Critical Wetting in Systems with Long-Range Forces

In a recent Letter,[1] Nightingale, Saam, and Schick studied wetting transitions in three dimensions in the framework of interface models. The pinning potential for the interface coordinate h was taken as

$$V(h) = \text{const} - Ah^{-2}, \quad h > 0, \tag{1}$$

with a hard wall for $h < 0$. It was found that no depinning transition occurs for (1) and $A > 0$. A continuous transition was only found as $A \rightarrow 0^+$. Thus, *critical wetting cannot occur for model* (1).[1]

The potential (1) is related to the effective interaction between the substrate and the interface. For temperature $T = 0$, this interaction is given by the difference between the interactions of the adatoms with the real substrate and with a hypothetical substrate composed of adatoms.[2] For general T, however, $V(h)$ is given by the local free-energy density of a rigid interface located a distance h from the substrate. In this Comment, we argue that, for large h, this free energy behaves as

$$V(h) = \text{const} - \{A + A^*(t)\}h^{-2}, \tag{2}$$

where A^* depends on the reduced temperature $t = 1 - T/T_c$. *For model (2), critical wetting occurs* at $A = -A^*(t)$.[3]

The potential (2) is obtained as follows. We start from the semi-infinite Ising model defined in Eq. (10) of Ref. 2. Using the standard procedure of a Gaussian transformation,[4] we obtain a continuum model for the order-parameter field ϕ. Since we will only study mean-field theory, we put $\phi = m(z)$. z is the distance from the substrate surface. This leads to the Landau-Ginzburg free energy

$$F(m) = \int_0^\infty dz\, \{m(z)\chi(z,m) - u(z)m(z) - \ln\cosh[2\chi(z,m)]\}, \tag{3}$$

with

$$\chi(z,m) = \int_0^\infty dz'\, K(z-z')m(z'), \tag{4}$$

$K(z-z')$ is the interaction between the adatoms due to the long-range van der Waals forces after integration over lateral directions. We take $K(x) = (J/T)\{1 + (x/a)^2\}^{-2}$ where J and a are the overall strength and the range of this interaction. The effective substrate potential $u(z)$ in (3) is taken to be

$$u(z) \rightarrow (B/T)z^{-3}, \quad z \rightarrow \infty \tag{5}$$

with $B > 0$.

The model (3)–(5) has a bulk critical point at $T_c = Ja/\pi$. As $T \rightarrow T_c$, the bulk order parameter behaves as $m_\pm = \pm M = \pm(3t)^{1/2}$ with $t = 1 - T/T_c$. In the following, "liquid" and "gas" mean $m > 0$ and $m < 0$, respectively. The substrate potential (5) can induce a liquid layer of thickness h near the surface while the bulk is still a gas. The free energy of this film is $F(m(z)) - F(m_-)$ where $m(z)$ solves $\delta F/\delta m = 0$. For large h,

$$\bar{m}(z) = m_+\theta(h-z) + m_-\theta(z-h) \tag{6}$$

should be a reasonable approximation for $m(z)$. Thus, the free energy of a thick film should be approximately equal to $F(\bar{m}(z)) - F(m_-) = V(h)$. If (6) is inserted into (3)–(5), one finds

$$V(h) = \text{const} + (M/T)\{B - \tfrac{2}{3}Ja^4M + O(M^3)\}h^{-2}$$

for large h. Thus, the phase boundary for critical wetting is

$$B^* = \tfrac{2}{3}Ja^4M + O(M^3) \rightarrow 2Ja^4t^{1/2}/\sqrt{3}$$

as $T \rightarrow T_c$. With $A = -MB/T$ and $A^* = MB^*/T$, one obtains (2).

Finally, note that (2) contains only the leading terms of $V(h)$ for large h. For short-range forces, next-to-leading terms are important in order to determine the complete phase diagram.[5] They are also necessary in order to explain the results of van der Waals theory for systems with long-range forces[6] as will be discussed elsewhere.

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[2] R. Pandit, M. Schick, and M. Wortis, Phys. Rev. B **26**, 5112 (1982).

[3] In Ref. 1, a short-range potential was included by $V(h) \equiv 0$ for small h. In this case, critical wetting is found if the constant in (2) is negative near T_c.

[4] D. J. Amit, *Field Theory, The Renormalization Group, and Critical Phenomena* (McGraw-Hill, New York, 1978).

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[6] D. M. Kroll and T. Meister, to be published.