Nonlinear Interaction of Convective Cells in Plasmas

H. L. Pécseli, J. Juul Rasmussen, and K. Thomsen
Physics Department, Association EURATOM–Risø National Laboratory, Risø, DK-4000 Roskilde, Denmark
(Received 29 February 1984)

The nonlinear interaction of externally excited convective cells was investigated experimentally. Two cells of the same polarity were observed to coalesce into one large cell provided their relative distance was sufficiently short. The nonlinear nature of the interaction was explicitly demonstrated. Two cells of opposite polarity interact through a mutual perturbation of orbits.

PACS numbers: 52.35.Mw, 52.35.Fp, 52.35.Ra

A flutelike convective cell can be visualized as a slight excess of charge in a magnetic flux tube giving rise to an electrostatic field.[1] The plasma particles flow along equipotential contours by the cross-field $\vec{E}\times\vec{B}/B^2$ convection velocity. Because of the assumption of strictly $\vec{B}$-field elongated structures the net charge cannot be neutralized by particle flow along field lines. Electrostatic convective cells are thus characterized by a significant potential perturbation with a corresponding very small density perturbation. A linear analysis for homogeneous ideally two-dimensional plasmas demonstrates that these modes are stationary (i.e., nonpropagating) and slowly damped by ion viscosity. A simple analysis of the *nonlinear* properties of convective cells can be based on the inviscid two-dimensional Navier-Stokes equation

$$(\partial_t + \vec{u}\cdot\nabla_\perp)\rho = 0, \tag{1}$$

where $\vec{u} = -\nabla_\perp\phi\times\vec{B}/B^2$ is the cross-field convection velocity and $\rho = e(n_i - n_e)/\epsilon_0 = -\nabla_\perp^2\phi$ is the charge separation associated with the cells. The electrostatic potential ϕ plays the role of a stream function and contours of constant ϕ at fixed t are plasma stream lines. The effect of finite Larmor radii and collisional interactions are ignored in this simple model. Equation (1) was solved[2] for a variety of initial conditions, in a finite domain of a two-dimensional space. It was found that an initial distribution of many vortices with both positive and negative polarities evolved into a pair of large counterrotating vortices, thus demonstrating a coalescence or condensation of all vortices having the same polarity while counterrotating vortices interact only through a mutual perturbation of orbits. A simple qualitative argument for the difference in behavior of the cells depending on their relative polarity can be given by reference to Fig. 1. It seems almost self-evident that the situation shown schematically in Fig. 1(a) allows the transformation of two cells into one by cancellation of the two opposite directions of plasma flow between the two cells. For two cells of opposite polarity [Fig. 1(b)] the corresponding flows are enhanced and a coalescence is implausible. The results of Ref. 2 refer to the situation described by Eq. (1) where both energy and enstrophy (i.e., mean-square vorticity) are conserved for this simple nondissipative equation. A laboratory experiment, to be described in the following, was carried out in order to investigate the interaction of convective cells in physically realistic conditions, where damping, finite Larmor radii, plasma inhomogeneities, boundaries, etc., cannot be ignored. The first evidence for vortex coalescence (or condensation) of externally excited convective cells is reported here.

The experiment was carried out in a linear plasma column produced by surface ionization of cesium on a 3-cm-diam hot (2200 K) tantalum plate, in a single-ended Q machine. The length of the plasma column was 125 cm, plasma densities were in the range 10^{10}–10^{11} cm^{-3}, and temperatures $T_i \approx T_e \approx 0.2$ eV were determined by the hot plate. Neutral pressures were below 10^{-5} Torr. The plasma column was confined radially by a magnetic field (variable in the range 0.2–0.6 T), and terminated by a cold metal plate of diameter 10 cm. Convective cells of positive polarity were excited in the

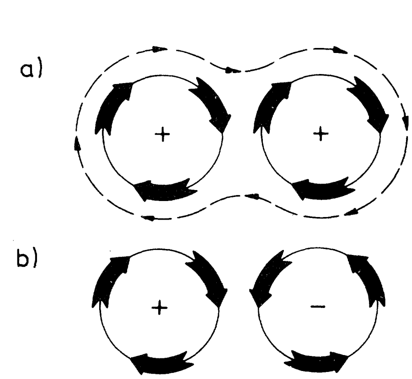

FIG. 1. Plasma $\vec{E}\times\vec{B}$ flow associated with interacting convective cells, shown schematically, for cells of (a) same polarity, and (b) opposite polarity.

