Order of the Finite-Temperature Phase Transition in the SU(4) Gauge Theory

Ghassan George Batrouni and Benjamin Svetitsky
Newman Laboratory for Nuclear Studies, Cornell University, Ithaca, New York 14853
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Using Monte Carlo methods, we conclude that the SU(4) gauge theory has a first-order confinement transition at finite temperature. On a lattice with small timelike extent, the bulk transition of the Wilson action drives the finite-temperature transition. We used an action with plaquettes in the fundamental and adjoint representations in order to avoid the bulk transition while keeping the lattice size manageable.

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The first-order nature of the finite-temperature phase transition in the four-dimensional SU(3) gauge theory[1] [in contrast[2] to SU(2)] can be understood in several ways. The original arguments, based on Landau theory[3] or on the renormalization group,[4] depended on the invariance of the cube of the order parameter under a global Z_3 symmetry. Among SU(N) theories, then, these arguments treated $N=3$ as a special case: They made *no* predictions for the order of the transition for $N\neq 3$.

More recently, it has been argued[5] that large-N theories possess first-order transitions. It is intriguing to suppose that $N=3$ is the border of the large-N regime, i.e., that all theories with $N\geqslant 3$ behave in accord with the large-N predictions. Calculations[6] based on mean-field theory in the strong-coupling approximation support this hypothesis, as does a model[7] of the phase transition which likens it to a percolation transition for $N\geqslant 3$. We have obtained Monte Carlo data for the SU(4) theory which fit this picture.

Our calculation was done on a lattice with timelike and spacelike dimensions $N_t=2$ and $N_s=5$. Study of the finite-temperature transition on such a small lattice is complicated by the presence of a bulk phase transition for the Wilson action[8] at $\beta_f\approx 10.2$. As discussed by McLerran and Svetitsky,[9] the finite-temperature transition occurs when the confinement length ξ, set by the string tension, crosses the scale set by the inverse temperature T^{-1}, which is N_t in lattice units. While the bulk transition has nothing to do with confinement, the confinement length does change discontinuously there; if the discontinuity makes ξ cross T^{-1}, the bulk transition will bring on a first-order confinement transition. We find that this is indeed the case when $N_t=2$.

The trouble is that such a bulk-driven confinement transition may have nothing to do with the behavior of the system in the continuum limit. As one studies systems with increasing N_t, the finite-temperature transition will move toward weaker coupling so as to yield a finite physical transition temperature. The bulk transition, on the other hand, will stay at finite bare coupling, and the two transitions will decouple. The finite-temperature transition, when it is no longer driven by the bulk transition, may then become continuous, and remain so in the continuum limit. It is this possibility which must be investigated.[10]

In order to avoid the bulk transition of the Wilson action, we have used the more general mixed fundamental-adjoint action. In the next section, we review its zero-temperature phase diagram and display our conjecture of how the finite-temperature transition is superimposed on it. Thereafter, we present Monte Carlo data for the Wilson theory, which show that the bulk transition drives the finite-temperature transition. We then present data for a mixed action, which exhibits a clear first-order confinement transition in the absence of the bulk transition. We add some remarks in the final section, and our calculation is described in an appendix.

Phase diagram of the mixed model.—The mixed action for the SU(N) gauge theory is given by the usual sum over plaquettes

$$S=-\sum_p\left\{\frac{\beta_f}{N}\mathrm{Re}\,\mathrm{tr}U_p+\frac{\beta_A}{N^2-1}|\mathrm{tr}U_p|^2\right\}. \qquad (1)$$

Based on study of the mixed-action SU(2) and SU(3) theories,[11,12] the mixed-action large-N theory,[13] and the Wilson-action SU(4) theory,[8] the phase diagram consisting of the solid lines in Fig. 1 has emerged for the SU(4) theory at $T=0$. As discussed in the introduction and demonstrated in the next section, the discontinuity in the confinement length ξ where the β_f axis crosses the phase boundary is sufficiently large to bring about the finite-temperature transition when $N_t=2$. As one follows the phase boundary towards the critical point, this discontinuity should decrease, until the two phase transitions decouple as indicated in the figure. The

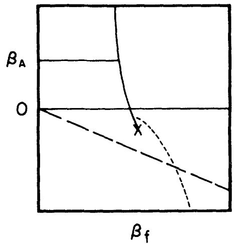

FIG. 1. Presumed phase diagram for the mixed-action SU(4) theory on an $N_t = 2$ lattice. Solid lines are bulk first-order transitions, ending in a critical point. The dotted line is the conjectured curve of the finite-temperature transition where it is decoupled from the bulk transition. The dashed line is $\beta_A = -\beta_f/2$.

