Flatness of the Universe: Reconciling Theoretical Prejudices with Observational Data

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(Received 2 March 1984)

Theoretical prejudices argue strongly for a flat Universe; however, observations do not support this view. We point out that this apparent conflict could be resolved if the mass density of the Universe today were dominated by (i) relativistic particles produced by the recent decay of a massive, relic particle species, or by (ii) a relic cosmological constant. Scenario (i) has several advantages in the context of galaxy formation, but must confront the problem of a young Universe.

PACS numbers: 98.80.–k, 12.10.En, 14.60.–z, 98.80.Bp

Theoretical prejudice (specifically, the "naturalness" of the $k=0$ Einstein–de Sitter model[1]) strongly suggests that today Ω should be 1 (more precisely that $k=0$). Inflationary Universe scenarios[1–3] provide a means for ensuring that the curvature term is negligible today. Although the observational data on the precise value of Ω are far from being conclusive, the data suggest that $\Omega \simeq 0.1$–0.3 and none suggest a value as high as 1.[4] However, all of the methods used are only sensitive to mass which clumps on scales $\lesssim 100$ Mpc, and would not have revealed the presence of mass which is smoothly distributed out to scales $>> 100$ Mpc.[5] Thus it is possible to reconcile theory and observation if the matter which clumps (say, e.g., baryons) provides $\Omega \simeq 0.1$–0.3 and the additional mass density required in a $k=0$ cosmology is provided either by relativistic particles, which by virtue of their high speeds are necessarily smooth on all scales up to the present horizon, or by a relic cosmological term, which by definition is spatially constant. We shall explore both possibilities, giving considerably more attention to the former. In fairness we mention that all of the observational techniques for determining Ω rely upon the assumption that galaxies provide a good tracer of mass. If this very nontrivial assumption is not valid, then the discrepancy which we are trying to resolve may not exist at all!

In a $k=0$ Friedmann-Robertson-Walker cosmological model the evolution of the cosmic scale factor $R(t)$ (which we normalize so that $R_{\rm today}=1$) is governed by

$$H^2 \equiv (\dot{R}/R)^2 = 8\pi G\rho/3 + \Lambda/3, \qquad (1)$$

where H is the expansion rate, ρ the total energy density, and Λ the cosmological term. For convenience we use $\Omega_{\rm R}$, $\Omega_{\rm NR}$, and Ω_Λ to refer to the fraction of critical energy density contributed today by relativistic (R) particles, nonrelativistic (NR) particles, and the cosmological term; the critical density is $\rho_{\rm crit} \equiv 3H_0^2/8\pi G$, where $H_0 = 50 h_{1/2}$ km s^{-1} Mpc^{-1} is the Hubble parameter. Note that $\Omega_{\rm R} + \Omega_{\rm NR} + \Omega_\Lambda = 1$.

(i) *Universe dominated by R particles*[6] ($\Omega_R = 1 - \Omega_{NR} >> \Omega_{NR}$).—There are two immediate concerns with such a model: the age of the Universe,[7] $t_U \simeq H_0^{-1}/2 \simeq 10 h_{1/2}^{-1}$ Gyr, and the growth of the density perturbations necessary for galaxy formation. Determinations of H_0 suggest that $h_{1/2} \gtrsim 1$,[8,9] implying that t_U could be at most 10 Gyr. Although the ages of globular clusters and nucleocosmochronology suggest a "best value" for the age around 15 Gyr,[10] it has been argued that t_U could be as low as 10 Gyr.[11] At present systematic uncertainties preclude a precise determination of H_0; the lowest values reported do not exclude a value for H_0 as small as 30 km s^{-1} Mpc^{-1} (when a 3σ uncertainty is taken into account).[8] For[7] $t_U \simeq 0.55 H_0^{-1}$, and $H_0 = 30$–45 km s^{-1} Mpc^{-1}, $t_U = 18$–12 Gyr. The age problem is a serious one, and this scenario can be falsified if $H_0 t_U$ is shown to be $\gtrsim 0.55$.

