Chaotic Phases and Magnetic Order in a Convective Fluid

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Studies of different types of transitions to chaos in Rayleigh-Bénard convection in a cylindrical container of mercury subjected to a horizontal magnetic field are reported. As the field is increased, structural changes of the spatial pattern are connected with different time-dependent behaviors and routes to chaos.

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A problem of interest is the onset of chaos in physical systems with many degrees of freedom, and an important question to investigate is whether and how dynamical system theory can be extended to that case. We report here an experimental investigation about the influence of the number of unstable modes on the transition to chaos in a layer of mercury heated from below and subjected to a horizontal magnetic field. For small magnetic fields ($B \lesssim 1000$ G) a random spatial pattern sets in at the convection onset and an abrupt transition to time-dependent chaotic behavior is observed. For large magnetic fields ($B \gtrsim 3000$ G) the convective flow consists of rolls parallel to the magnetic field axis, and the route to chaos involves a limit cycle that undergoes subharmonic bifurcations. In the intermediate range of magnetic field amplitude, several bands of periodic states exist within the chaotic region.

Chaotic regimes just above the onset of convection have been observed in experiments with small Prandtl number fluids, liquid helium[1] ($P \sim 1$) and mercury[2] ($P \simeq 0.025$), with large aspect-ratio containers. (The aspect ratio is $\Gamma = L/d$ where L is the characteristic horizontal scale and d the fluid layer height.) Qualitative differences in the sequence of events leading to chaos have been observed by decreasing the aspect ratio.[1,3] Experiments with mercury[2] have shown that the container shape plays also an essential role: in parallelepiped containers of dimensions $50\times30\times4.8$ mm and $32\times32\times5.3$ mm a periodic regime follows a stationary one as the Rayleigh number is increased, whereas for a smaller aspect ratio but cylindrical container of 18 mm radius and 6 mm height, a chaotic time dependence is observed just above the onset of convection.

We present here the effect of a horizontal magnetic field on convective regimes in this container. In our experimental setup, described elsewhere,[4] we have obtained local temperature measurements with a negative-temperature-coefficient thermistor, and convective pattern visualization with a layer of cholesteric liquid crystal.

The different flow regimes are plotted in Fig. 1, in a two-parameter space R, Q. R is the Rayleigh number, $R = g\alpha d^3 \Delta T/\nu K$, where g is the acceleration of gravity, α is the isobaric thermal expansion coefficient, ν is the kinematic viscosity, K is the heat diffusivity, and ΔT is the temperature difference across the fluid layer. The Chandrasekhar number Q measures the ratio between the anisotropic viscosity due to the magnetic field B, and the kinematic viscosity; $Q = \sigma B^2 d^2/\rho\nu$, where σ is the electrical conductivity and ρ the fluid density.

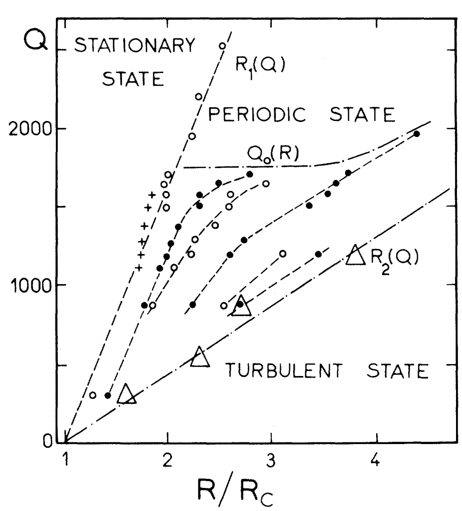

FIG. 1. Nature of the time-dependent states as a function of the Rayleigh and Chandrasekhar numbers. The dashed curves indicate a transition between two different regimes. $R_1(Q)$ corresponds to the onset of time-dependent flow. Different time dependences observed are damped transient oscillations (plusses), sustained oscillations (open circles), or chaotic time dependence (closed circles). For $R = R_2(Q)$, the heat fluxes with and without magnetic field join together (triangles). Above the dash-dotted line $Q_0(R)$, the time dependence is periodic (oscillation between simple patterns).

In the absence of magnetic field ($Q=0$), above the critical Rayleigh number R_c for the convection onset, the convective pattern consists of hot ascending and cold descending fluid regions, randomly distributed in space, and moving on a slow time scale compared with the vertical heat diffusion characteristic time ($d^2/K \simeq 10$ sec). A chaotic time dependence with a broadband frequency spectrum is associated with these motions.

When a horizontal magnetic field is applied, the convective pattern takes the form of rolls parallel to the field axis, and the flow becomes stationary. The field inhibits convective modes with velocity variation along its axis, but does not affect rolls parallel to its axis.[5] Consequently the critical Rayleigh number R_c for the onset of convection is unchanged (see Fig. 2). For $R > R_c$, the heat flux initial slope changes discontinuously from the slope without magnetic field [curve labeled (1) in Fig. 2] to the slope with magnetic field [curve labeled (2)]. This is in agreement with nonlinear perturbation theory at the convection onset,[6] demonstrating that parallel rolls are associated with the highest convective heat transport when the top and bottom boundaries have a much higher heat conductivity than the fluid. For a given magnetic field (for instance $Q=1200$ in Fig. 2), one follows the curve (2) at convection onset and leaves it for $R > R_1(Q)$. This corresponds to the first time dependence always associated with three-dimensional patterns. With increasing Rayleigh number different time-dependent regimes (periodic or chaotic) are observed. They depend on the Chandrasekhar number and are associated with different spatial patterns we shall describe below. When the Rayleigh number reaches the value labeled $R_2(Q)$ in Fig. 1, the heat fluxes with and without magnetic field become equal (see Fig. 2, $Q=300$ and 1200); the spatial order is completely lost.

