Solitons versus Parametric Instabilities during Ionospheric Heating

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Ionospheric heating is studied by numerical solution of the modified Zakharov equations for typical ionospheric heating parameters. It is shown that modulational instability followed by soliton formation can be more important than the previously considered parametric decay instability.

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Since before 1970 the ionosphere has been heated from the ground by radio transmitters operating at 1–10 MHz with powers in the hundreds of kilowatts to one megawatt range (see Fejer[1,2] and Gurevich[3] as well as the November 1974 issue of Radio Science and the December 1982 issue of Journal of Atmospheric and Terrestrial Physics). The frequency is usually adjusted so that it is below the ionospheric cutoff frequency and thus the heating wave is reflected. Near the reflection region, the electric field becomes quite intense and a substantial heating of electrons takes place, primarily through the collisions of electrons with neutral atoms.

In addition to the collisional heating of electrons the experiments indicate several anomalous effects, such as the production of intense levels of high-frequency Langmuir waves and the production of fast electrons. If was suggested[4] in 1971 that these anomalous effects could be due to a class of nonlinear processes called parametric instabilities which convert the heating-wave energy into plasma oscillations in a range of wave numbers for both high frequencies near the heating frequency (electron plasma frequency) and low frequencies below the ion plasma frequency.

A number of theoretical and experimental investigations have lent support to this idea, but many questions remain. The theoretical investigations have concentrated[5-7] on the three-wave parametric instability, which saturates nonlinearly through a repeated cascade of Langmuir wave energy to lower wave numbers. There has been less work on the modulational instability,[8-11] which saturates nonlinearly through the formation of solitons.

In this paper we investigate the competition among these various effects by numerically solving the modified Zakharov equations[8,12]

$$i\partial_t E(x,t) + i\nu_e E + \partial_x^2 E = nE + nE_0 \exp(-i\Delta\omega t) - \langle nE \rangle, \quad (1)$$

$$(\partial_t^2 + 2\nu_i \partial_t - \partial_x^2) n = \partial_x^2 [\,|E|^2 + E_0 E^* \exp(-i\Delta\omega t) + E_0^* E \exp(i\Delta\omega t)\,], \quad (2)$$

where dimensionless variables are used as defined in Ref. 9. The constant E_0 is the dimensionless form of the heater electric field amplitude, $E(x,t)$ is the slowly varying (in time) amplitude of the high-frequency electric field, and $n(x,t)$ is the slowly varying density deviation. The x direction is along the geomagnetic field near the exact reflection point of an ordinary-mode[13] heater wave launched vertically from the ground, and is thus along the electric field of the heater wave in this spatial region. The symbol $\langle nE \rangle$ means the spatial average of nE; the plasma is treated as though it were homogeneous in the x direction. Note that the definition of the high- and low-frequency damping coefficients ν_e and ν_i used in Ref. 9 differs by a factor of 2 from that used here. The symbol $\Delta\omega$ is the dimensionless form of $\omega_0 - \omega_e$, the difference between the heater frequency and the local electron plasma frequency. The value of E_0 is held fixed throughout the calculation. Holding E_0 fixed in time is a somewhat crude model of the sum of two competing effects, the depletion of E_0 due to the growth of the internal high-frequency electric field $E(x,t)$, and the replenishment of E_0 due to the flow of energy from the transmitter. Equations (1) and (2) are supplemented by the condition that the spatial average of $E(x,t)$ vanishes for all time. The high-frequency damping coefficient ν_e represents[9] collisions of electrons with ions and neutrals; in physical units $\tilde{\nu}_e/\omega_e = 2\times 10^{-5}$. The low-frequency damping operator ν_i is chosen such that in spatial Fourier space the k component of $\nu_i n(x,t)$ is $\nu_i(k) n(k,t) = |k|\, n(k,t)$. This model is intended to reproduce qualitatively the effect known as induced scattering in plasma with equal electron and ion temperatures and has been used in previous studies of strong Langmuir turbulence.[9,14-16] The basic modified Zakharov equations (1) and (2) have been solved elsewhere[17,18] in other contexts. A detailed justification for the use of this model in the present context, together with the associated features just discussed, will be presented in a related publication.[19]

Our earlier work[9,10] along the present lines was appropriate to the exact reflection point of the heating wave (the point $z=0$ in Fig. 1 of Ref. 9), where $\Delta\omega=\omega_0-\omega_e=0$. At that location, there can be no parametric decay instability and our two-dimensional numerical solution of equations similar to (1) and (2) demonstrated that the dominant effect was modulational instability followed by soliton formation and collapse. By contrast, the present paper considers the spatial location of the maximum electric field amplitude of the reflected heating wave (the Airy maximum shown in Fig. 1 of Ref. 9), some one or two hundred meters below the exact reflection point. At this location, $\Delta\omega=\omega_0-\omega_e>0$ and both kinds of parametric instability are possible.