Now consider the growth of density perturbations. Perturbations in a NR component do not grow significantly while the mass density of the Universe is dominated by R particles.[12] To avoid this difficulty we suppose that the Universe became radiation dominated only recently, as a result of the decay of a massive, relic species (denoted by X) into light ($m << 10$ eV) particles which are still relativistic today. Denote the epoch of X decay by $R \simeq R_D$ and $t \simeq t_D \simeq \tau_X$ (= lifetime of the X); as we shall see we will be interested in $R_D \simeq 0.1$–0.5. In such a scenario, as we shall show, there is no difficulty with the growth of density perturbations —they grow during the epoch in which the mass density of the Universe is dominated by NR X's. In order to avoid the very stringent constraints on the abundance of a relic species which decays radiatively, we will need to insure that the branching ratio to radiative channels is very small ($\lesssim 10^{-6}$).[13]

If we make the simple assumption that the NR component today is baryons, then through primordial nucleosynthesis[14] the observed abundances of the light elements constrain $\Omega_{\rm NR}$:

$$\Omega_{\rm NR} \simeq (0.042\text{–}0.14) h_{1/2}^{-2} \theta^3, \qquad (2)$$

where $\theta = T_\gamma/(2.7\ {\rm K})$ ($3.0\ {\rm K} \gtrsim T_\gamma \gtrsim 2.7\ {\rm K}$). For $h_{1/2} = 1$ (0.8), $\Omega_{\rm NR}$ can be as large as 0.19 (0.30) —large enough to account for the dark (and luminous) matter in galactic haloes, galaxy clusters, etc.[4]

In this scenario the Universe becomes matter dominated (by NR X's; $\rho_X \simeq \rho_\gamma + \rho_{\nu\bar\nu}$) at the epoch $R_{\rm eq} \simeq 1.4 \times 10^{-4} R_D (\Omega_{\rm R} h_{1/2}^2)^{-1} \theta^4$ and $T_{\rm eq} \simeq (1.7\ {\rm eV}) R_D^{-1} (\Omega_{\rm R} h_{1/2}^2) \theta^{-3}$, and radiation dominated again at the epoch $R = R_D$,

$$t_D \simeq \tau_X \simeq (4.1 \times 10^{17}\ {\rm sec}) R_D^2 (\Omega_{\rm R} h_{1/2}^2)^{-1/2}.$$

During the time the Universe is matter dominated, $R \simeq R_{\rm eq}$ to $R \simeq R_D$, density perturbations grow, $\delta\rho/\rho \propto t^{2/3} \propto R(t)$. When the Universe becomes radiation dominated, the growth of those perturbations still in the linear regime effectively ceases,[12] so that the potential "growth factor" is

$$\gamma \simeq R_D/R_{\rm eq} \simeq 7.3 \times 10^3 (\Omega_{\rm R} h_{1/2}^2) \theta^{-4}. \qquad (3)$$

Note that γ is independent of R_D—it is the same as it would have been in a scenario where the Universe is *still* dominated by relic, NR X's. This is because although perturbations cease growing when $R \simeq R_D$, the epoch of matter domination begins earlier (at a value of R smaller by a factor of R_D).

From the standpoint of the growth of density perturbations, it appears that R_D can take on any value; however, there are other considerations. The horizon at the epoch of matter domination today corresponds to a scale $\lambda_{\rm eq} \simeq (64\ {\rm Mpc}) R_D \theta^2 / \Omega_R h_{1/2}^2$, and contains a baryonic mass of

$$M_{\rm eq} \simeq 10^{16} M_\odot R_D^3 \Omega_{\rm R}^{-3} \Omega_{\rm NR} h_{1/2}^{-4} \theta^6. \qquad (4)$$

A perturbation which enters the horizon before the epoch of matter domination, with an amplitude $(\delta\rho/\rho)_H$, will grow[12] by about a factor of 2 by $R \simeq R_{\rm eq}$; from $R \simeq R_{\rm eq}$ until today it will grow by a factor of about γ, so that for scales $\lambda << \lambda_{\rm eq}$, $(\delta\rho/\rho)_0 \simeq 2\gamma (\delta\rho/\rho)_H$. A scale which enters the horizon after $R \simeq R_{\rm eq}$ will grow by less than this amount; for $\lambda \gtrsim \lambda_{\rm eq}$, $(\delta\rho/\rho)_0 \simeq \gamma (\lambda/\lambda_{\rm eq})^{-2} \times (\delta\rho/\rho)_H$. Therein lies a potential difficulty; $\lambda_{\rm eq} \propto R_D$ and so for R_D too small, the scales important for galaxy formation will not be able to undergo sufficient growth.

