Levitation of Extended-State Bands in a Strong Magnetic Field

In a recent Letter,[1] Levine, Libby, and Pruisken have reported a new theoretical analysis of the ordinary quantum Hall effect which leads to a plausible[2] scaling theory of localization in two dimensions in the presence of a magnetic field. In this Comment I point out two things: (1) This scaling theory leads to a quantification of a prediction made by Halperin[3] that extended-state bands are in the following sense conserved: If disorder is increased so as to push the system toward complete localization, the extended-state bands cannot disappear discontinuously, but must float upward in energy, like bubbles. The quantification is that if τ is the classical elastic collision time, that is appropriate for describing the system when its size is the magnetic length $a_0 = (\hbar c/eH_0)^{1/2}$, where H_0 is the magnetic field strength, then the nth extended-state band lies at energy

$$E_n = (n + \tfrac{1}{2})\hbar\omega_c \left[\frac{1 + (\omega_c\tau)^2}{(\omega_c\tau)^2}\right], \tag{1}$$

where $\omega_c = eH_0/mc$ is the cyclotron frequency. (2) This behavior can be deduced from elementary principles without resorting to field-theoretic arguments.

I address the second point first. The existence of extended states and their levitation by disorder may be demonstrated in a thought experiment[3,4] in which a ribbon of two-dimensional metal is assumed to be disordered everywhere except within a distance d of each edge. The ribbon is then bent into a loop while maintaining a magnetic field H_0 normal to its surface. If a flux quantum $\Delta\phi = hc/e$ is inserted adiabatically through this loop, the disorder-free zone at one edge "injects" one state per Landau level into the disordered region, while the disorder-free zone at the other edge "removes" a state. Since localized states are unaffected by the addition of $\Delta\phi$, injected extended states must have been able to "get through" the disordered region via a conduit of extended states. Increasing disorder must eventually localize all the states below an arbitrary energy E. The only way it can achieve this without annihilating the conduit is to float the conduit upward in energy past E, thereby creating an "edge."

The quantitative result follows[2,5] from scaling of the classical conductances

$$\sigma_{xx}^0 = (\rho e^2\tau/m)[1 + (\omega_c\tau)^2]^{-1} \tag{2}$$

and

$$\sigma_{xy}^0 = (\omega_c\tau)\sigma_{xx}^0, \tag{3}$$

where ρ is the areal electron density, from length scales comparable with a_0, where they are appropriate, to length scales the size of the sample, where they are not appropriate. Scaling maps conductances to the quantum values

$$\sigma_{xx} = 0 \tag{4}$$

and

$$\sigma_{xy} = ne^2/h, \tag{5}$$

with n an integer, unless the density ρ is such that[2,5]

$$\sigma_{xy}^0 = (n + \tfrac{1}{2})(e^2/h). \tag{6}$$

Equating Eqs. (3) and (6) one obtains as the condition for the presence of an extended-state band at the Fermi surface

$$\rho = (n + \tfrac{1}{2})\frac{1}{2\pi a_0^2}\left[\frac{1 + (\omega_c\tau)^2}{(\omega_c\tau)^2}\right], \tag{7}$$

which is equivalent to Eq. (1).

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