Resistance Oscillations and Electron Localization in Cylindrical Mg Films

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Longitudinal magnetoresistance measurements of hollow Mg cylinders between 1.5 and 10 K are reported and analyzed within the framework of weak electron-localization theories. The observed resistance oscillations decrease with increasing cylinder diameter and temperature, in excellent agreement with the theory of Altshuler *et al.* The phase-breaking and spin-orbit-interaction times are comparable to the values obtained in plane Mg films.

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The transport properties of two-dimensional electronic systems in the presence of disorder have been extensively investigated in recent years.[1-4] It has been shown that the induced anomalous resistance behavior of weakly localized metallic films is very sensitive to electronic scattering processes. Hence accurate values can be determined for the phase-breaking time $\tau_\phi(T)$ due to inelastic or magnetic impurity scattering and for the spin-orbit scattering time τ_{so}. Also scattering by superconducting fluctuations has been studied.[5]

As shown by Bergmann,[6] weak localization can be represented as an interference experiment with conduction electrons split into pairs of waves interfering in the back-scattering direction. In a plane metal film the phase coherence of the two partial waves is destroyed by a magnetic field. When the two partial waves surround a constant area the relative change of the two phases[7] is an oscillating function of the magnetic flux with period $h/2e$, the superconducting flux quantum. This produces an oscillatory behavior of the film resistance at low temperatures.

In 1981 Altshuler, Aronov, and Spivak[8] predicted that these oscillations can be detected in a cylindrical metal film subjected to a magnetic field parallel to the cylinder axis. The condition for observing the effect is a diffusion length $L_\phi = (D\tau_\phi)^{1/2}$ comparable to the cylinder circumference $2\pi r$ (D is the electron diffusion coefficient). The oscillations were subsequently observed in cylindrical Mg[9] and Li[10] films, indicating the importance of the spin-orbit interaction. These experiments have to our knowledge not been confirmed by other groups or by studies on very small normal metal rings.[11]

In this Letter we report for the first time on a systematic and quantitative analysis of the quantum-interference effect in hollow Mg cylinders. The main results of our study can be summarized as follows: (i) The period and the temperature dependence of the resistance oscillations are in agreement with the theory of Altshuler, Aronov, and Spirak[8]; (ii) the oscillation amplitude decreases exponentially when $L_\phi < 2\pi r$ and depends on the Mg film sheet resistance as predicted; (iii) the phase-breaking time $\tau_\phi(T)$ and the spin-orbit time τ_{so} derived from the resistance oscillation data at $B < 10^{-2}$ T are consistent with the longitudinal magnetoresistance (MR) measurements at higher fields; (iv) the τ_ϕ and τ_{so} values are comparable to the values obtained from perpendicular MR measurements in plane Mg films, in both cases the temperature dependence of $(\tau_\phi)^{-1}$ being approximately linear below 4.2 K.

Samples were prepared by evaporating 99.99% pure Mg in a partial pressure of 10^{-2} Torr pure He onto a rotating quartz fiber held at room temperature. The quartz fiber was stretched over a hole cut in a glass substrate and attached with glue onto two Cu strips. Reliable and low-resistance current and voltage contacts were made on the Cu strips using silver paint. The average film thickness was measured with a quartz-crystal thickness monitor mounted near the fiber holder. The deposition rate was of the order of 0.2 nm s^{-1}. After the condensation and in order to minimize contamination all the fiber samples were constantly kept in an atmosphere of technical-grade helium. The continuity of the Mg layer and the diameter of the quartz fiber were controlled by scanning electron microscopy (SEM). The conventional dc four-terminal resistance measurements enabled us to detect resistance changes $\Delta R/R \leqslant 10^{-5}$ at low currents ($< 10\ \mu$A).

