Background Terms for Second-Sound Damping near T_λ

In a recent Letter Ferrell and Bhattacharjee[1] (FB) attempted to explain the disagreement between their previous theory[2] of second-sound damping D_2 and experiment[3] in the background region $[t=(T-T_\lambda)/T_\lambda \approx -0.02]$ in terms of a correction which was missed originally[2] because the background value of D_2 had been determined by extrapolation from above T_λ. Their estimate of this correction led to a 30% effect,[1] thus bringing their earlier calculations[2] into agreement with the experiments of Crooks and Robinson.[3] Moreover, since this correction was left out of the theory based on the renormalization group,[4,5] its inclusion would spoil the agreement with experiment[3] claimed for that theory.[4,5] While we agree that the correction discussed by FB should appear below T_λ (along with many other analytic corrections), we show in this Comment that its size has been severely overestimated.[1] It follows that this term does not change the theoretical values of D_2 appreciably at $t=-0.02$, and so cannot be used to improve one calculation[2] or invalidate another.[4-6]

To summarize the argument of FB, we separate D_2 into a singular and a background contribution $D_2 = D_2^S + D_2^B = D_2^S + \lambda_B/C_p' + \overline{B}_\psi(t)$. The singular part D_2^S was earlier[2] evaluated to be $D_2^S = (\lambda - \lambda_B)/C_p'$, where λ is the thermal conductivity above T_λ, λ_B its background value, and C_p' the specific heat below T_λ. The background order-parameter relaxation is given by

$$\overline{B}_\psi(t) = B_\psi + B_\psi'(t) = B_\psi(1 + b\rho_s/\rho_n), \tag{1}$$

where the constant $B_\psi = 1.05\times10^{-4}$ cm^2/s was determined[2] from data above T_λ, and the new term B_ψ' depends on the existence of a superfluid density ρ_s. To estimate it, we may appeal to the exact hydrodynamic formulas

$$D_2 = (\rho_s/\rho_n\rho)(\tfrac{4}{3}\eta + \zeta_2 + \rho^2\zeta_3 - 2\rho\zeta_1) + \lambda'/C_p',$$

$$D_1 = \rho^{-1}(\tfrac{4}{3}\eta + \zeta_2).$$

In the range $|t| \gtrsim 0.02$ FB assumed that the singular part $D_2^S(t)$ was negligible, which implies $\overline{B}_\psi(t) = (\rho_s/\rho_n\rho)(\frac{4}{3}\eta + \zeta_2 + \rho^2\zeta_3 - 2\rho\zeta_1)$. A crude estimate of the right-hand side of this equation is obtained by neglecting the term $\rho^2\zeta_3 - 2\rho\zeta_1$, which yields

$$\overline{B}_\psi(t) \approx (\rho_s/\rho_n)D_1, \tag{2}$$

corresponding to Eq. (11) of Ref. 1. In the temperature range $10^{-3} < |t| < 5\times10^{-2}$, D_2 is reasonably well approximated by its background value, but D_1 has a strong divergence. If the full experimental value[7] of D_1 is inserted into the right-hand side of (2) the result is *independent of t* in this range and equal to 1.2×10^{-4} cm^2/sec. The experimental result for $D_1\rho_s/\rho_n$ agrees with (1) and (2) and the estimate[2,8] $B_\psi = 1.05\times10^{-4}$ cm^2/s only if $b<1$. Thus the B_ψ' term is negligible compared to B_ψ for $|t|\approx 0.02$, and the earlier procedure[2,4,5] used to estimate the background terms in D_2 is consistent. FB arbitrarily pick out the contribution to (2) from the background part of D_1, and equate it to B_ψ', thus obtaining the large value $b\approx 5$. They have not explained[1,9] how their procedure is consistent with the exact hydrodynamic formulas for D_1 and D_2 which lead to Eq. (2).

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[6]More precise recent experiments by R. Mehrotra and G. Ahlers [Phys. Rev. Lett. 51, 2116 (1983)] have revealed some discrepancies with theory (Refs. 4 and 5), but these are of opposite sign to those discussed by FB, and their explanation awaits further theoretical developments.
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