Unitary Flavor Unification through Higher Dimensionality

Paul H. Frampton and Katsuji Yamamoto
Institute of Field Physics, Department of Physics and Astronomy, University of North Carolina, Chapel Hill, North Carolina 27514
(Received 14 March 1984)

We study the phenomenon of spontaneous compactification for the case of anomaly-free gauge theories in six dimensions with manifold $M_4 \times S_2$ and in eight dimensions with manifold $M_4 \times S_4$. For the latter case, we show how to obtain, for example, Georgi's three-family SU(11) theory in Minkowski space, M_4.

PACS numbers: 11.15.Tk, 04.50.+h, 11.30.Ly, 11.30.Rd

Of all the fundamental interactions, gravity appears most elusive theoretically. We have candidate theories for strong and electroweak forces, both based on the notion of renormalizable local gauge symmetry. One attractive approach to include gravity is through supersymmetry and supergravity, and this topic is currently attracting a great amount of attention. Another approach which we shall follow here, not necessarily incompatible with the first, is to consider higher-dimensional space-times and their "spontaneous" compactification to four dimensions by choosing stable solutions to the generalized Einstein equations with less than the maximum available symmetry. By compactifying the extra dimensions to a sufficiently small unobservable size, one may hope to generate all interactions from essentially one. This ambitious program has a long and tempestuous history. Presently, the major questions are of stability of the solutions, and of the occurrence of massless states such as chiral fermions.

In particular, there has been much recent interest in gravity-coupled Yang-Mills gauge theories.[1-4] For consistency of the quantum version, one must start with a classical theory which avoids both gauge[5] and mixed gauge-gravity anomalies[6,7]; this severely restricts the choice of fermion representation.[8,9] In order to obtain chiral fermions in four dimensions, it is necessary[2,10] to compactify with gauge fields having a topologically nontrivial vacuum value in the extra dimensions. The vacuum configuration of the gauge field in the compact space may be, for example, a monopole[1,2] or an instanton.[4] Here we consider both these cases and discuss how to obtain interesting chiral fermions in four-dimensional Minkowski space.

The inclusion of gauge fields is contrary to the original spirit of Kaluza[11] and Klein[12] whose aim was unification with gravity. Although this "non-unified" approach seems necessary to obtain chiral fermions, we may hope that the gauge fields might be obtainable from a still higher dimension, for example, by use of quasi Riemannian geometry where the tangent group is other than the generalized Lorentz group.[13] If this were correct, then the present type of theory could be contained within a fully unified framework including gravity.

Let us consider an SU(N) gauge theory in some even-dimensional (pseudo) Riemannian space-time coupled to spin-$\frac{1}{2}$ Weyl fermions, to gravity, and possibly to scalar fields. The fermions must be chosen such that if the dimension is $d=2n$, then symmetrized traces over the products of $(n+1-2p)$ generators of the gauge group written in the fermion representation vanish for $p=0,1,2,\ldots$. The solutions of this algebraic problem are known[8] and are most conveniently written[9] in terms of superalgebras SU(N/M). Super Young tableaux $[m]^{(N/M)}$ correspond to the sums

$$\sum_{p=0}^{m}(-1)^p([m-p]^{(N)},\ \{p\}^{(M)}),$$

where $[m-p]^{(N)}$, $\{p\}^{(M)}$ denote totally antisymmetric, symmetric representations of SU(N), SU(M), respectively. In this article, we do not consider possible physical implications of SU(M) which we regard as merely a classification group.

The nature of the solution depends on whether d is of the form $d=4k$ or $d=(4k+2)$. In $d=(4k+2)$, we must chose an imaginary representation; for example, we may choose

$$[m]_{\mathscr{L}}^{(N/M)}-[N-M-m]_{\mathscr{L}}^{(N/M)}, \tag{1}$$

where $\mathscr{L}$ denotes left-handed chirality. In $d=4k$, a real representation is necessary; for example, with $(N-M)$ even,

$$[\tfrac{1}{2}(N-M)]_{\mathscr{L}}^{(N/M)} \tag{2}$$

is anomaly free in $d=4k$.

It is possible to show (as was conjectured in Ref.

9) that this procedure generates all possible solutions for the fundamental totally antisymmetric representations of SU(N).

