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Chiral Symmetry Breaking in Supersymmetric Yang-Mills Theory

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It is proved that the contribution of self-dual twisted gauge configurations (torons) to the gluino condensate $\langle\lambda\lambda\rangle$ in $N=1$ supersymmetric Yang-Mills theory remains finite in the thermodynamic limit. Using the richer vacuum structure of the theory, defined in a twisted box, the authors explain the constant value of the instanton contribution to the condensate $\langle\lambda\lambda(x)\lambda\lambda(y)\rangle$. The physical picture obtained is consistent with $\mathrm{tr}(-1)^F$ and the effective-Lagrangian approach.

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Effective Lagrangians[1] and the value of $\mathrm{Tr}(-1)^F$ [2] for $N=1$ supersymmetric Yang-Mills theory strongly suggest a vacuum structure of N states characterized by different values of the gluino condensate $\langle\lambda\lambda\rangle$. Instantons are not sufficient in order to generate such a condensate. This is similar to QCD where instantons cannot break the $\mathrm{SU}_L(N_f)\otimes\mathrm{SU}_R(N_f)$ symmetry. If the gauge group is SU(2), for example, an instanton can give rise to condensates of the form $\langle\lambda\lambda(x)\lambda\lambda(y)\rangle$ due to the existence of four gluino zero modes.[3] The instanton size must be integrated over in order to preserve supersymmetry.[4]

The instanton contribution to the above Green's function has been calculated recently[5] with the peculiar result that as $|x-y|\to\infty$ it does not fall off but rather approaches a constant limit. The puzzle is that WKB theory with instantons does not provide us with a state $|S\rangle$ with nonzero

$$\langle\theta|\lambda\lambda(x)|S\rangle\langle S|\lambda\lambda(y)|\theta\rangle, \tag{1}$$

where θ denotes the vacuum angle.

In this Letter we will show that the special topology of Yang-Mills theories allows the construction of such states $|S\rangle$, and provides us with a classical tunneling configuration which induces condensates as Eq. (1) in WKB theory. The mechanism is similar to the one which we have recently suggested for chiral symmetry breaking in QCD.[6]

We consider a finite four-dimensional box with a nontrivial $Z(N)$ twist in the (1, 2) plane. This box represents a unit of magnetic flux in the 3 direction.[7] The boundary condition is the same for gluons and gluinos in order to preserve supersymmetry. In the temporal gauge $A^4=0$, the residual gauge transformations $\Omega(\vec{x})$ can be topologically classified by π_3 and π_1 of $\mathrm{SU}(N)/Z(N)$ which are given by Z and $Z(N)$, respectively.

The perturbative vacua in the box are therefore given by

$$|n,k\rangle=\int(d\Omega_0)|A_i^{(n,k)}(\vec{x})\rangle, \tag{2}$$

where the integration runs over trivial gauge transformations and $A_i^{(n,k)}(\vec{x})$ are zero action gauge configurations

$$A_i^{(n,k)}(\vec{x})=\Omega^{(n,k)}(\vec{x})\,\partial_i[\Omega^{(n,k)}(\vec{x})]^{-1}, \tag{3}$$

$n\in Z$ is the instanton number, and k is the number of times that Witten's operator $T_z{}^2$ has acted on the trivial vacuum $|A_i=0\rangle$.

The explicit topological definition of k and n can be given in terms of the corresponding Chern-

Simons invariant:

$$K(x^4=0) \equiv \frac{1}{16\pi^2}\int d^3x\left(2\epsilon_{\nu\alpha\beta 4}\,\mathrm{Tr}A_\nu{}^{(n,k)}\partial_\alpha A_\beta{}^{(n,k)} - \frac{2}{3i}A_\nu{}^{(n,k)}A_\alpha{}^{(n,k)}A_\beta{}^{(n,k)}\right) = -\frac{N-1}{N}k + n. \tag{4}$$

Tunneling between the states $|n,k\rangle$ and $|n+1,k+1\rangle$ can be produced by the self-dual gauge configurations with Pontryagin number $1/N$ (torons).[8] In fact,

$$P = K(x^4=\beta) - K(x^4=0), \tag{5}$$

with β the period in time.

The states in the physical Hilbert space are given by

$$|\theta, e\rangle = \sum_{n,k}\exp\left[\frac{2\pi i(k\cdot e)}{N} + \theta i\left(-\frac{N-1}{N}k+n\right)\right]|n,k\rangle, \tag{6}$$

where e is 't Hooft's electric flux in the 3 direction; $e=1,\ldots,N$.