To be more quantitative, the galaxy-galaxy correlation function $\xi(r)$ indicates that the scale which is just entering the nonlinear regime today $[(\delta\rho/\rho)_0 \simeq 1]$ has a size[15] $\lambda_c \simeq 10 h_{1/2}^{-1}$ Mpc and contains a baryonic mass of $M_c \simeq 3.6 \times 10^{13} \times M_\odot h_{1/2}^{-1} \Omega_{\rm NR}$. For an initial spectrum of adiabatic perturbations [specified by $(\delta\rho/\rho)_H$] which is consistent with the observed anisotropy of the 3-K background, a growth factor $O(2\gamma)$ is about sufficient for the scale λ_c to achieve $(\delta\rho/\rho)_0 \simeq 1$.[16,17] Since $\lambda_{\rm eq} \propto R_D$, if $R_D \lesssim 0.16 h_{1/2} \Omega_{\rm R} \theta^{-2}$ the scale λ_c will enter the horizon *after* $R \simeq R_{\rm eq}$ and will undergo less growth than $O(2\gamma)$. For example, for $R_D \simeq 0.05$, $\lambda_{\rm eq} \simeq 3$ Mpc, and the growth factor for the scale λ_c is $\simeq 0.1\gamma$. Therefore, we will require that R_D be $\gtrsim 0.1$ or so.[18]

Now let us discuss some candidates for the X; its mass and abundance (before it decays) must be related by

$$m_X \simeq (6.6\ {\rm eV}) (n_X/n_\gamma)^{-1} R_D^{-1} (\Omega_{\rm R} h_{1/2}^2/\theta^3). \qquad (5)$$

First consider the possibility that X is a massive neutrino species. In this case $n_X/n_\gamma = \frac{3}{11}$ and[19] $m_\nu \simeq (24\ {\rm eV}) R_D^{-1} (\Omega_{\rm R} h_{1/2}^2/\theta^3)$. The decay cannot be via the weak interaction, because the branching ratio to radiative modes would be too large and the lifetime too long.[20] However, a suitable lifetime and negligibly small radiative branching ratio can be obtained if the decay is due to some horizontal interaction (e.g., as in the familon[21] or majoron[22] models). For the familon model,[21] we have $\nu \to \nu' + f$ (ν' a very light neutrino species, f a massless Nambu-Goldstone boson associated with a spontaneously broken "family symmetry"), and

$$\tau_\nu \simeq (10^9\ {\rm yr}) [(100\ {\rm eV})/m_\nu]^3 [F/(10^9\ {\rm GeV})]^2$$

(F = scale of spontaneous symmetry breaking). The symmetry-breaking scale needed for our scenario is

$$F \simeq (0.5 \times 10^9\ {\rm GeV}) R_D^{-1/2} (\Omega_{\rm R} h_{1/2}^2)^{5/4} \theta^{-9/2}.$$

As a result of the neutrino free streaming all perturbations on scales less than[23] $\lambda_\nu \simeq (52\ \text{Mpc}) \times R_D(\Omega_R h_{1/2}^2)^{-1}\theta^2$ will be strongly damped. The baryonic mass associated with the scale λ_ν is $M_\nu \simeq 2\times 10^{15} M_\odot R_D^3 (\Omega_R h_{1/2}^2)^{-2} \Omega_{NR}/\Omega_R$. As in the usual neutrino scenario the first structures to form (pancakes) will have mass of order M_ν; these pancakes must then fragment to form galaxies.

In the usual neutrino scenario numerical simulations[24] indicate that it is difficult both to form galaxies early enough [$R_{GF} \lesssim 0.25$ since quasistellar objects with red shifts $z \simeq 3$ are seen; $R(z) = (1+z)^{-1}$] and to achieve the observed galaxy-galaxy correlation function. Our scenario may help to alleviate these difficulties. Since all of the growth (γ) occurs by R_D, galaxies must form early on ($R_{GF} \leq R_D$); in fact in our scenario the "usable growth factor" is effectively a factor of 4 or so larger (compared to the usual scenario), since in the usual scenario after galaxies form ($R \simeq R_{GF}$), linear perturbations could have still grown by an additional factor of $R_{GF}^{-1} \simeq 4$. In the unstable-neutrino scenario λ_ν is smaller (since $\lambda_\nu \propto R_D$); this may be beneficial since the difficulty with reproducing $\xi(r)$ is in part traceable to the fact that λ_ν is much larger than λ_c. More specifically, to achieve galaxy formation by $z \simeq 3$ and match the observed $\xi(r)$ the simulations require[24] $\Omega \simeq (2\text{–}3) h^{-1}\theta^2$. For our scenario $\Omega_{NR} + \Omega_R/R_D$ is to be identified with Ω, suggesting that with $R_D \simeq 0.2$ it might result in reasonable agreement with the observed structure.

