Dissipation in Computation

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(Received 8 June 1983)

The question of the energy dissipation in the computational process is considered. Contrary to previous studies, dissipation is found to be an integral part of computation. A complementarity is suggested between systems that are describable in thermodynamic terms and systems that can be used for computation.

PACS numbers: 89.80.+h, 89.70.+c, 06.50.-x

Every numerical computation, no matter how abstract, is ultimately bound to limits imposed by physical processes that occur in the real world. A question therefore arises as to whether or not the physical laws that govern the appropriate processes impose constraints on computation. This question has recently attracted considerable attention with regard to the minimum energy required for a bit manipulation.[1,2] All known computational systems, including biological ones, are dissipative, and it was suggested quite early that the computational (or physical) processes which really require energy dissipation lead to a minimum energy loss per step of[3,4]

$$kT \log_e 2. \tag{1}$$

Landauer[3] arrived at (1) through an argument that most computation is logically irreversible and this necessarily imposes physical irreversibility *due to a loss of phase space.* The earliest approach which yields (1) uses an analogy between logic gates and communication channels.[5,6] In this analogy, sometimes attributed to von Neumann, (1) represents the minimum energy required per bit for accurate transmission of the bit to the next gate in the presence of noise. While no flaw has been found which would cause this analogy to produce an inaccurate measure of the minimum dissipation, later workers have largely ignored it.

Contrary to this latter interpretation, there is a current belief that favors the possibility of dissipationless computation; i.e., dissipating less than (1) per bit operation. These recent models rely upon Landauer's interpretation that (1) arises from the loss of phase space in erasure, and therefore suggest that logically reversible Turing machines are possible.[7] This led to the conclusion[8] that computation can be carried out at no expense of energy, although the information-theory arguments have never been refuted.

In this paper, we point out that logical irreversibility is irrelevant for the question of the energy requirements of computation, and that the efforts based upon logical reversibility lack a physical basis. In reconsidering the concepts of computation and measurement, we conclude that computation requires a nonequilibrium system and requires dissipation. Our approach is to consider the energy requirements of single bit operations rather than the overall logical structure of the computation.

The physics of computation involves an element of measurement and interpretation at its very foundation. While the time evolution of any system can in principle be viewed as representing a numerical process, computational systems are those which implement a Turing machine,[9] the general-purpose computer. In the classical sense, a Turing machine is composed of an automaton and a finite-dimensioned tape. The automaton, or reading head, has several internal parameters and can change its state as a result of an external interaction, such as in response to a symbol read from the tape. In addition, it can change the symbol recorded on the tape. In particular, we note that the automaton changes its state in response to inputs from the tape during computation.

Let us now turn to the questions of logical reversibility in the literature. Landauer[3] has argued that dissipation enters the computational process when a random tape (a tape with unknown content) is erased. At the end of every one-tape Turing machine computation, information is thrown away by erasing all intermediate results (the random tape) and this is presumably the operation that results in dissipation. In deleting a random bit, he argued that the phase volume is reduced by a factor of 2 corresponding to the logically irreversible mapping of the 0 or 1 onto either as a final state. According to Landauer, this decrease in phase volume is accompanied by dissipation according to usual equilibrium thermodynamic arguments regarding entropy. We argue below that this interpretation is wrong. However, Bennett[7] adopted Landauer's line of reasoning and cured the computational process of this logical irreversibility by constructing a three-tape Turing machine, in which strictly logically reversible bit operations are used. In the first phase of this construction, the computation is performed. The result is then copied to a separate tape and the final phase causes the machine to compute backward thus erasing all intermediate results. Because of the logical 1:1 mappings, this construction avoids the erasure of random tapes, and is viewed as proof[8] that dissipationless computation is possible.

The above lines of reasoning, which are supposedly based upon the thermodynamic arguments, must be reexamined. Although the erasure of a random bit is a logically irreversible operation, it does not imply a reduction in physical phase space. Storage of an information bit requires the presence of a barrier to prevent thermalization of the system.[3] Erasure returns all bits to a common state. From the point of view of the bit, it is always in a single state (either 0 or 1). Bit operations are always physical 1:1 mappings and are not accompanied by dissipation (if done slowly enough) because of arguments regarding entropy. On the other hand, Landauer's argument would apply if a bit was physically random, i.e., the 0 and 1 states were randomly occupied in the presence of thermal noise. Then, however, these bits could not be used for computation as they are changed stochastically rather than according to a defined program.