Let us consider the case of $d=6$ dimensions, the lowest even number greater than four. To obtain chiral fermions in $d=4$, we must start with chiral fermions in $d=6$ in one of the above anomaly-free representations, and arrange that for the gauge field of a U(1) subgroup of SU(N) there is a monopole-like vacuum value in the 56-space.[14] With monopole charge n the number of massless Weyl spinors in $d=4$ can be computed, by use of monopole harmonics[1,2]; the answer is n times the eigenvalue of the U(1) generator, written such that the entries are relatively prime integers (i.e., no common factor). We assume an unbroken SU(5) naturally embedded so that the U(1) generated is $Y\sim(p,p,p,p,p,a_1,a_2,\ldots,a_{N-5})$ with $\sum_{i=1}^{N-5}a_1=-5p$. One can then show that[15]

$$[m]_{\mathscr{L}}^{(N/M)}\rightarrow np(m[m]_L^{(N-1/M)}+(m-N+M)[m-1]_L^{(N-1/M)}) \tag{3}$$

under spontaneous compactification. The subscripts $\mathscr{L}$, L denote left chirality in $d=6$, 4, respectively.

For minimal multiplicity we first consider $n=p=1$ in Eq. (3). To analyze the SU(5) content it is convenient to note that under SU(L) $\rightarrow$ SU($L-1$), with natural embedding,

$$[m]^{(L/M)}\rightarrow[m]^{(L-1/M-1)}. \tag{4}$$

In this notation, an SU(5) family in $d=4$ corresponds to an anomaly-free combination which may be written

$$[2]_L^{(5/1)}\quad\text{or}\quad([12]_L^{(5/2)}+[1]_L^{(5/2)}) \tag{5}$$

or in other forms (like $[1]_L^{(5/1)}+[3]_L^{(5/1)}$) equivalent up to real representations. The forms exhibited in Eq. (5) are connected by relations belonging to the type

$$[m]^{(L/M)}+[m-1]^{(L/M)}\equiv[m]^{(L/M-1)}. \tag{6}$$

Consequently, we may consider just $[2]_L^{(5/1)}$ (one family). Considering Eqs. (3) and (4) this can arise only by starting from the anomaly-free $d=6$ combination

$$[3]_{\mathscr{L}}^{(N/N-5)}-[2]_{\mathscr{L}}^{(N/N-5)}. \tag{7}$$

But after compactification, this, according to Eq. (3), gives $-10[2]_L^{(5/1)}$ up to real representations and hence the number of families obtained in this way is always a multiple of ten.[16] Because of this, such monopole-induced compactification looks unpromising and we are motivated to look at the next possible even dimension, $d=8$.

In $d=8$, we consider the compactification on to $M_4\times S_4$ and arrange a vacuum value for gauge fields in an SU(2) subgroup to be in an instanton configuration.[4] In this case, when we take only totally antisymmetric representations of SU(N), the chiral fermions follow the rule

$$[m]_{\mathscr{L}}^{(N/M)}\rightarrow[m-1]_L^{(N-2/M)} \tag{8}$$

in spontaneous compactification from $d=8$ to $d=4$. Note that Eq. (8) is so simple because only the doublets of the SU(2) subgroup survive. To obtain one family in the form $[2]_L^{(5/1)}$ we may start from $[3]_{\mathscr{L}}^{(7/1)}$ with an SU(7) gauge group, for example.

This makes it interesting to study previously suggested unitary flavor unifications.[17,18] To obtain Georgi's SU(11) model, we start from the fermion representation

$$[5]_{\mathscr{L}}^{(13/3)}+2[4]_{\mathscr{L}}^{(13/5)}+3[3]_{\mathscr{L}}^{(13/7)} \tag{9}$$

which reduces to the three-family combination, with unit coefficients, $[4]_L-[3]_L-[2]_L-[1]_L$ under SU(11) in $d=4$. The other models of a similar type[18] can likewise be reproduced in SU(7) and SU(9).

It is important to require that the vacuum field configuration correspond to at least a perturbatively stable solution. The stability of spontaneously compactified models has been studied by several authors[1,2,4,19-22] but the results are still incomplete. It has been conjectured,[19] on the basis of several explicit examples, that the spontaneously compactified solutions of gravity-coupled gauge theories are stable if the gauge group is contained in the isotropy group of the compact space. For example, the compactification from $d=8$ with gauge group SU(2) would be stable but with larger SU(N) would be unstable.

We should, therefore, use as the initial gauge group in $d=8$, SU(N) $\otimes$ SU(2) and keep only doublets of SU(2). To obtain the Georgi model[17] we therefore use

$$([4]_{\mathscr{L}}^{(11/3)}+2[3]_{\mathscr{L}}^{(11/5)}+3[2]_{\mathscr{L}}^{(11/7)},\ 2)$$

$$+3([3]_{\mathscr{L}}^{(11/7)}+[1]_{\mathscr{L}}^{(11/7)},\ 1)$$

under SU($11/M$) $\otimes$ SU(2). This solution has been shown to be perturbatively stable.[4]

After spontaneous compactification, the local gauge symmetry is SU(N) ⊗ O(5) where O(5) is the isometry group. The isometry group is unbroken but in the present case there are no massless fermions with nontrivial O(5) couplings. Thus the O(5) acts purely as a spectator in the light sector. There are massive particles which have nontrivial O(5) transformations, but these states may be very heavy.

