Sustainment Dynamo Reexamined: Nonlocal Electrical Conductivity of a Plasma in a Stochastic Magnetic Field

Abram R. Jacobson and Ronald W. Moses
Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Received 13 March 1984)

We show that sustained plasma discharges in the ZT-40M reversed-field pinch experiment can be explained in terms of electron wander in a stochastic magnetic field. We indicate results of a Fokker-Planck calculation for nonlocal electrical conductivity in slab geometry and show that this approach can account for the key anomalies of reversed-field pinch behavior, without needing to invoke a "plasma dynamo."

PACS numbers: 52.55.Ez, 51.10.+y, 52.25.Fi

The "plasma dynamo" is both an intriguing and a practical concept. The intrigue derives from attempts to explain natural[1] and manmade[2,3] plasmas whose strong field-aligned currents $j_{\parallel}$ disobey the most naive Ohm's law $j_{\parallel} = \sigma_{\parallel} E_{\parallel}$. The practical importance arises from the dynamo's role both in formation and in sustainment of reversed-field pinch (RFP)[2] and Spheromak[3] fusion plasmas. We will examine certain features of the quasisteady discharges[2] on ZT-40M, an RFP in apparent need[4] of a sustainment dynamo. We will show that the tail electrons (which carry $j_{\parallel}$) are probably wandering (along stochastic magnetic field lines) over much of the minor radius in one mean free path. This will void *any* local Ohm's law, whether naive $(j_{\parallel} = \sigma_{\parallel} E_{\parallel})$ or containing additional terms (such as the $\langle \vec{v} \times \vec{B} \rangle_{\parallel}$ of nonlinear dynamo theory). Instead, this suggests that observed quasisteady RFP discharges in ZT-40M are explainable in simple terms $(f = ma)$ of electron-momentum diffusion in a stochastic field, using a stochasticity inferred from observed nonradiative electron heat-loss time τ_{Ee}. We will then present results of a formal model of this momentum diffusion. The model predicts the key observed anomalies of sustained RFP behavior (excess loop resistance, slower-than-classical current decay) in terms of electron dynamics in a stochastic magnetic field. Absent from our model are the usual turbulent-dynamo concepts: magnetic-helicity conservation, mode-mode interactions, relaxation, wave-number cascades, etc.

Quasisteady discharges that defy a naive Ohm's law[4] have been reported[2] on ZT-40M. Their parameter regime is low density $(n \lesssim 2\times 10^{19}\ \mathrm{m}^{-3})$ and high temperature $(T_e \gtrsim 150\ \mathrm{eV})$, and the nonradiative electron heat-loss time is $\tau_{Ee} \approx 10^{-4}$ s. At moderate pinch parameter $(\Theta \lesssim 1.5)$ these RFP discharges show very little poloidal variation of the reversed toroidal field $[B_\phi(a)]$ apart from the factor $1/R$: $[\Delta B_z(a)/B_z(a)]_{\mathrm{rms}} \lesssim 0.1$ and $[\Delta B_z(a)/B_\theta(a)]_{\mathrm{rms}} \lesssim 0.01$. This observed laminarity does not appear to be consistent with the sustainment dynamo's properties seen in magnetohydrodynamic calculations by Sykes and Wesson[5] and by Aydemir and Barnes,[6] both of which calculations predict[7,8] such large-scale poloidal asymmetry that $B_z(a)$ is not even everywhere reversed, i.e., $\Delta B_z(a)/B_z(a) \sim 1$.

Rechester and Rosenbluth[9] showed that a typical tokamak can be driven stochastic (i.e., islands overlap everywhere) with $(B_r^{\mathrm{local}}/B_0)_{\mathrm{rms}} \gtrsim 10^{-5}$ if a wave-number spectrum populated out to $k_\perp \rho_{ci} \approx 1$ is assumed. Repeating their exercise for a typical RFP indicates that $(B_r^{\mathrm{local}}/B_0)_{\mathrm{rms}} \gtrsim 10^{-4}$ would produce stochasticity. The point we make is that even such a level is undetectable, so that Ockham's razor would favor stochasticity as the cause of observed $\tau_{Ee} \approx 10^{-4}$ s in ZT-40M.

If we assume that ZT-40M *is* stochastic, then the electron-heat diffusivity[9] D_e inferred from τ_{Ee} can be used to estimate the magnetic field-line diffusivity D_F. (Krommes, Oberman, and Kleva[10] suggest that this estimate be a *lower bound* for D_F.) If we write $\tau_{Ee} \approx a^2/D_e$, the electron-heat diffusivity (with $a = 0.2$ m) is $D_e \approx 4\times 10^2\ \mathrm{m^2\ s^{-1}}$. Thus an upper bound[10] on the stochasticity-induced electron-heat diffusivity[9] is $D_e \approx v_{T_e} D_F$. Using $T_e \approx 200$ eV so that the electron thermal speed is $v_{T_e} \approx 6\times 10^6\ \mathrm{m\ s^{-1}}$, we get $D_F \approx 7\times 10^{-5}$ m as a lower bound on the magnetic-field-line diffusivity.