There appears to be no physical reason to stress the operation of erasing a single bit or of throwing away logical information. For the reading head of a Turing machine, all successive bits are random, regardless of whether the operation is "compute" or "erase." In particular, a finite random tape is not essentially different from any other tape in this regard. Therefore, a Turing machine can be programmed to erase a random tape.[10] Moreover, Bennett's machine is no more than a specialized three-tape Turing machine, and any such machine can be cast as a one-tape machine.[9,11] The portion of the machine that erases the intermediate results is just the needed algorithm to erase the particular "random" tape. We are therefore led to the conclusion that Bennett's construction shows that any one-tape machine can be made logically reversible, but the discussion of logical reversibility is irrelevant to considerations of physical reversibility and dissipation in computers, a conclusion reached earlier by Mead and Conway.[12] If Bennett's machine is physically reversible, it is because the individual bit operations are reversible, but the rationale for believing that their bit operations are physically different from others has never been given. However, if bit manipulations are dissipative, the entire computation is dissipative.

A realization of a three-tape machine has been proposed, the so-called "Brownian" computer[8] which operates close to thermal equilibrium. An external force still is needed that drives the computation and, as a consequence, time-reversal symmetry is broken. *It is assumed, but not demonstrated,* that for arbitrarily low computation speeds, arbitrarily little energy has to be dissipated, although this may be incompatible with the earlier requirement.[12] In any case, only computation at finite speed is of interest. Toffoli's "billiard ball" computer,[13] which operates at finite speed and consumes no energy, assumes zero noise and no friction and is therefore dissipationless *by construction.* However, the slightest noise (even in the initial condition) disrupts the computation completely; it is an example of a strongly mixing system.[14] We turn now to a discussion of dissipation in computation which considers bit operations rather than the overall logical reversibility of the process. We will find that dissipation is an integral part of computation.

An integral part of every computational step of the Turing machine is the process of reading information from the tape, and this in fact is just the process of making a measurement on the tape. The essential ingredient of a computer is the reading operation that is necessary for the Turing machine. The state of the latter system cannot be allowed to evolve freely, since state transitions within the reading head must be made to accommodate different computations. In this sense, the computer must be forced along the desired logical path. At each step of the Turing machine, we carry out a reading operation and the subsequent application of a rule which governs transitions within the automaton and symbols on the tape. Computation, therefore, consists of a series of steps, which force the system from one state to its logical successor. It is the competition between measurement and the thermalization process that requires energy to be dissipated. In this respect, the measurement in the presence of noise and the need to keep the system from thermalizing are just the standard arguments of information theory.[15] The amount of energy dissipated in each reading operation has to increase for increasing accuracy. In general, the measurement requires an action away from thermal equilibrium which consequently leads to dissipation. As a result, computation now emerges as dissipative at least because of the reading operation, a price that has to be paid in expended energy in order to keep the computation on its desired track in the presence of thermal noise. Again, we see that only systems that are forced along a nonthermodynamic path by virtue of measurements and subsequent decisions can be applied to computation. We are therefore led to the conclusion that any Turing machine, even a logically reversible one, is physically irreversible because of the competition from noise. Equation (1) arises from just these considerations. Thus, discussions which would use logical reversibility to deduce physical reversibility are restricted to the irrelevant case of $T=0$.

In this respect, a computer resembles Maxwell's demon.[16] The evolution of the system is made dependent upon the outcome of a measurement thereby manipulating the development in time. The demon violates the microscopic reversibility of the trajectories in phase space by means of measurements and thus forces the system into a nonthermodynamic evolution. However, a price in dissipated energy has to be paid. A system that does not contain these measurements cannot be prevented from thermalizing. Compare, for example, Feynman's[17] construction of a purely mechanical demon (no decisions on its own) by ratchets and pawls which cannot prevent the system from thermalizing. Only systems that are forced along a nonthermodynamic path by virtue of measurements, and thus dissipation, can be used for computation.

Our discussion here is confined to the domain of classical physics. We abstain from a discussion of quantum mechanical models in view of the unsettled question of the quantum mechanical measurement process which has to be considered at every step of computation. In the literature, there exists a quantum mechanical model[18] of Bennett's three-tape machine which claims to be physically reversible when operated at finite speed. However, the crucial question of the measurement process has not been addressed. In that respect, this model in also subject to our criticism.