residual plasma outside the main plasma column by controlling the end losses to small metal disks (diameter 8 mm) placed parallel to the cold end plate at a distance of 2 mm. Cells of negative polarity could subsequently be excited by the interaction of the positive cell and the plasma column. Sugai *et al.*[3] and Pécseli *et al.*[4] describe the excitation and basic characteristics of the convective cells in our setup. In the present experiment *two* cells could be excited independently by two exciters which can be positioned arbitrarily in the plane perpendicular to the plasma column.[3,4] Each exciter had a separate dc bias and pulse generator with variable time delay. The potential response in the plasma was mapped by a movable Langmuir probe in a plane perpendicular to the plasma column at a distance of 60 cm from the end plate (see Refs. 3 and 4 for details). The signals were sampled and averaged by a boxcar integrator and processed numerically.[5]

An isolated convective cell propagates as a result of the bulk $\vec{E}_r \times \vec{B}_0$ flow of the plasma and damps primarily because of end losses[3] and velocity shear.[4] We observed that *two* cells could move either independently or coalesce to one large cell. An example of the latter process is shown in Fig. 2 where equipotential contours for various times after the excitation are given. In Figs. 3(a) and 3(b) we show the angular positions and amplitudes of the cells before and after coalescence. The position is defined by the angle between a radial reference line and the line connecting the center of the plasma column with the maximum of the cell. An estimated azimuthal half-width is indicated in Fig. 3(a) by the vertical bars. The larger cell attracts and finally absorbs the smaller one. The interaction with the main plasma column is important for the overall energy and momentum conservation. By varying the distance between the two exciters we observed that two cells interacted only when the distance between them was less than their average diameter.

In order to demonstrate that the interaction is indeed *nonlinear,* we measured the potential in the plasma at a given time delay first with exciter 1 and then exciter 2 turned off. By numerically adding the two signals and subtracting them from the sig-