How far does an electron wander during one mean free path across the flux surfaces, if indeed $D_F \approx 7\times 10^{-5}$ m? On ZT-40M the effective nuclear charge may be[2] $Z_{\mathrm{eff}} \approx 2$. Under these circumstances the mean free path (for 90° scattering) is dominated by electron-ion encounters, so that a Lorentz-gas model may be used. The most probable electron $(v = v_{T_e}\sqrt{2} \equiv v_0)$ has a mean free path (in a Lorentz plasma with $Z_{\mathrm{eff}} = 2$, $n = 2\times 10^{19}\ \mathrm{m}^{-3}$, and $T_e = 200$ eV) $\lambda_0 = 10$ m. The more relevant length, though, is λ averaged over j, and this can be shown[11] to be $\lambda_j \equiv \int \lambda \, dj / \int dj = 20\lambda_0$ for a Lorentz plasma because of the weighting of suprathermal electrons in carrying j. Using $\lambda_j = 200$ m, we obtain an electron wander $(\Delta x)_j = (2D_F\lambda_j)^{1/2} = 0.17$ m. Thus, in one mean free path the j-weighted electron radial wander is similar to the plasma radius!

Consider a slab-geometry RFP with x the normal to "flux surfaces" (like r in a cylinder). The configuration is sustained by a steady, uniform applied E_z. The local magnetic-field-aligned electric field is $E_{\parallel}(x) = E_z B_z(x)/B$. The average gradient length $E_{\parallel}/(\partial E_{\parallel}/\partial x)$ in an RFP will be smaller than the radius a. Thus in our example of ZT-40M, tail electrons wander all over the $E_{\parallel}$ gradient in one mean free path. This voids a local Ohm's law. More importantly, it suggests that RFP sustainment on ZT-40M may be due to export of electron field-aligned momentum from the core (where $E_{\parallel} > j_{\parallel}/\sigma_{\parallel}$) to the outer region (where $E_{\parallel} \lesssim 0 < j_{\parallel}/\sigma_{\parallel}$).

We have recently developed[11] a formal procedure for treating electron-momentum export down the $E_{\parallel}$ gradient. The treatment is facilitated by some simplifying assumptions (none of which, though, is required for the basic mechanism to be viable): (1) The plasma is isothermal and isodense, and $f^{(0)}(\vec{v})$ is a Maxwellian. (2) Slab geometry is employed, and $|\vec{B}|$ is unifom. (3) Coulomb scattering is approximated by electron collisions only with massive ions (Lorentz gas). (4) The applied electric field is weak: $E_{\parallel} << E_c$, where E_c is the critical (runaway) field.[12] (5) $L_F << \lambda$ where L_F is the (Kolmogorov) correlation length[9] and λ is the electron mean free path for cumulative 90° scattering. (6) Electrons may not wander across the plasma boundary (at $x = a$); thus $\partial f^{(1)}(\vec{v},x)/\partial x$ vanishes at boundary.

Under these conditions we have obtained[11] the following results: First, the perturbation $f^{(1)}(\vec{v},x)$ in the electron distribution function is laminar, depending on x (the normal to "flux surfaces") but not on y or z. Second, the linearized perturbation $f^{(1)}(\vec{v},x)$ is purely odd in $\cos\theta$ (where θ is the angle between $\vec{v}$ and $\vec{B}$); this leads to export of field-aligned momentum, *but not of electron number density,* down the $E_{\parallel}$ gradient. Third, the spatial gradient $\partial f^{(1)}(\vec{v},x)/\partial x$ causes a Fick's-law flux $-D_e \partial f^{(1)}(\vec{v},x)/\partial x$, which carries the electron momentum exported down the $E_{\parallel}$ gradient. Fourth, for each electron velocity $\vec{v}$, $f^{(1)}(\vec{v},x)$ is a solution of a separate Boltzmann equation:

$$f^{(1)}(\vec{v},x) = -\frac{E_{\parallel}(x)}{E_c}\left(\frac{v}{v_0}\right)^4 \cos\theta f^{(0)}(\vec{v}) + 2\lambda_0\left(\frac{v}{v_0}\right)^4 |\cos\theta| \frac{\partial}{\partial_x}\left[D_F(x)\frac{\partial f^{(1)}(\vec{v},x)}{\partial x}\right]. \quad (1)$$

The first term on the right-hand side of Eq. (1) is the local Spitzer-Härm[13] Lorentz-gas solution. The second term on the right-hand side is (minus) the divergence of the Fick's-law flux down the spatial gradient of $f^{(1)}(\vec{v},x)$. The $(v/v_0)^4|\cos\theta|$ weighting is caused by the mean free path's dependence on $\vec{v}$.