In summary, the previous approaches to dissipationless computation fail because equilibrium thermodynamics cannot be employed for systems that compute. In particular, the tape in the Turing machine represents stored information. If this information is to be preserved, the system must be kept in an ordered, far-from-equilibrium state. Such systems are often called dissipative structures.[3,19,20] For computation, individual symbols on the tape should only be changed according to logical rules (following a read operation), and must be secured from being changed by thermal noise. Consequently, there appears to be a contrast, or complementarity, between systems that can be used for computation and systems that are describable in equilibrium thermodynamic terms. The more a system satisfies equilibrium thermodynamics, the less it is usable for computation, and conversely. Only systems with a deterministic time evolution, and that are secured from external noise by being maintained in an ordered nonequilibrium state, can be used to represent physically the deterministic process of computation. Dissipation enters through the need to preserve the ordered state of the tape and the need to make measurements on the tape in the presence of noise. The minimum energy dissipation is thus defined by the ability to distinguish a signal from noise, and the long-established information-theoretic arguments lead to the lower limit (1).[5,6]

We would like to thank S. E. Goodnick for discussions and comments. We also acknowledge communications with R. Landauer and C. H. Bennett, which led to further clarification of their work but does not imply their acceptance of the position presented here. This work was supported in part by the U. S. Office of Naval Research.

[1]For a review, see, e.g., Int. J. Theor. Phys. 21, Nos. 3/4, 6/7, and 12 (1982), which contain the Proceedings of the Conference on Physics of Computation.

[2]The general thrust is reviewed by R. Landauer, to be published.

[3]R. Landauer, IBM J. Res. Dev. 3, 183 (1961).

[4]R. W. Keyes and R. Landauer, IBM J. Res. Dev. 14, 152 (1970).

[5]L. Brillouin, *Science and Information Theory* (Academic, New York, 1962).

[6]J. D. Bekenstein, Phys. Rev. Lett. 46, 623 (1981).

[7]C. H. Bennett, IBM J. Res. Dev. 17, 525 (1973).

[8]C. H. Bennett, Int. J. Theor. Phys. 21, 905 (1982).

[9]A. Turing, Proc. London Math. Soc. 42, 230 (1936), and 43, 544 (1937).

[10]Both 0 and 1, or any other symbol, are computable numbers in the sense of Turing (Ref. 9). The resetting of a tape to a standard symbol must be regarded as erasing, as this is in fact all that Bennett's machine (Ref. 7) can do as well.

[11]A universal Turing machine can be built to carry out any process that can naturally be called an effective procedure. This is usually called Church's thesis after A. Church, Am. J. Math. 58, 345 (1936). Its applicability here lies in the fact that the third phase of Bennett's construction actually calculates the inverse Turing function of the first phase. If such an inverse function exists, it can be shown that another Turing machine, computing forward, exists which computes the same result. Therefore, Bennett's construction is a normal three-tape Turing machine, and can be recast as a one-tape machine. Inverse Turing functions are discussed by J. McCarthy, in *Automata Studies*, edited by C. E. Shannon and J. McCarthy (Princeton Univ. Press, Princeton, 1956), p. 177.

[12]C. Mead and L. Conway, *Introduction to VLSI Systems* (Addison-Wesley, Reading, Mass., 1981), p. 349. These authors argue that a nonequilibrium state is necessary to have the time asymmetry required for computation.

[13]T. Toffoli, Int. J. Theor. Phys. 21, 165 (1982).

[14]I. Sinai, in *Statistical Mechanics, Foundations, and Applications*, edited by T. Bak (Copenhagen, 1966). See also the discussion by M. V. Berry, Ann. Phys. (N.Y.) 131, 163 (1981).

[15]R. W. Keyes, IEEE Trans. Comp. C-26, 1017 (1977).

[16]L. Szilard, Z. Phys. 53, 840 (1929).

[17]R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, Mass., 1963), Vol. 1, Chap. 46.

[18]P. Benioff, Phys. Rev. Lett. 48, 1581 (1982).

[19]G. Nicolis and I. Prigogine, *Self-Organization in Non-Equilibrium Systems* (Wiley, New York, 1977).

[20]I. Prigogine, *Thermodynamics of Irreversible Processes* (Wiley, New York, 1955).