As parameters typical of the ionospheric modification facilities at Arecibo, Puerto Rico, and Platteville, Colorado, we choose most of those used in Refs. 9 and 10, namely electron density 3×10^5 cm^{-3}, electron and ion temperature 0.1 eV, ionosphere density scale length 50 km, heater frequency $\omega_0=2\pi\times5\times10^6$ s^{-1}, ion-to-electron mass ratio 16 $\times1836$ (O^+ ions), and maximum heater electric field strength 2 V/m. These imply a frequency difference $(\omega_0-\omega_e)/\omega_e=1.7\times10^{-3}$ occurring at the Airy maximum[13] 170 m below the exact reflection point.

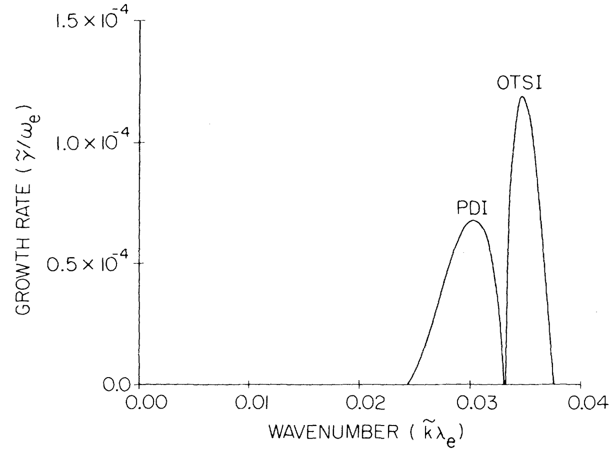

FIG. 1. Positive growth rates for the linearized Zakharov equations as a function of the wave number, $\tilde{k}$. Since the dispersion equation is an even function of the wave number, only the growth rates for positive values of $\tilde{k}$ are shown. The portion of the curve marked OTSI corresponds to the oscillating two-stream instability which is a purely growing mode. The portion marked PDI corresponds to the parametric decay instability which has a real frequency.

If we assume that the initial state consists of the pump (heater) wave plus small high- and low-frequency fluctuations at all wave numbers, the standard parametric instability analysis[20] leads to the dispersion relation

$$[\omega^2+2i\nu_i(k)\omega-k^2][(\omega+i\nu_e)^2-(\Delta\omega-k^2)^2]+2k^2|E_0|^2(\Delta\omega-k^2)=0, \tag{3}$$

where (ω,k) are associated with the low-frequency density fluctuation. Equation (3) is solved numerically with the parameters discussed above, and the positive growth rates $\mathrm{Re}[\omega(k)]>0$ are shown in Fig. 1. It is clear that the branch marked OTSI (for oscillating two-stream instability) exhibits a higher growth rate than the branch marked PDI (for parametric decay instability).

To obtain the fully nonlinear evolution we solve (1) and (2) numerically. The additional terms in (1) and (2) are easily incorporated into our numerical algorithm[21] for solving the usual Zakharov equations.

In order to obtain accurate solutions of the Zakharov equations for the parameters associated with the ionosphere one must use many Fourier components. There are two reasons for this: (1) The narrow range of growing modes requires that we use a value of the length L of the system such that a significant number of modes lie in the range corresponding to the two instabilities; (2) during the nonlinear evolution of the equations modes with a large wave number become important. For the results shown in this paper we used 8192 Fourier components for a system with a length of 1.8 $\times10^4\lambda_e$. After the aliasing terms have been removed, this corresponds to maximum wavenumber of $\tilde{k}\lambda_e=0.93$, where we use a tilde to designate a physical variable.

For our numerical investigation of the time evolution of the ionosphere during the heating process, we solve equations (1) and (2) with small random initial fluctuations for the electric field and the density variations. During the initial stage all of the modes were observed to be damped except those with a positive growth rate shown in Fig. 1. During the early linear stage both the modes corresponding to the modulational instability and the parametric decay instability had the growth rates shown in Fig. 1. As the electric field and the density variations became larger the nonlinear effects became more