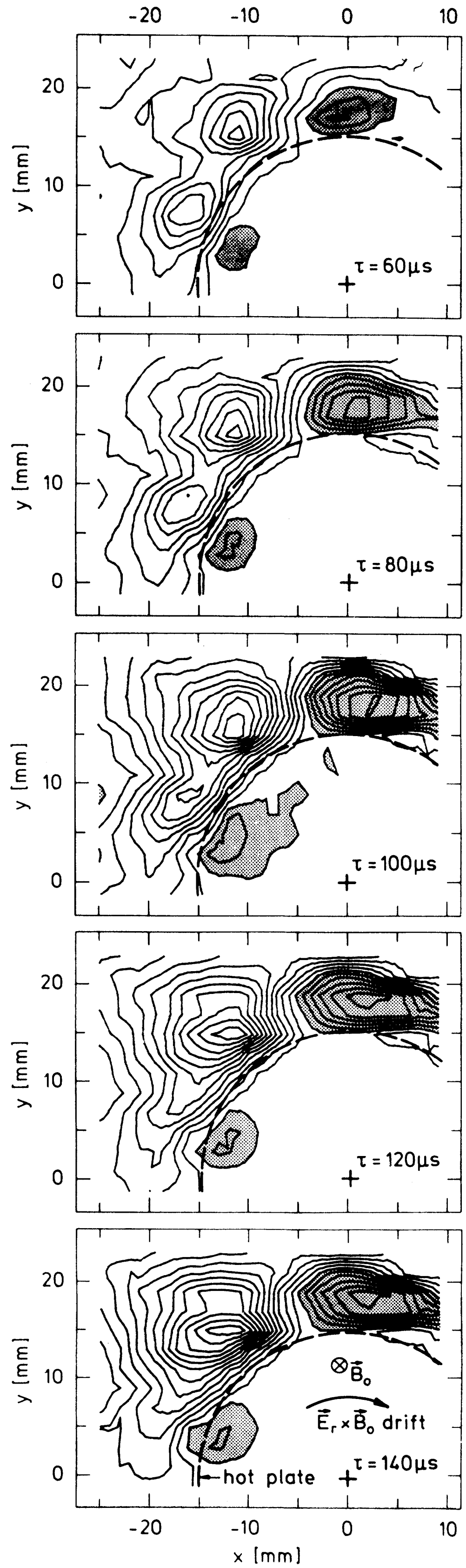

FIG. 2. Temporal evolution of two interacting cells measured at different times τ after the turn-on of the exciters. The pulse durations were 15 μs; amplitudes, 1 V. The potential difference between two adjacent contours is 4 mV. Negative potential regions are denoted by shading. The dashed arc shows the projection of the hot plate.

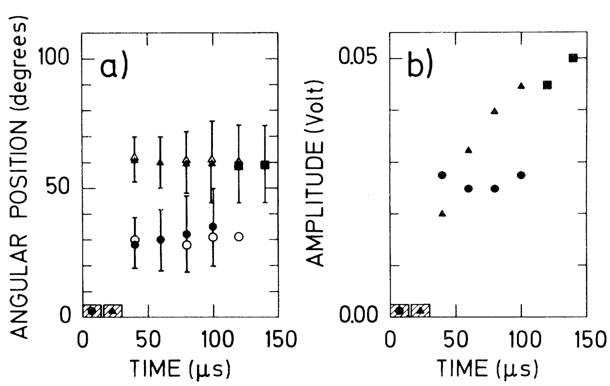

FIG. 3. Temporal evolution of two interacting convective cells, referring to Fig. 2. The two cells are marked by the closed circles and triangles, and after coalescence by the closed squares. The time durations and the delays of the pulses applied to the exciters are given by shaded rectangles. In (a) we give the angular positions at various times. The angle is positive in the $\vec{E}_r \times \vec{B}_0$ direction (see Fig. 2). The bars indicate an estimated azimuthal width of the cells. Open symbols refer to measurements of individually excited cells. In (b) we give the amplitude variation of the cells.

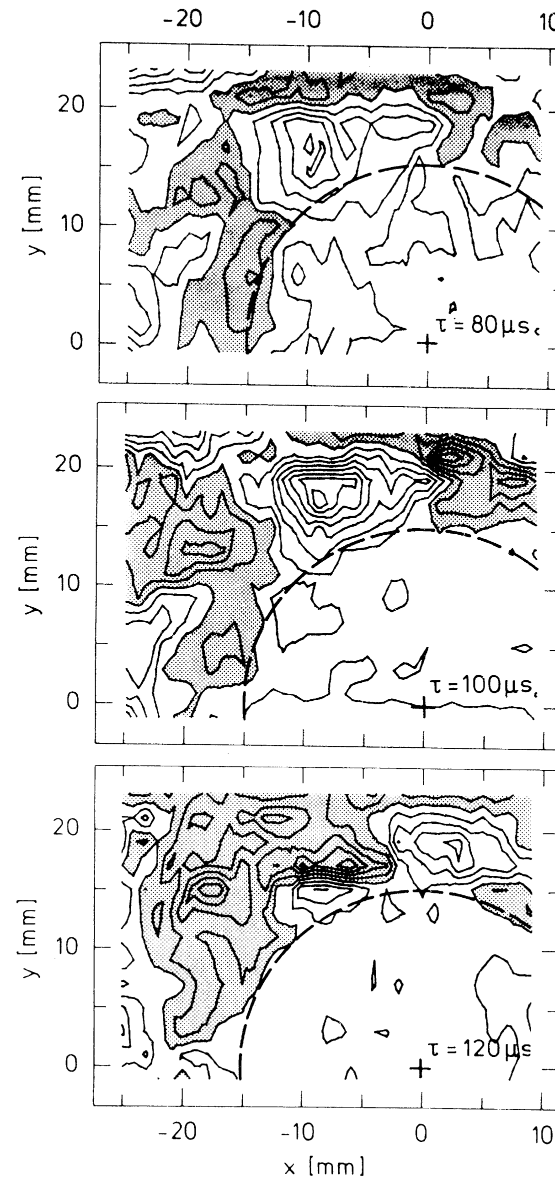

FIG. 4. Demonstration of the nonlinearity of the convective-cell interaction in Fig. 2 for different times. The figure is obtained by first measuring the potential contours associated with the two cells excited individually. The two resulting potential maps are numerically superimposed and subsequently subtracted from results given in Fig. 2. Thus shaded areas denote a *decrease* in potential induced by the nonlinear interaction. The potential difference between adjacent contours is 2 mV.

