \approx 0.04$ mm. No size effect was seen; $d\rho/dT$ increased linearly with T and had the same magnitude as data for thicker samples of the same composition.

The data of Figs. 1 and 2 are incompatible with most of the models proposed to explain either previous data on K below 1 K or "size effects" in other metals. The anisotropic electron-electron scattering model[6] requires a simple T^2 variation of $\rho(T)$ that is inconsistent with the more complex behavior we see. The particular electron-phason model[5] used to explain the $T^{3/2}$ behavior reported by Rowlands, Duvvury, and Woods cannot describe either the general form of our data or the appearance of a negative $d\rho/dT$ in the thinnest samples. The Knudsen flow model of Rowlands, Duvvury, and Woods is also incompatible with our data, because if the thickest samples have $d\rho/dT > 0$ as a result of the Knudsen flow mechanism, then the Monte Carlo calculations of Black[8] show that merely thinning the samples cannot give rise to $d\rho/dT < 0$. Finally, none of the models used to describe size effects in other metals can generate a negative $d\rho/dT$.[15]

The only model we know of which might describe what we see involves the combination of contributions from (a) UEES, giving a positive term, $2AT$, in $d\rho/dT$; and (b) NEES plus surface scattering, giving a negative term in $d\rho/dT$ which increases in magnitude with increasing temperature.

The primary difficulty in directly comparing this model with our data is that the best estimates of $l_{ee}{}^N$ for K yield $l_{ee}{}^N \sim 10-100$ mm at 1 K.[16] This means that we are not in the limit $l_{ee}{}^N \ll d$, but rather in the opposite limit $l_{ee}{}^N \gg d$. However, the Monte Carlo calculations of Black[8] suggest that when $l_{ei} \sim d$, which is the situation for our thinnest samples, negative values of $d\rho/dT$ should persist to at least $l_{ee}{}^N/d \gtrsim 5$. His calculations showed reductions in the total resistivity ρ of about 1% (the limit of accuracy of his calculations) from $l_{ee}{}^N = \infty$ to $l_{ee}{}^N/d = 5$. For our thinnest samples, we see reductions from the presumed UEES contribution of only about 0.003% of the total ρ from $T = 0$ K (i.e., $l_{ee}{}^N = \infty$) to $T = 1$ K. Such small reductions could well be consistent with values of the ratio $l_{ee}{}^N/d$ as large as 100 or even 1000, in which case the theory would be consistent with our data. The fundamental theoretical questions are whether for $l_{ei} \sim d$ the Gurzhi effect dominates the Knudsen-flow effect at such large values of $l_{ee}{}^N/d$ and, if so, whether the Gurzhi effect is large enough to be seen. A much more accurate Monte Carlo calculation will be needed to answer these questions. If the Gurzhi effect is *not* the source of the behavior we observe, then there currently exists no satisfactory explanation for what we see.

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