We solve Eq. (1), with $E_{\parallel}(x)$ and $D_F(x)$ profiles as inputs, at each of 39 velocities (three angles, θ, at each of thirteen speeds, v). The solutions are multiplied by $-ev\cos\theta$ and integrated over d^3v with splines to give $j_{\parallel}(x)$. The contrived boundary condition at the wall is $\partial f^{(1)}/\partial x_{x=a} = 0$, corresponding to zero momentum export from the plasma to the wall. The $E_{\parallel}(x)$ profile shape is affected by the $j_{\parallel}(x)$ result, because $j_{\parallel}(x)$ controls the magnetic field orientation (via Ampere's law), and $E_{\parallel}(x) = E_z B_z(x)/B$. Thus we iterate the solution of Eq. (1), at each step using an updated $E_{\parallel}(x)$ profile, unitl the current $j_{\parallel}(x)$ satisfies both $f = ma$ [Eq. (1)] and Ampere's law.

The parameters which we may choose are $\lambda_0 D_F/a^2$ (characterizing the electron wander) and $j_{\parallel}(0)/B$ (corresponding to how hard we push the system). In order to compare with RFP phenomenology we may use $B_y(a)/\langle B_z \rangle$ (corresponding to the pinch parameter, Θ) as the second parameter instead of $j_{\parallel}(0)/B$. For the ZT-40M ex-

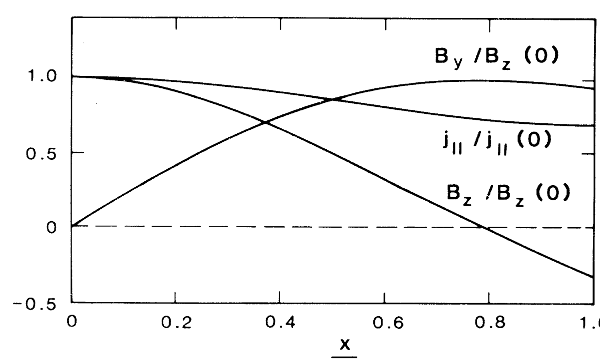

FIG. 1. Normalized profiles of magnetic fields and field-aligned current density for uniform diffusivity. $\lambda_0 D_F/a^2 = 0.05$; $B_y(a)/\langle B_z \rangle = 2.10$.

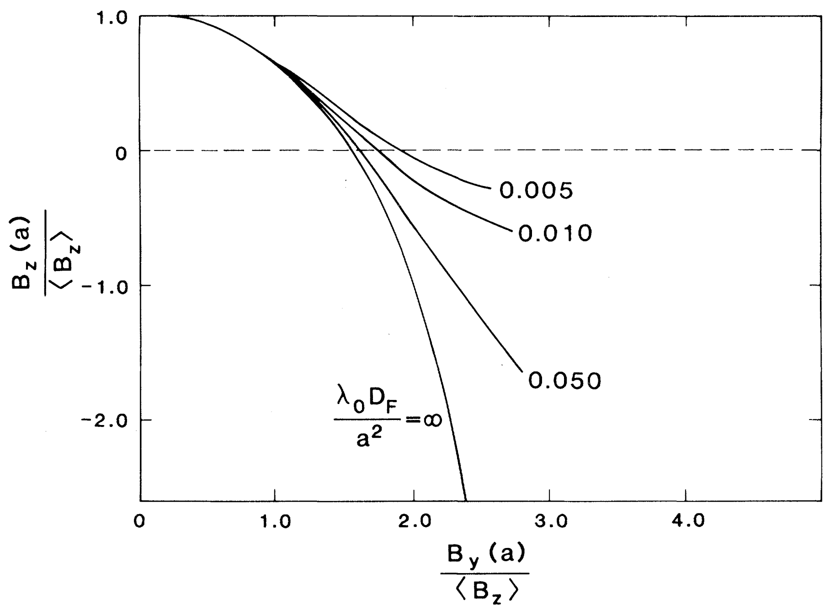

FIG. 2. F-Θ trajectories for various diffusivities, in slab geometry.

ample (above), $\lambda_0 D_F/a^2 \lesssim 0.035$.