Table I contains the essential parameters of three cylindrical and two plane Mg films. Δr is the variation of the fiber radius over the length l, d is the average Mg film thickness, $R^{4.2}$ and $R_\square^{4.2}$ are respectively the film resistance and sheet resistance at 4.2 K, the resistance ratio is $R_{300}/R_{4.2}$, and $D = \frac{1}{3}v_F l_{el}$ where the effective mean free path l_{el} at 4.2 K is obtained from the resistance ratio and

TABLE I. Relevant parameters for the studied cylindrical and plane Mg films.

Sample	r (μm)	Δr (%)	l (mm)	d (nm)	$R^{4.2}$ (Ω)	$R_{\square}^{4.2}$ ($\Omega/\square$)	Resistance ratio	D (cm^2/s)	τ_{so} (10^{-11} s)	$\tau_{\phi}^{1.5}$ (10^{-10} s)	θ (deg)
Fiber 1	0.60	16	5.3	14.5	5056	3.60	1.35	24.8	12	16	1.3
Fiber 2	0.72	20	5	14	8465	7.66	1.38	26.9	7.0	8.0	2.5
Fiber 3	1.03	27	3.5	14	4269	7.89	1.41	28.4	8.0	8.8	1.3
Film 1[a]	...	...	4	13.5	170	10.1	1.36	25.5	1.1	3.1	...
Film 2[b]	...	...	...	40.1	...	22.3	...	3.86	1.27	7.1	...

[a]Ref. 15. [b]Sample 5 of Ref. 16.

$\rho l = 6.52\times10^{-12}\ \Omega\ \text{cm}^2$.

A typical set of resistance oscillations in a longitudinal field (parallel to the fiber axis) at $T=1.45$ K is shown in Fig. 1. The oscillation period and amplitude decrease with increasing cylinder diameter and are practically not detectable for Fiber 3. The observed positive MR at low fields consists of three main contributions: (i) resistance oscillations caused by quantum interference in the small cylindrical geometry; (ii) a low-field longitudinal MR[12]; (iii) a fraction of the low-field perpendicular MR caused by a misalignment between the fiber axis and the field.

According to Altshuler, Aronov, and Spivak[8] the first two contributions are taken into account by the expression[13]

$$\frac{R(B)-R(0)}{R^2(0)} = \frac{e^2}{2\pi^2\hbar}\frac{2\pi r}{l}\left[\tfrac{3}{2}Z_\Phi(L_\phi^*(B)) - \tfrac{1}{2}Z_\Phi(L_\phi(B))\right], \tag{1}$$

where

$$Z_\Phi(L_\phi(B)) = 2\ln\frac{L_\phi(B)}{L_\phi(0)} + 4\sum_{n=1}^{\infty}\left[K_0\left(n\frac{2\pi r}{L_\phi(B)}\right)\cos\left(n\frac{2e}{\hbar}\Phi\right) - K_0\left(n\frac{2\pi r}{L_\phi(0)}\right)\right]$$

and $1/L_\phi^2(B) = 1/D\tau_\phi + \frac{1}{3}(deB/\hbar)^2$; $1/L_\phi^{*2}(B) = 1/L_\phi^2(B) + 2/D\tau_{so}$. Here $\Phi=\pi r^2 B$ is the magnetic flux and $K_0(x)$ is the McDonald function.

The orthogonal MR is given by[2]

$$\frac{R(B)-R(0)}{R^2(0)} = \frac{e^2}{2\pi^2\hbar}\frac{2\pi r}{l}\left[\frac{1}{2}f\left(\frac{4De\tau_\phi}{\hbar}B\sin\theta\right) - \frac{3}{2}f\left(\frac{4De\tau_\phi^*}{\hbar}B\sin\theta\right)\right], \tag{2}$$

