Chaotic Flow Regimes in a Convection Loop

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In our experiments on a loop of fluid heated with a constant flux on the bottom half and cooled at a constant temperature on the top half, we have observed three chaotic flow regimes: a globally chaotic regime whose essential features can be described by a one-dimensional cusp-shaped map, a subcritical regime in which the flow can be either chaotic or steady, and a transient regime in which the flow remains chaotic for a time and then decays into a steady flow.

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In 1974 Gollub and Swinney[1] conjectured that their experimental results on the transition to turbulence in circular Couette flow could be described by a low-dimensional chaotic attractor; however, they were not able to identify any characteristics of this attractor. In the intervening years a number of techniques have emerged to determine the characteristics of chaotic attractors from experimental time series.[2] These techniques have been applied to determine the characteristics of chaotic attractors in a variety of physical systems; however, there have been only a few hydrodynamic systems which have been shown to be described by chaotic attractors.[3] The chaotic behavior of these systems is typically limited to a small parameter range and there is little, if any, connection between the chaotic dynamics and the underlying hydrodynamics.[4]

We will show that the flow in a convection loop heated from below is qualitatively described by the nonlinear dynamics of the Lorenz model[5] over a wide range of parameters encompassing three different chaotic regimes. In a future publication[6] we will show the connection between the parameters of the Lorenz model and the fluid dynamics parameters.

The flow in a rectangular loop of fluid heated at the bottom was first discussed by Welander[7] who showed that the flow would undergo oscillations which increased in amplitude until the flow reversed direction. Malkus and Howard[8] considered a circular loop of fluid subject to a uniform temperature gradient and showed that the resulting equations were isomorphic to the Lorenz equations. They made the simplifying assumptions that the velocity and temperature were uniform across the loop, that the fluid flow was opposed by a friction force, which is proportional to the instantaneous flow rate, and that the rate of heat transfer between the walls of the tube and the surrounding fluid was proportional to the temperature difference between wall and fluid. When the temperature of the fluid is expanded in a Fourier series in θ, the equations for the low modes decouple from those of the higher modes and the infinite set of equations can be truncated without loss of information.[9]

Creveling *et al.*,[10] in previous experiments with fluid loops using a constant heat flux over the bottom half and a constant temperature over the top half, observed a flow regime in which the fluid oscillated with increasing amplitude about the mean flow and then reversed direction and resumed oscillation. Both the analysis by Creveling *et al.*[10] and a subsequent one by Greif, Zvirin, and Mertol[11] identified a regime of unstable oscillations but did not characterize the dynamics of this regime or identify any other chaotic regimes. The boundary conditions corresponding to the experiment violate the assumptions which lead to exact truncation; however, we will show that the qualitative characteristics of the flow can be described by the nonlinear dynamics of the Lorenz model. The flow can be categorized into five regimes: (absolutely) stable, steady (clockwise or counterclockwise) circulation, transient, subcritical, and globally chaotic.[12]

Determination of flow regimes.—The flow in the convection loop can be represented by the motion of a point in an abstract three-dimensional state space whose axes are the fluid velocity and the sine and cosine Fourier components of the temperature of the fluid (x, y, and z axes, respectively, in Fig. 1). Trajectories in state space correspond to flows in real space.

The absolutely stable regime corresponds to a fixed point (point A in Fig. 1) in state space. No

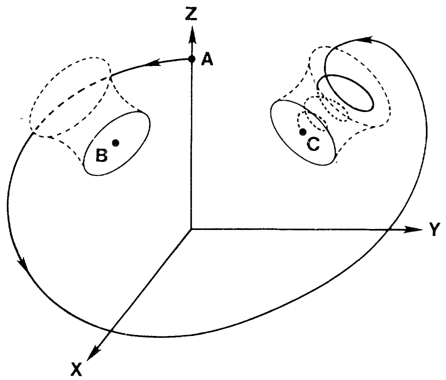

FIG. 1. Schematic diagram of a trajectory in state space corresponding to a transient state (see text).

fluid motion is observed and a steady temperature field is maintained in the fluid. The transition to steady circulation occurs when this state loses stability at a critical value of the driving parameter and two stable steady solutions (fixed points B and C in Fig. 1) bifurcate from it. This transition was detected experimentally by allowing the system to come to equilibrium in the absolutely stable regime and slowly increasing the heat flux until circulation was observed.