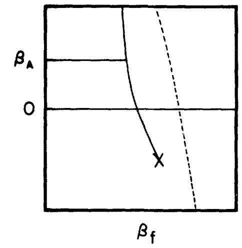

FIG. 2. As in Fig. 1, but with N_t large.

bulk phase boundary ends at the critical point, while the confinement phase boundary should continue in the negative β_A direction, roughly along one of the curves of constant string tension discussed by Bhanot and Dashen.[14]

If we were to increase N_t, we should see the confinement transition line moving to the right, until for sufficiently large N_t the two transitions would decouple entirely and the situation would be as shown in Fig. 2. In the interest of faster computation, we have chosen to avoid the bulk transition by working along the dashed line in Fig. 1, where $\beta_A = -\beta_f/2$. According to Ref. 13, this line should bypass the critical point, and we show in the next section that the bulk transition does not appear in the region of interest.

Monte Carlo data.—The order parameter for the finite-temperature phase transition is the Wilson line

$$W_{\vec{n}} = \mathrm{tr} \prod_{n_0=1}^{N_t} U^0_{n_0, \vec{n}}. \tag{2}$$

We display in Fig. 3 our Monte Carlo data for the average plaquette

$$P = 1 - N^{-1} \mathrm{Re}\,\mathrm{tr}\, U_p \tag{3}$$

and for the mean square magnetization

$$W^2 = |N_s^{-3} \textstyle\sum_{\vec{n}} W_{\vec{n}}|^2 \tag{4}$$

in the neighborhood of the transition on the β_f axis. There is marked hysteresis in both observables. At $\beta_f = 9.8$ and $\beta_f = 10.0$, nonzero magnetization in $\langle W^2 \rangle$ is always associated with the lower values of $\langle P \rangle$, zero magnetization with the upper. At $\beta_f = 9.5$ and $\beta_f = 10.5$, we observed tunneling from the metastable to the equilibrium phase, in the course of which $\langle W^2 \rangle$ switched phases during the same passes as $\langle P \rangle$. These observations show that the bulk transition and the finite-temperature transition are identical on the β_f axis.

Proceeding to our calculations on the $\beta_A = -\beta_f/2$ line, we present in Fig. 4 data for $\langle P \rangle$ and $\langle W^2 \rangle$ near the finite-temperature transition. Rather than present hysteresis data, we show values for the equilibrium phases only, where these have been determined from the mixed-start runs shown in Fig. 5. These runs show clearly the first-order nature of the transition.

Finally, to show that the bulk transition is not present, we show in Fig. 6 the results of heating and cooling runs for an $N_t = 4$, $N_s = 4$ lattice in the neighborhood of the transition found for $N_t = 2$.

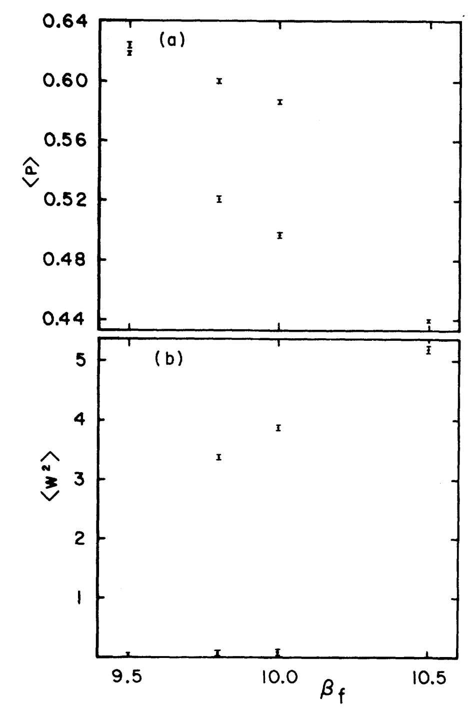

FIG. 3. (a) Average plaquette $\langle P \rangle$ and (b) magnitude of the order parameter $\langle W^2 \rangle$ along the β_f axis on a 2×5^3 lattice. Lower points in (a) and upper points in (b) are from runs with decreasing β_f; their counterparts are from runs with increasing β_f.