where $f(x) = \ln x + \psi(\frac{1}{2}+1/x)$, $\psi(y)$ is the digamma function, $\tau_\phi^{-1} = \tau_i^{-1} + 2\tau_s^{-1}$, and $\tau_\phi^{*-1} = \tau_i^{-1} + \frac{2}{3}\tau_s^{-1} + \frac{4}{3}\tau_{so}^{-1}$ with $\tau_i(T)$ the inelastic scattering time and τ_s the magnetic scattering time. θ is the angle between the fiber axis and the field. An excellent agreement with the experimental results could be obtained by a summation of Eqs. (1) and (2) using $\tau_\phi(T)$, τ_{so}, and θ as fitting parameters (see Fig. 1). The damping of the oscillations with increasing field is due to the averaging of Eq. (1) over a distribution of fiber radii as measured by SEM (see Δr in Table I).

The behavior of the magnetoresistance at different temperatures (Fig. 2) is characteristic of a weakly localized electron system with spin-orbit interactions. With the same $\tau_\phi(T)$, τ_{so}, and θ fitting parameters as in Fig. 1 a good agreement with theory [Eqs. (1) and (2)] is obtained up to $B\simeq0.4$ T, the normal positive MR being important at higher field. It should be remarked that above $B\simeq 2\times10^{-2}$ T the theoretical curves in Fig. 2 are the mean value of a rapidly oscillating behavior on a logarithmic scale (period $<2\times10^{-3}$ T). Analysis of the MR data at each temperature produces a phase-breaking time $\tau_\phi(T)\propto T^{-1}$ (see inset in Fig. 2) as predicted by Abrahams *et al.*[14] for $\tau_s >> \tau_i(T)$. The diffusion length $L_\phi = (D\tau_\phi)^{1/2}$ (calculated with $D=26$ cm^2/s) is comparable to the fiber diameter.

The observed differences between the values of $\tau_\phi(T)$ and τ_{so} for, respectively, the Mg cylinders and plane films (see also Table I) is probably due to (i) a different film structure, since the Mg plane

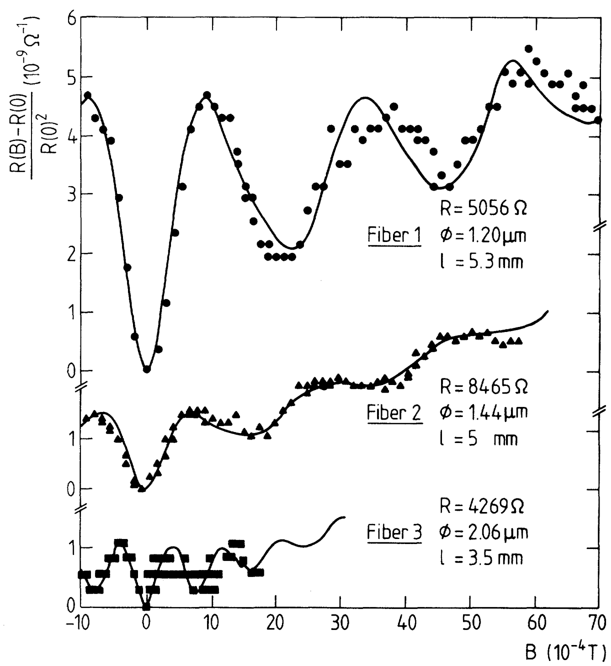

FIG. 1. Resistance oscillations in three cylindrical ($\varnothing = 2r$) Mg films at $T = 1.45$ K. The full curves are calculated with Eqs. (1) and (2).

film 1 is exposed to air during photolithographic processing[15] and film 2 is evaporated and measured *in situ* on a liquid He-cooled substrate[16]; (ii) the different sheet resistance which influences the phase-breaking time $\tau_\phi^{-1} \propto R_\square^{4.2}$ (Ref. 14).

The temperature dependence of the oscillating MR at very low fields ($B < 10^{-2}$ T) is also in excellent agreement with theory (Fig. 3) and is consistent with the decrease of $L_\phi(T)$ as a function of increasing temperature.