A self-consistent solution with uniform diffusivity ($\lambda_0 D_F/a^2 = 0.05$) and pinch parameter $B_y(a)/\langle B_z\rangle = 2.10$ is shown in Fig. 1. The $E_{\parallel}(x)$ profile has the same shape as the $B_z(x)$ profile. Despite the $E_{\parallel}(x)$ profile's sign reversal (at $x \simeq 0.8a$), the field-aligned current $j_{\parallel}(x)$ is almost flat, and never reverses sign.

An "F-Θ diagram" for slab geometry is shown in Fig. 2, using various spatially uniform diffusivities $\lambda_0 D_F/a^2$. The extreme case ($\lambda_0 D_F/a^2 = \infty$) would be called "fully relaxed," and the others "partially relaxed" in dynamo parlance. In our theory of nonlocal conductivity, however, "relaxation" plays no role; instead, the F-Θ trajectory is controlled by the range of electron wander, measured by $\lambda_0 D_F/a^2$.

The nonlocal conductivity process actually occurring in ZT-40M may be even more robust than indicated by our analysis, which is linearized in E. The electric field on axis ($r=0$) is sufficient to cause $v_c \approx 2v_0$, when v_c is the critical speed for electron runaway.[12] The resultant runaway electrons must achieve longer mean free path λ than appeared in our model, thus requiring less D_F than we indicated. Measurements of limiter damage[14] in the edge of ZT-40M show that the current (mainly poloidal there) is carried by suprathermal electrons (kinetic energy > 1 keV). This is consistent with runaways' being produced in the core (where $E_{\parallel}$ is strong) and spreading radially on account of their longer mean free path ($\lambda \sim v^4$).

The tangled discharge model (TDM), originated by Rusbridge[15] and later elaborated by Miller,[16] was an earlier theory of RFP sustainment based on field-line stochasticity. The TDM differs from our kinetic model in using a fluid description and a local Ohm's law. Thus the TDM and our kinetic model must apply to dissimilar regimes of radial electron wander (in one mean free path), i.e., to different regimes of $\lambda_0 D_F/a^2$.

[1]G. L. Siscoe, D. G. Sibeck, E. J. Smith, B. T. Tsurutani, J. A. Slavin, and D. E. Jones, in Proceedings of the AGU Chapman Conference on Magnetic Reconnection, Los Alamos National Laboratory, 1983 (to be published), session E.

[2]D. A. Baker, M. D. Bausman, C. J. Buchenauer, L. C. Burkhardt, G. Chandler, J. N. DiMarco, J. N. Downing, P. R. Forman, R. F. Gribble, A. Haberstich, R. B. Howell, J. C. Ingraham, A. R. Jacobson, F. C. Jahoda, K. A. Klare, E. M. Little, L. W. Mann, R. S. Massey, J. Melton, G. Miller, R. Moses, J. A. Phillips, A. E. Schofield, K. F. Schoenberg, K. S. Thoma, R. G. Watt, P. G. Weber, and R. Wilkins, in *Proceedings of the Ninth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, 1982* (International Atomic Energy Agency, Vienna, 1983), Vol. 1, p. 587.

[3]T. R. Jarboe, I. Henins, A. R. Sherwood, Cris W. Barnes, and H. W. Hoida, Phys. Rev. Lett. **51**, 39 (1983).

[4]E. J. Caramana, D. A. Baker, and R. W. Moses, Los Alamos National Laboratory Report No. LA-UR-83-2284, 1983 (unpublished).

[5]A. Sykes and J. A. Wesson, in *Proceedings of the*

Eighth European Conference on Controlled Fusion and Plasma Physics, Prague, 1977 (International Atomic Energy Agency, Vienna, 1978).

[6]A. Y. Aydemir and D. C. Barnes, Institute for Fusion Studies Report No. 102, 1983 (unpublished).

[7]J. A. Wesson, private communication.

[8]D. C. Barnes, private communication.

[9]A. B. Rechester and M. N. Rosenbluth, Phys. Rev. Lett. **40**, 38 (1978).

[10]J. A. Krommes, C. Oberman, and R. G. Kleva, J. Plasma Phys. **30**, 11 (1983).

[11]Abram R. Jacobson and Ronald W. Moses, Phys. Rev. A **29**, 3335 (1984).

[12]H. Knopfel and D. A. Spong, Nucl. Fusion **19**, 785 (1979).

[13]L. Spitzer and R. Härm, Phys. Rev. **89**, 977 (1953).

[14]J. N. Downing, R. A. Gordon, K. S. Thomas, and R. G. Watt, Los Alamos National Laboratory Report No. LA-9774-MS, 1983 (unpublished).

[15]M. G. Rusbridge, Nucl. Fusion **22**, 1291 (1982).

[16]Guthrie Miller, Los Alamos National Laboratory Report No. LA-UR-83-2688, 1983 (to be published).