If the X is heavy ($m_X \geq$ keV), then the damping scale ($M_D \propto m_{Pl}^3/m_X^2$) corresponds to a mass $\lesssim 10^{12} M_\odot$—leading to the so-called "cold dark matter" scenario.[25] With the exception that $\Omega_{NR} \simeq 0.1\text{–}0.3$, things should proceed as they do in the usual "cold-dark matter" scenario, where structure forms on smaller scales first (e.g., galaxies), and then develops in a hierarchical fashion on up to the larger scales.

Finally, let us mention two potential difficulties with this scenario. When $R \simeq R_D$, the ratio ρ_X/ρ_{NR} is $\simeq \Omega_R/\Omega_{NR} R_D >> 1$. Any systems (e.g., clusters, galactic haloes) that exist then and have their mass dominated by X's may be disrupted when the X's decay and the mass in X's disperses. Roughly speaking, if the dynamical time scale for the system t_{dyn} is less than or approximately the decay time scale τ_X, the system should respond adiabatically (increasing in linear size by a factor[26] $\simeq 1 + \Omega_R/\Omega_{NR} R_D$) and avoid disruption; on the other hand if $t_{dyn} \gtrsim \tau_X$, then only a small part of the original system may remain bound. For $R_D \simeq \frac{1}{3}$, $\tau_X \simeq 10^9$ yr, which is much longer than t_{dyn} for a galaxy ($\simeq 10^8$ yr) and comparable to t_{dyn} for a cluster of galaxies. Note that for systems less massive than M_ν (or M_D), the mass should be dominated by baryons (since the neutrinos, or X's, are initially smooth on these scales), and so systems less massive than this will not be disrupted in any case. The second difficulty is the peculiar-velocity field. Peculiar velocities induced recently ($R \gtrsim R_D$) by the matter distribution will be small and characteristic of an "$\Omega \simeq \Omega_{NR}$" Universe.[5] However, peculiar velocities induced by the perturbed matter distribution just before the X's decay (which themselves subsequently decay $\propto R^{-1}$) may today be large and characteristic of an "$\Omega = 1$" Universe.

(ii) *Universe dominated by a relic cosmological constant*[27]—One of the outstanding puzzles in physics is the smallness of the present cosmological term ($\Lambda/m_{Pl}^2 \lesssim 10^{-121}$). Although an understanding of its extreme smallness is still lacking, perhaps there is a mechanism by which a primordial Λ relaxes to a very small value, which fortuitously happens to be significant today. In our notation this corresponds to $\Omega_R = 0$, $\Omega_{NR} + \Omega_\Lambda = 1$. The age of the Universe is $H_0 t_U = \frac{2}{3} \ln[(1 + \Omega_\Lambda^{1/2})/\Omega_{NR}^{1/2}]/\Omega_\Lambda^{1/2}$; for $\Omega_{NR} = 0.05$, 0.1, 0.15, 0.20, $H_0 t_U \simeq 1.5$, 1.3, 1.2, 1.1. This scenario can easily accommodate a 15-Gyr-old Universe, and if $H_0 t_U$ should be shown to be $> \frac{2}{3}$, would be the only way to salvage the $k=0$ model.

Once $\rho_\Lambda \gtrsim \rho_{NR}$ [$R \gtrsim R_\Lambda \equiv (\Omega_{NR}/\Omega_\Lambda)^{1/3}$], linear perturbations in the matter cease growing, i.e, $\delta\rho_{NR}/\rho_{NR} \simeq$ const. [From $R = R_\Lambda$ to $R >> 1$, $\delta\rho_{NR}/\rho_{NR}$ grows by only a factor $\simeq 1.65$.] For $\Omega_{NR} \gtrsim 0.05$ the growth only ceased recently ($R_\Lambda \gtrsim 0.4$); in the usual $\Omega_{NR} << 1$ scenario (i.e., $\Omega_\Lambda = 0$, $k < 0$) $\delta\rho_{NR}/\rho_{NR}$ stops growing when $R \simeq \Omega_{NR}$—thus in the $\Lambda \neq 0$ scenario there is an additional growth factor of $O(\Omega_{NR}^{-1})$.[28] If we do not invoke another form of NR matter[29] (besides baryons), then galaxy formation should proceed as in the original "pancake" picture and must be complete by $R_{GF} \lesssim \frac{1}{4}$. Even with $\Omega_\Lambda \neq 0$ the pancake scenario with $\Omega_{NR} << 1$ may already be in conflict with the anisotropy of the 3-K background.[17]

To summarize, we have proposed two possibilities for reconciling theory and observation with regard to the value of Ω. The more attractive of the two, that the Universe is today dominated by the relativistic decay products of a massive relic species which decayed in the recent past ($R_D \simeq 0.1\text{–}0.5$), seems to improve the viability of the "neutrino-dominated" Universe scenario, and makes two potentially testable predictions: that the deceleration parameter is $q_0 = 1 - \Omega_{NR}/2$; and that very interesting events were taking place at a rather modest red shift, $z \simeq R_D^{-1} - 1$. This scenario, however, faces the very formidable difficulty of a youthful Universe.