In the transient and subcritical regimes the fixed points are locally stable to small perturbations but unstable to finite-amplitude disturbances. The transient regime is characterized by states exhibiting nonperiodic motion which abruptly decay to steady circulation. This phenomenon arises in the Lorenz model when two unstable periodic solutions of large amplitude are born from a homoclinic orbit. Each periodic solution rings one of the stable solutions and its stable manifold effectively forms a barrier tube (dotted line in Fig. 1) for trajectories. There is a special trajectory[13] which all observable trajectories pass close to. In the transient regime this special trajectory passes through the barrier tubes. Other trajectories which begin outside of these tubes remain outside for a time, t_{ko},[14] called the kickout time. When these trajectories penetrate a tube, they spiral into a fixed point. The trajectory shown in Fig. 1 corresponds to a flow which reverses direction and undergoes three decaying oscillations before steady flow.

In the subcritical regime, the space of possible flows is divided into disjoint regions because the special trajectory has moved completely outside the barrier tubes. Trajectories which start outside the tubes stay outside and are chaotic. Trajectories which are inside a tube will decay to steady circulation. Experimentally the transiton between the transient and subcritical regimes was determined by *abruptly* applying the heat flux and finding the lowest value at which sustained chaotic behavior is observed.

As the driving parameter is increased, the unstable periodic solutions shrink and the barrier tubes enclose a smaller region of phase space. The loss of stability of the steady solution occurs when each unstable periodic solution merges with the steady solution it surrounds. This transition corresponds to a subcritical Hopf bifurcation of the steady solution. The transition from the subcritical to the globally chaotic regime was measured experimentally by *slowly* increasing the heat flux until steady circulation became unstable and was replaced by a chaotic flow.

The experimentally determined flow regimes are as follows: $Q < Q_c$, stable; $Q_c < Q < 4.1Q_c$, steady (clockwise or counterclockwise); $4.1Q_c < Q < 8.1Q_c$, transient; $8.1Q_c < Q < 14.5Q_c$, subcritical; $Q > 14.5Q_c$, globally chaotic.[15]

Experiment.—The experimental apparatus was constructed to reproduce the results of Creveling *et al.*[10] A loop, 76 cm in diameter, made from 2.5-cm Pyrex tubing, was encased with a Plexiglas jacket over the top half and wrapped first with heating tape and then with asbestos tape (to reduce heat loss) over the bottom half. Water from a temperature-controlled bath (± 0.05°C) was circulated through the jacket; a Variac provided the constant heat flux over the bottom half of the loop. Thermistors with 1-sec response times were placed at three, six, and nine o'clock. The temperature at six o'clock, T_6, is proportional to the cosine Fourier coefficient of the temperature, and the temperature difference between nine o'clock and three o'clock, $T_9 - T_3$, is proportional to the sine Fourier coefficient of the temperature. The output from the thermistors was digitized by a 12-bit analog-to-digital converter, stored in a microcomputer, and later sent to a mainframe where it was digitally filtered and analyzed. The working fluid was water.

Each chaotic flow regime has characteristics which can be compared with the predictions of nonlinear dynamics.

Globally chaotic regime.—Figure 2 shows representative traces of $T_9 - T_3$ and T_6 in the globally chaotic regime. A number of techniques have been developed in recent years to analyze and characterize chaotic flows. Lorenz[5] plotted the amplitude of the Nth maximum versus the amplitude of the $(N+1)$st maximum and obtained a one-dimensional cusp-shaped map which characterized the dynamics. A topologically equivalent map can