Now with respect to states $|\theta,e\rangle$ the condensate $\langle\lambda\lambda(x)\lambda\lambda(y)\rangle$ corresponding to the contribution of gauge configurations with Pontryagin number 1 involves terms like

$$\langle n+1, k+1|\lambda\lambda(x)|nk\rangle\langle nk|\lambda\lambda(y)|n-1,k-1\rangle \tag{7}$$

which we can evaluate in WKB theory around self-dual twisted gauge configurations with Pontryagin number $\frac{1}{2}$. In general and for any SU(N) we will have that the contribution of gauge configurations with Pontryagin number 1 to condensates $\langle\lambda\lambda(x_1)\cdots\lambda\lambda(x_N)\rangle$ (in the adjoint representation the number of zero modes is $2NP$) involves terms like $\Pi_{i=1}^N\langle\lambda\lambda(x_i)\rangle$, where $\langle\lambda\lambda\rangle$ is the effect of gauge configurations with Pontryagin number $1/N$. Now on states $|\theta,e\rangle$ the constant correlation function $\langle\lambda\lambda(x)\lambda\lambda(y)\rangle$ can, in principle, cluster as the product of two $\lambda\lambda$ condensates. In order to see if this is what really happens we will take the thermodynamic limit of $\langle\theta e|\lambda\lambda(x)|\theta e\rangle$ for $e=0$, assuming that the theory lives in the confinement phase.

The method of computation for fermionic amplitudes in theories with Weyl fermions in real representations of the gauge group is given by Vainshtein and Zakharov[9] and amounts simply to squaring the determinant and then using a four-component (Dirac) spinor formulation.

Consider now specifically SU(2). The toron which induces the condensate in Eq. (7) is a configuration in a four-dimensional box with the twist $n_{12}=-n_{34}=1$, the rest vanishing. The transition functions are given by

$$\Omega_\mu(x) = \exp(i\omega\alpha_{\mu\lambda}x_\lambda/L) \tag{8}$$

with $\omega = 2\pi\sigma_z$ and $\alpha_{12}=-\alpha_{21}=\alpha_{34}=-\alpha_{43}=\frac{1}{4}$. L is the size of the box. The gauge configuration is

$$A_\lambda{}^{\mathrm{cl}}(x) = -\omega_{\mu\lambda}(x_\mu - z_\mu)/L^2. \tag{9}$$

It is self-dual and its Pontryagin number is $\frac{1}{2}$. z_μ is an arbitrary origin, leading to four translation zero modes. Note that with a fixed box size, there are no dilation zero modes.

In a toron background there are two gluino zero modes given by the supersymmetry transformations of the toron (in a Majorana representation),

$$\lambda = \tfrac{1}{2}\sigma_{\mu\nu}\boldsymbol{F}_{\mu\nu}{}^{\mathrm{cl}}(x)\alpha. \tag{10}$$

α is the Grassman parameter of the transformation and

$$F_{\mu\nu}{}^{\mathrm{cl}} = -2\omega\alpha_{\mu\nu}/L^2. \tag{11}$$

Note that there are no superconformal zero modes and that the number of fermionic zero modes is in agreement with the Atiyah-Singer index theorem.

We now embed $F^{\mu\nu}$ and φ (the Weyl representation of λ) in a superfield $W_\alpha(x_L,\theta)$ with $x_{L\mu}=x_\mu - i\theta\sigma_\mu\bar\theta$ and compute

$$\langle\epsilon_{\alpha\beta}W^\alpha W^\beta\rangle_{\mathrm{toron}} \sim \frac{1}{L^3}\exp\left(\frac{8\pi^2}{2g^2(\mu)} + \ln(\mu L)^{4-1/2\times 2}\right)\int_V d^4z\, d^2\theta_0\,(F_{\mu\nu}{}^{\mathrm{cl}})^2(\theta-\theta_0)^2. \tag{12}$$

θ_0 stands for the supersymmetric partner of the toron origin z. In Eq. (12) we have substituted the gluino zero modes and integrated over the collective coordinates with the supersymmetric measure. The power of μL in the logarithm is a result of the four gluonic zero modes, two Weyl-gluino zero modes, and the cancellation of nonzero modes between gluons and gluinos. We find

$$\langle\lambda\lambda\rangle_{\mathrm{toron}} \sim \mu^3\exp[4\pi^2/g^2(\mu)] \tag{13}$$

which is renormalization-group invariant.

As in the case of QCD the thermodynamic limit of the chirality-breaking condensate can be easily taken with $\mu > \Lambda_c$ fixed and $L \to \infty$, and it is finite.

We conclude that torons present us with exactly the right characteristics needed to explain physically the results of the instanton computation. An important ingredient is the contribution of configurations of large size. Large instantons must be taken into account in order to obtain a supersymmetric result—a cutoff in the integration over the instanton size will lead to supersymmetry breakdown in the effective Lagrangian. On the other hand, the configuration which induces $\langle\lambda\lambda\rangle$ must have a fixed scale—dilatation symmetry would necessarily lead to too many gluino zero modes. The toron has this property, with the scale being the box size. However, as the volume of the box increases and the toron field strength decreases its effect does not disappear, signaling the spontaneous nature of the symmetry breakdown.[6]

Our computation is done in a state with nonvanishing $Z(N)$ magnetic flux. This state can represent the vacuum only in a phase where magnetic flux does not cost energy, i.e., confinement (and perhaps Coulomb). Witten has presented arguments which suggest that supersymmetric Yang-Mills theory confines.[2] If a Higgs phase does exist, we expect chiral symmetry to be broken there also, for the following reason: Since supersymmetry is unbroken, a massless physical gluino state must be accompanied by a massless gluonic state incompatible with a Higgs phase which is characterized by a mass gap.

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