We thank M. Davis, J. Fry, J. Gunn, and J. Silk for valuable discussions. This work was initiated at the Aspen Center for Physics, and supported by the U. S. Department of Energy and the National Aeronautics and Space Administration.

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[6]The possibility of a Universe which is presently dominated by R particles was first proposed by D. Dicus, E. Kolb, and V. Teplitz, Astrophys. J. **223**, 327 (1978), and has been discussed by G. Gelmini, D. N. Schramm, and J. Valle, to be published. We we use "dominated by R particles" and "radiation dominated" interchangeably.

[7]Taking into account the NR contribution to ρ, and the earlier matter-dominated epoch ($R \lesssim R_D$), we find that $H_0 t_U = \frac{2}{3}\{R_D^2 a^{-1/2} + [(1-a^{3/2}) + 3\Omega_R(a^{1/2}-1)]/\Omega_{NR}^2\}$, where $a = \Omega_R + \Omega_{NR} R_D$. For $\Omega_{NR} \simeq 0.1{-}0.3$ and $R_D \simeq 0.1{-}0.5$, $H_0 t_U \simeq 0.51{-}0.57$.

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[18]There is another potential difficulty with $R_D << 1$: For $R \gtrsim R_D$, $\delta\rho/\rho$ on scales which have gone nonlinear will continue to grow $\propto R^3$, while $\delta\rho/\rho$ on scales which have not achieved nonlinearity will remain constant. J. Gott and M. Rees [Astron. Astrophys. **45**, 365 (1975)] have argued that this will lead to a galaxy-galaxy correlation function $\xi \propto r^{-3}$ for $R_D^{-3} \gtrsim \xi \gtrsim 1$, whereas the observed behavior is $\xi \propto r^{-1.8}$. Note that in scenario (ii) this is not a difficulty since the interval where this would occur is $R_\Lambda^{-3} \gtrsim \xi \gtrsim 1$, and $R_\Lambda^{-3} \simeq \Omega_\Lambda/\Omega_{NR} \simeq 1$.

[19]It is simple to see why $m_\nu \propto R_D^{-1}$. Since the decay products of the X are relativistic, their average energy $\propto m_\nu R_D/R(t)$. In order to have $\Omega_R \simeq 1$, this average energy must be $\simeq 30$ eV today, and thus $m_\nu \simeq 30 \text{ eV } R_D^{-1}$.

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[26]Systems whose mass was originally dominated by X's formed at higher densities, by a factor of $(1 + \Omega_R/\Omega_{NR} R_D)^4$, than in usual scenarios. This fact may be of some importance when one considers the formation and early evolution of galaxies and clusters.

[27]Model universes with $\Lambda \neq 0$ have been discussed; see, e.g., J. Gunn and B. Tinsley, Nature (London) **257**, 454 (1975), and references therein. With regard to reconciling $k = 0$ with the observational data, $\Lambda \neq 0$ has also been discussed by P. H. Frampton and G. B. Lipton, University of North Carolina Report No. IFP-203-UNC, 1983 (unpublished); P. J. E Peebles (see Ref. 5).

[28]We have investigated the growth of perturbations in this scenario numerically. From $R = R_i$ until $R = R_\Lambda$ linear perturbations grow by a factor of $0.873 R_\Lambda/R_i$; in a model with $\Lambda = 0$ and $k = 0$ the growth would be a factor of R_Λ/R_i (for the same interval in R). R_i can be no smaller than R_{eq} ($\propto \Omega_{NR}^{-1}$); if the NR component is baryons then R_i must be $\gtrsim R_{\text{decoupling}} \simeq 10^{-3}$.

[29]If in addition to baryons one also invoked another form of cold, NR matter (X) such that $\Omega_X >> \Omega_B$ and $\Omega_X + \Omega_B + \Omega_\Lambda = 1$, one could again have the "cold, dark matter scenario" in which structure forms hierarchically. Such a scenario is less likely to be in conflict with the anisotropy of the microwave background (Ref. 17).