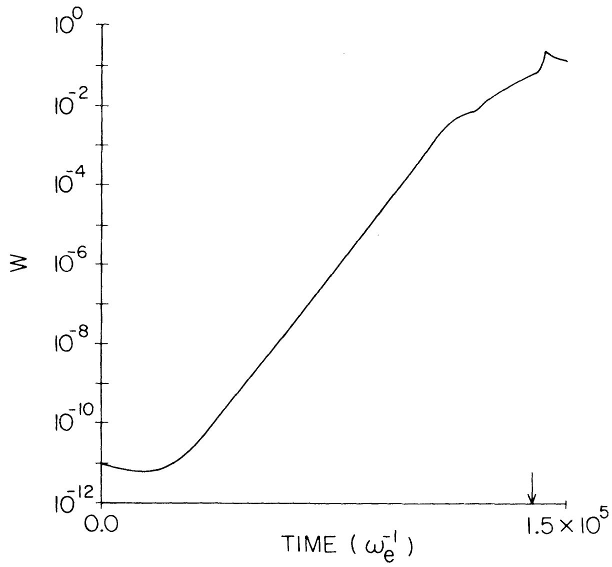

FIG. 2. Value of W as a function of time for $0 \leqslant \tilde{t} \leqslant 1.5 \times 10^5 \omega_e^{-1} = 5.0$ ms. The electric field and the density variation are shown in Fig. 3 at the time indicated by the arrow.

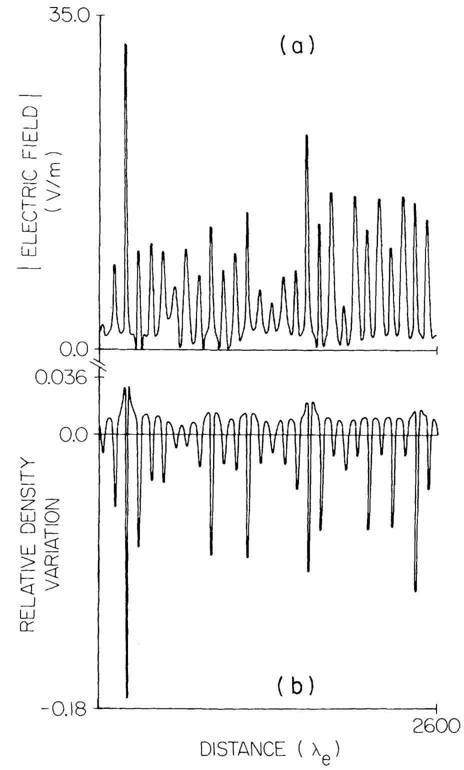

FIG. 3. (a) Absolute value of the electric field at $\tilde{t} = 1.4 \times 10^5 \omega_e^{-1} = 4.7$ ms. Only the values for $\tilde{x} \leqslant 2600 \times \lambda_e = 11.0$ m are shown. (b) Relative density variation for the ions at $\tilde{t} = 1.4 \times 10^5 \omega_e^{-1} = 4.7$ ms.

important. These effects led to the saturation of the field and the formation of solitons.

To demonstrate these time-dependent effects we plot in Fig. 2 the time dependence of $W \equiv \langle |\tilde{E}|^2 \rangle / 4\pi n_0 T$. From Fig. 2 one can see that initially W grows exponentially with a growth rate approximately equal to the maximum growth rate for the modulational instability. A detailed study of the Fourier components of the fields shows that because of the difference in growth rates for the two instabilities the Fourier components of the parametric decay instability soon become insignificant compared to those of the modulational instability.

Also, one finds that as W increases the linear growth rate eventually stops and the solution saturates. This behavior is coincident with the formation of solitonlike features in the fields. To show this structure we plot in Fig. 3 $|\tilde{E}(\tilde{x})|$ and $\tilde{n}(\tilde{x})$ for $\tilde{t} = 1.4 \times 10^5 \omega_e^{-1}$ (marked by the arrow in Fig. 2). For a soliton one finds a density depression resulting from the ponderomotive force in the region where $\tilde{E}$ is large. A comparison of Figs. 3(a) and 3(b) shows that in those regions where $|\tilde{E}|$ is large one finds a corresponding depression in the ion density (as indicated by negative values of the relative density variation).

Once saturation has occurred, this solitonlike structure persists. The intense electric fields associated with the solitons can accelerate electrons, thus resulting in the production of fast electrons. This process will be studied in a future paper.

We conclude that for some typical ionospheric parameters the modulational instability is more important than the parametric decay instability in the spatial region of strongest heater electric field, as tacitly assumed in the original work on this subject.[4] The modulational instability leads to the formation of solitons, as originally predicted by Petviashvili.[22] This scenario invalidates the concept of nonlinear saturation via repeated scattering of Langmuir waves, which, however, may occur for parameters other than those considered here. There are also other effects, such as filamentation instabilities,[1-3] stimulated diffusion scattering,[1-3] and mode conversion,[23] which can coexist and compete with the effects considered here.

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