In the light sector the gauge group SU(N) needs to be broken down to the phenomenological groups SU(3) ⊗ [SU(2) ⊗ U(1)] and SU(3) ⊗ U(1). As candidate Higgs scalars there are the components 5,6,7,8 of the gauge fields in the adjoint of SU(N). To decrease rank, however, we need scalars with nonzero N-ality. It seems that these must be put in by hand unless we hypothesize a dynamical symmetry-breaking mechanism.

To summarize, we have seen how through spontaneous compactification anomaly-free gauge theories lead to certain chiral fermions in four dimensions. It appears most attractive to begin with $d=8$, and then use an instanton-induced compactification. In this way, we can reproduce in $d=4$ flavor unification schemes that have been previously studied. We have given only a summary of results; details will be given elsewhere.[15] Our main point is that the chiral fermions of a gauge theory in a higher-dimensional space-time are severely restricted by the requirement of survival of acceptable massless fermions in four space-time dimensions.

[1]Z. Horvath, L. Palla, E. Cremmer, and J. Scherk, Nucl. Phys. **B127**, 57 (1977).

[2]S. Randjbar-Daemi, A. Salam, and J. Strathdee, Nucl. Phys. **B214**, 491 (1983).

[3]S. Randjbar-Daemi, A. Salam, and J. Strathdee, Phys. Lett. **124B**, 349 (1983).

[4]S. Randjbar-Daemi, A. Salam, and J. Strathdee, Phys. Lett. **132B**, 56 (1983).

[5]P. H. Frampton and T. W. Kephart, Phys. Rev. Lett. **50**, 1343, 1347 (1983), and Phys. Rev. D **28**, 1010 (1983), and Phys. Lett. **131B**, 80 (1983); P. K. Townsend and G. Sierra, Nucl. Phys. **B222**, 493 (1983).

[6]R. Delbourgo and A. Salam, Phys. Lett. **40B**, 381 (1972); T. Eguchi and P. G. O. Freund, Phys. Rev. Lett. **34**, 1251 (1976).

[7]L. Alvarez-Gaumé and E. Witten, Nucl. Phys. **B234**, 269 (1984).

[8]P. H. Frampton, University of Texas at Austin Report No. UTTG-13-83, 1983, Phys. Lett. (to be published).

[9]L. Caneschi, G. R. Farrar, and A. Schwimmer, Phys. Lett. **138B**, 386 (1984).

[10]G. F. Chapline and B. Grossmann, Phys. Lett. **135B**, 109 (1984); E. Witten, to be published.

[11]Th. Kaluza, Sitzunsgber, Preuss. Akad. Wiss. Math. Phys. Kl. **1921**, 966.

[12]O. Klein, Z. Phys. **37**, 895 (1926).

[13]S. Weinberg, Phys. Lett. **138B**, 47 (1984).

[14]We are here ignoring the requirement of stability; this can, however, easily be rectified without affecting the counting of families by use of SU($N-1$) ⊗ U(1) with the monopole in U(1) and with SU($N-1$) as a spectator gauge group.

[15]P. H. Frampton and K. Yamamoto, University of North Carolina at Chapel Hill Report No. IFP 222-UNC, 1984 (to be published).

[16]The present discussion is somewhat heuristic because complete proofs are lengthy and complicated. Details are provided in Ref. 15.

[17]H. Georgi, Nucl. Phys. **B156**, 126 (1979).

[18]S. Dimopoulos, P. H. Frampton, H. Georgi, and M. B. Wise, Phys. Lett. **117B**, 185 (1982); P. H. Frampton, Phys. Lett. **88B**, 299 (1979), and **89B**, 352 (1980).

[19]S. Randjbar-Daemi, A. Salam, and J. Strathdee, Phys. Lett. **124B**, 345 (1983).

[20]T. Appelquist and A. Chodos, Phys. Rev. Lett. **50**, 141 (1983), and Phys. Rev. D **28**, 772 (1983); T. Appelquist, A. Chodos, and E. Myers, Phys. Lett. **127B**, 51 (1983).

[21]E. Witten, Nucl. Phys. **B195**, 481 (1982).

[22]P. Candelas and S. Weinberg, Nucl. Phys. **B237**, 397 (1984).