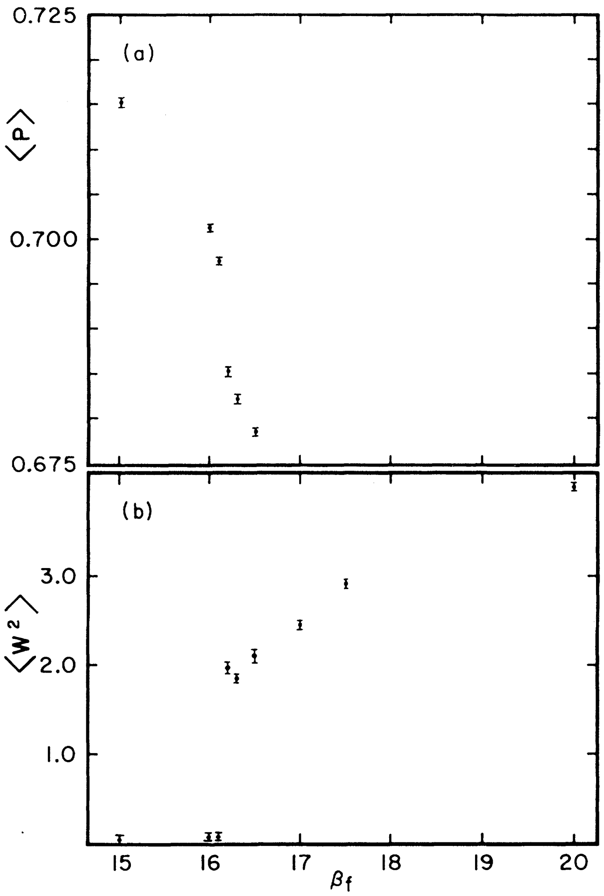

FIG. 4. (a) Average plaquette $\langle P \rangle$ and (b) magnitude of the order parameter $\langle W^2 \rangle$ along the line $\beta_A = -\beta_f/2$.

No hysteresis is evident, showing that no bulk transition was left behind when the finite-temperature transition indeed moved away to weaker coupling.

It may be argued that our first-order transition, while not coincident with the bulk transition, is affected by its proximity. Certainly we do not claim to have approached the continuum limit, where all effects of the bulk transition will be insignificant.[15] However, a change in the order of the transition as N_t is increased would be a very interesting effect in itself. If, in place of varying N_t, one were to make the gauge couplings anisotropic, a change in the order as one moved along the phase boundary would occur at a tricritical point. The tricritical exponents would not fit into the simple picture of Ref. 4, and would presumably be due to competing interactions in the effective action for the Wilson line.

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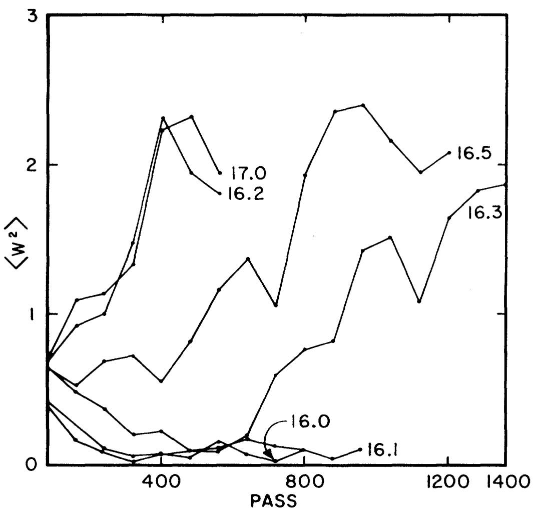

FIG. 5. Mixed-start runs near the phase transition in Fig. 4. Values of β_f are indicated.

Appendix.—Our computer program employed a Metropolis algorithm, with ten hits per link per pass and an acceptance between 0.5 and 0.6. The data in Figs. 3 and 4 are averages over between 200 and 400 passes after reaching equilibrium. For the heating run (that with decreasing β_f) in Fig. 6, we started at $\beta_f = 16.5$ with 600 passes from a cold start, and then ran 100 passes at each β_f value, averaging over the last 50; the cooling run (that with increasing β_f) was similar.

The program for the pure Wilson action was written in FORTRAN and C and ran on a VAX-11/750; it did ten hits on a link in 250 ms. The program for mixed action ran on a Floating Points Systems FPS-164 Array Processor, and did ten hits on a link in 8.25 ms; it was written in FORTRAN and used library subroutines for matrix multiplication. The mixed-action algorithm was of course slower than that for the Wilson action; in comparing runs for the same algorithm we found that the Array Proces-

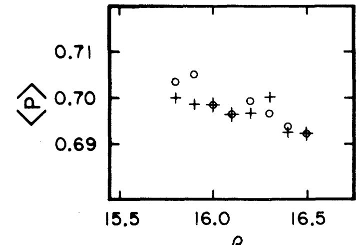

FIG. 6. Heating (crosses) and cooling (circles) runs on a 4×4^3 lattice with $\beta_A = -\beta_f/2$.

sor was 50 times as fast as the VAX.