nal where *both* exciters are activated, we obtain results as shown in Fig. 4. Clearly in the case of simple *linear* superposition this figure would show only noise. Instead the figure explicitly demonstrates that the overall amplitude of the smaller cell is decreased (as evidenced by the shading, see Fig. 4 caption) while the larger cell shows an overall *increase* and expansion (contours *without* shading). The nonlinearity gives rise to a change in amplitude of up to 20% as compared with a simple linear superposition. In other cases where the two cells had almost identical amplitudes the nonlinear interaction gave rise to a "filling-in" between the two cells and a reduction of peak amplitudes as compared with independently excited cells. In such cases the coalesced cells were located roughly at the geometric mean position (see Fig. 5), while for cells of different amplitude generally the final stage is dominated by the larger one, as in Fig. 3. We have in no cases observed "breakup" of a large cell.

The interaction between cells of *opposite* polarity was also investigated. In these cases the only observable effect is a slight perturbation of orbits from the simple motion induced by the bulk plasma flow in the residual plasma.[3,4] Our experimental results are thus in good qualitative agreement with the numerical solutions of Eq. (1) in spite of all the simplifying assumptions inherent in its derivation.

The coalescence of essentially two-dimensional convective cells described in this work is important for the understanding of the evolution of two-dimensional turbulence.[2] For this case a dual energy cascade is expected on theoretical grounds. Vortex coalescence provides the physical mechanism for the cascade towards *long* wavelengths. Indica-

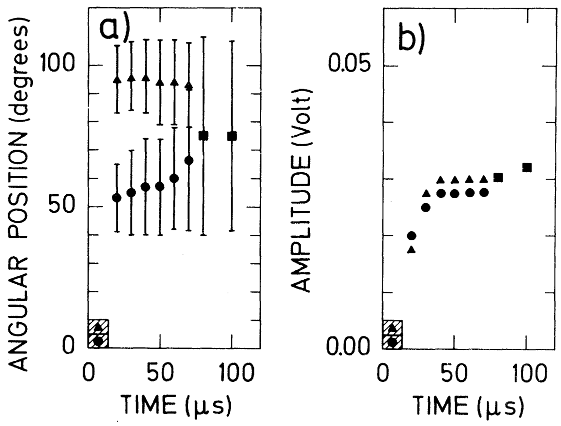

FIG. 5. Temporal evolution of two interacting convective cells of comparable size. Explanation of symbols as in Fig. 3.

tions for coalescence of convective cells were obtained in a levitated octupole.[6] However, in this case the cells were spontaneously excited and a direct observation of their interaction and evolution was not possible.

The authors thank H. Sugai for many stimulating discussions in the initial stages of these experiments on convective cells. The skilled technical assistance of M. Nielsen, B. Reher, and K.-V. Weisberg is gratefully acknowledged. Thanks is also given to J. P. Lynov and P. Michelsen for developing the computerized data processing facility.

[1]H. Okuda and J. M. Dawson, Phys. Fluids **16**, 408 (1973); P. K. Shukla, M. Y. Yu, H. V. Rahmann, and K. W. Spatschek, Phys. Rep. (to be published).

[2]C. E. Seyler, Y. Salu, D. Montgomery, and G. Knorr, Phys. Fluids **18**, 803 (1975); Y. Salu and G. Knorr, Plasma Phys. **18**, 769 (1976).

[3]H. Sugai, H. L. Pécseli, J. J. Rasmussen, and K. Thomsen, Phys. Fluids **26**, 1388 (1983).

[4]H. L. Pécseli, J. J. Rasmussen, H. Sugai, and K. Thomsen, Plasma Phys. (to be published).

[5]J. P. Lynov and P. Michelsen, Risø National Laboratory Report No. Risø-M-2393, 1983 (unpublished).

[6]A. Butcher Ehrhardt and R. S. Post, Phys. Fluids **24**, 1625 (1981).