Finally, measuring the fiber resistance versus temperature indicated that (i) at $B = 0$ T and $T \lesssim 5$ K the resistance is nearly constant, probably because of the interplay[4] between electron-electron interactions and weak antilocalization ($\tau_{so} << \tau_i$); (ii) at $B > 1$ T and $T \lesssim 5$ K weak localization is destroyed and the resistance increases logarithmically with decreasing temperature

$$\{[R_\square(1\text{ K}) - R_\square(10\text{ K})]/R_\square^2 \simeq 2.5 \times 10^{-5}\,(\Omega/\square)^{-1}\}$$

as a result of electron-electron interactions. A discussion of these phenomena will be given elsewhere.

In conclusion, a detailed analysis of the resistance oscillations in cylindrical metal films has shown that the theory of Altshuler *et al.* is relevant to the interpretation of the observed quantum interference of the conduction electrons. These experiments clearly indicate that the main concepts of the weak localization theory in quasi two-dimensional electron systems are correct.

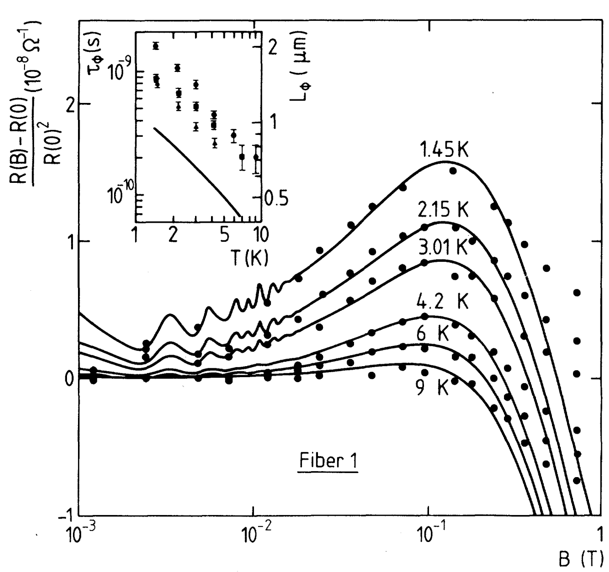

FIG. 2. Longitudinal MR curves of a cylindrical Mg film at different temperatures. The full curves are calculated using Eqs. (1) and (2). The inset shows τ_ϕ and L_ϕ vs T for Fiber 1 (circles), Fiber 2 (triangles), Fiber 3 (squares), and plane Mg film 1 (solid line).

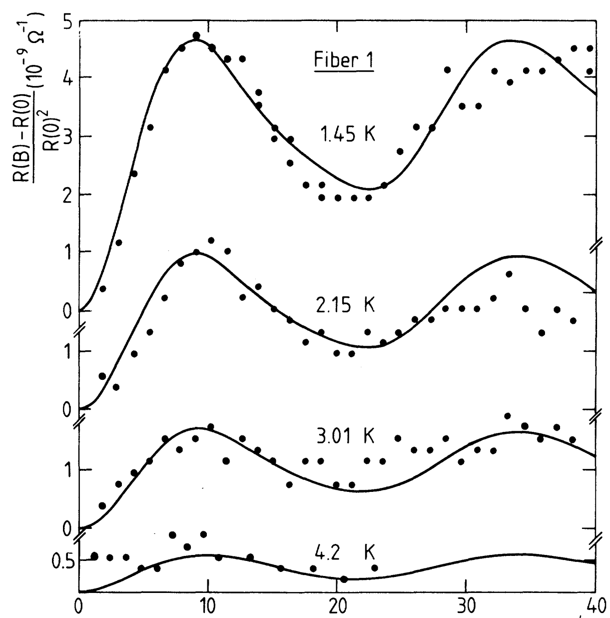

FIG. 3. Comparison between the experimental (dots) and theoretical (full curve) temperature dependence of the resistance oscillations in a cylindrical Mg film.

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