Note added.—Papers have reached us from Gocksch and Okawa[16] and from Wheater and Gross[17] which report a first-order transition for the fundamental action with $N_t = 4$, where it appears that the two transitions have decoupled as in Fig. 2.

[1]J. Kogut *et al.*, Phys. Rev. Lett. **50**, 393 (1983); T. Celik, J. Engels, and H. Satz, Phys. Lett. **125B**, 411 (1983); B. Svetitsky and F. Fucito, Phys. Lett. **131B**, 165 (1983).

[2]L. McLerran and B. Svetitsky, Phys. Lett. **98B**, 195 (1981), and Phys. Rev. D **24**, 450 (1981); J. Kuti, J. Polonyi, and K. Szlachanyi, Phys. Lett. **98B**, 199 (1981).

[3]B. Svetitsky, Nucl. Phys. A **418**, 477c (1984).

[4]L. Yaffe and B. Svetitsky, Phys. Rev. D **26**, 963 (1982); B. Svetitsky and L. Yaffe, Nucl. Phys. **B210** [FS6], 423 (1982).

[5]A. Gocksch and F. Neri, Phys. Rev. Lett. **50**, 1099 (1983), and New York University Report No. NYU/TR1/83, 1983 (unpublished). But see also H. Neuberger, Nucl. Phys. **B220** [FS8], 237 (1983); R. Pisarski, University of California, Santa Barbara Report No. NSF-ITP-83-71, 1983 (unpublished); S. R. Das and J. B. Kogut, Fermilab Report No. FERMILAB-Pub-84/14-T, 1984 (unpublished); F. Klinkhamer, in Proceedings of the Argonne Workshop on Lattice Gauge Theory, April 1984 (to be published).

[6]F. Green and F. Karsch, CERN Report No. TH.3748, 1983 (unpublished); M. Gross and J. Wheater, unpublished, and references therein; M. Ogilvie, Phys. Rev. Lett. **52**, 1369 (1984).

[7]A. Patel, Caltech. Report No. CALT-68-1073, 1983 (unpublished).

[8]M. Creutz, Phys. Rev. Lett. **46**, 1441 (1981); K. Moriarty, Phys. Lett. **106B**, 130 (1981); D. Barkai, M. Creutz, and K. Moriarty, Nucl. Phys. **B225** [FS9], 156 (1983).

[9]L. McLerran and B. Svetitsky, Phys. Rev. D **24**, 450 (1981).

[10]This point seems to have been missed by the authors of Ref. 6, and casts doubt on the validity of their results. Their strong-coupling approximations are relevant to the phase transition only if it occurs in the strong-coupling region, and this will be true only for small N_t. Small N_t, however, is precisely when the bulk transition will interfere with the finite-temperature transition and drive it first-order. For sufficiently small N_t, the finite-temperature transition may move to the strong-coupling side of the bulk transition.

[11]G. Bhanot and M. Creutz, Phys. Rev. D **24**, 3212 (1981); R. Brower, D. Kessler, and H. Levine, Phys. Rev. Lett. **47**, 621 (1981), and Nucl. Phys. **B205** [FS5], 77 (1982); R. Dashen, U. Heller, and H. Neuberger, Nucl. Phys. **B215** [FS7], 360 (1983).

[12]G. Bhanot, Phys. Lett. **108B**, 337 (1982).

[13]Yu. Makeenko and M. Polikarpov, Nucl. Phys. **B205** [FS5], 386 (1982).

[14]G. Bhanot and R. Dashen, Phys. Lett. **113B**, 299 (1982).

[15]In fact, one cannot take the continuum limit along the $\beta_A = -\beta_f/2$ line, because the $U^\mu = 1$ saddle point is unstable for $\beta_A < -15\beta_f/32$. (Perturbation theory then gives $1/g^2 < 0$.) One has to tune the bare couplings along a curve which eventually moves above the $1/g^2 = 0$ line in order to reach the conventional continuum theory. This $1/g^2 = 0$ line, however, has no significance outside the weak-coupling limit, and we would not expect the order of the finite-temperature transition to change as one crosses it.

[16]A. Gocksch and M. Okawa, Phys. Rev. Lett. **52**, 1751 (1984).

[17]J. Wheater and M. Gross, to be published.