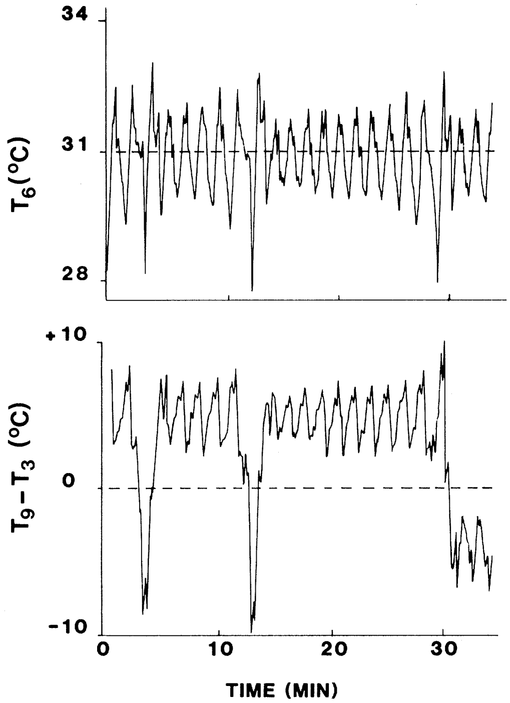

FIG. 2. Representative time series of voltages proportional to the sine and cosine Fourier coefficients of the temperature. A change in sign of T_9-T_3 corresponds to a flow reversal.

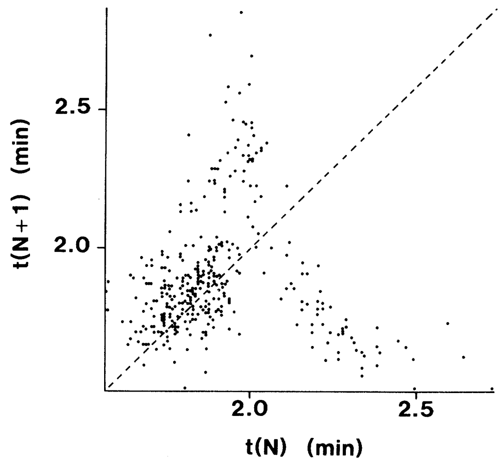

FIG. 3. A plot of $t(N+1)$ vs $t(N)$ where $t(N)$ is the time between the Nth and $(N+1)$st maxima in the time series of T_6.

be obtained by plotting the *time* between the Nth and $(N+1)$st maxima versus the *time* between the $(N+1)$st and $(N+2)$nd maxima. If this plot yields a quasi one-dimensional map, then the time of the next maximum can be found by iterating the map. In Fig. 3 we have plotted the experimentally observed times between maxima measured by T_6. Although there is a considerable scatter, the points fall on a one-dimensional cusp-shaped curve similar to that of the Lorenz model. The scatter on the right-hand side of the map is caused by contributions from two- and three-dimensional aspects of the flow which are not included in the simplified model. The larger scatter on the left-hand side of map is caused by the presence of two closely spaced branches which are not resolved. These aspects of data analysis will be discussed more extensively in Ref. 6. Maps constructed from time series in the transient and subcritical regimes have a similar cusp shape but differ in certain details.

Subcritical regime.—In the subcritical regime the probability of observing a chaotic flow, P_c, increases as the driving parameter is increased. Experimental values of P_c were obtained for three values of the (abruptly applied) heat flux; 0.05 at $Q=8.9Q_c$, 0.50 at $Q=9.5Q_c$, and 0.90 at $Q=11.1Q_c$.

Transient regime.—If the initial state of the system is chaotic and the heat flux is abruptly switched, changing the state of the system into the stable regime, the system decays immediately. If the initial state of the system is chaotic and the heat flux is abruply switched changing the state of the system into the transient regime, $4.1Q_c < Q8.1Q_c$, the system can remain chaotic for a time, $t_{\rm ko}$, and then will decay. The kickout time has a distribution of values for an ensemble of identical experiments. Our experiments indicate that the mean kickout time is 20 min and the probability of a given kickout time decreases monotonically from zero time in qualitative agreement with the numerical studies of Yorke and Yorke.[16]

The ideas of nonlinear dynamics have been used to establish experimental criteria for describing the characteristics of the flow in a convection loop over five regimes: stable, steady, transient, subcritical, and globally chaotic. Transient and subcritical regimes have not previously been identified in hydrodynamic systems.

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