Let us now describe the scenario leading to chaos at high Chandrasekhar number. In the diagram of Fig. 1 we follow the horizontal line $Q=2200$. For $R=R_1=2.3R_c$, the stationary parallel-rolls state loses its stability, and the temperature measurements show the occurrence of a periodic state [Fig. 3(b)]. This transition has no hysteresis and the limit cycle appears for $R=R_1$ with finite amplitude and vanishing frequency. More precisely, the portion of the limit cycle designated by 1 increases indefinitely when $R \rightarrow R_1$ from above. This transition corresponds to a saddle-node bifurcation. We have sketched in Fig. 3(a) the spatial patterns that correspond to the temperature recording of Fig. 3(b). The system oscillates between two parallel-rolls patterns, with hot (cold) fluid ascending (descending) in the center of the container. The transitions between these states involve three-dimensional patterns, generated after the tilting of the rolls, and corresponding to the peaks of the temperature recording. For $R > R_1$ subharmonic bifurcations occur ($R/R_c=3.95$ and 4.05) and the

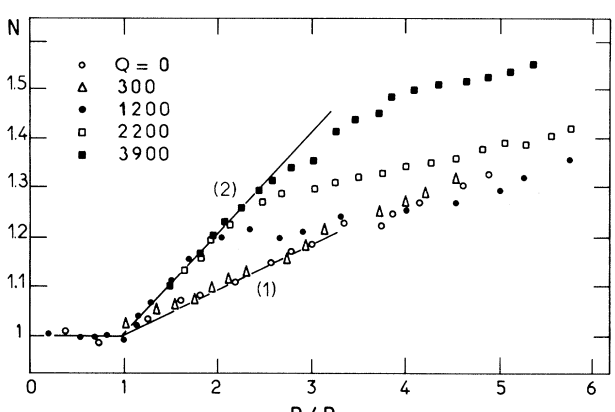

FIG. 2. Heat flux characterized by the Nusselt number vs the Rayleigh number, for different values of the Chandrasekhar number.

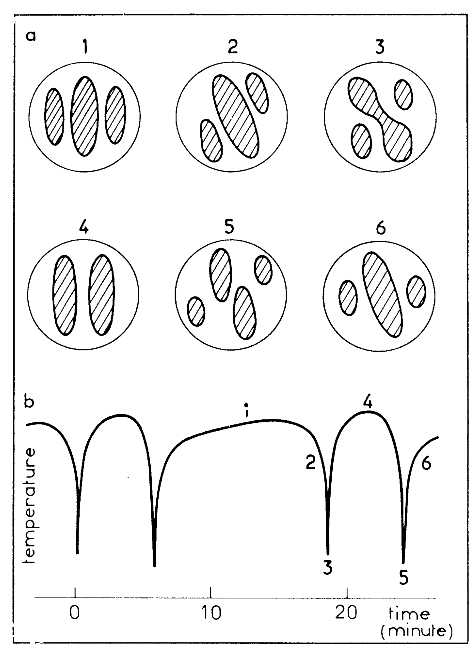

FIG. 3. (a) Sketches of convective patterns associated with the periodic state for $Q=2200$. The shaded regions correspond to hot ascending fluid visualized by a layer of cholesteric liquid crystal placed under the container top boundary made of sapphire. (b) Direct temperature recording for $Q=2200$ associated with the patterns of (a).

time dependence becomes chaotic for $R=4.4R_c$, where noise, exponentially decreasing with frequency, appears in the power spectrum.

In the intermediate range of Chandrasekhar number $(300 \lesssim Q \lesssim 2000)$, other types of transition to chaos are observed. For $R \lesssim R_1(Q)$, a small step in the heating current causes a damped transient oscillation with a characteristic period $T_0 \simeq 10d^2/K$. As the Rayleigh number is increased above $R_1(Q)$, let us describe two characteristic behaviors for $Q=300$ and $Q=1200$.

For $Q=300$ and $R=R_1=1.3R_c$, a small-amplitude nearly sinusoidal limit cycle is sustained. This transition is a supercritical Hopf bifurcation. Our visualization setup shows that it corresponds to a transverse oscillation of the parallel rolls. As R is increased Fourier analysis shows the noise amplitude increases in the temperature frequency spectrum. However, oscillating rolls remain visible indicating that a large amount of the flow kinetic energy is contained in a small spatial wave-number range. The spatial pattern order is destroyed only at higher Rayleigh number.

For higher Chandrasekhar numbers $(Q=1200)$, the evolution to turbulence involves an apparently more complex process, with alternation of chaotic and periodic bands. For $R \lesssim R_1(Q)=1.8R_c$ the damped oscillation described above is observed.

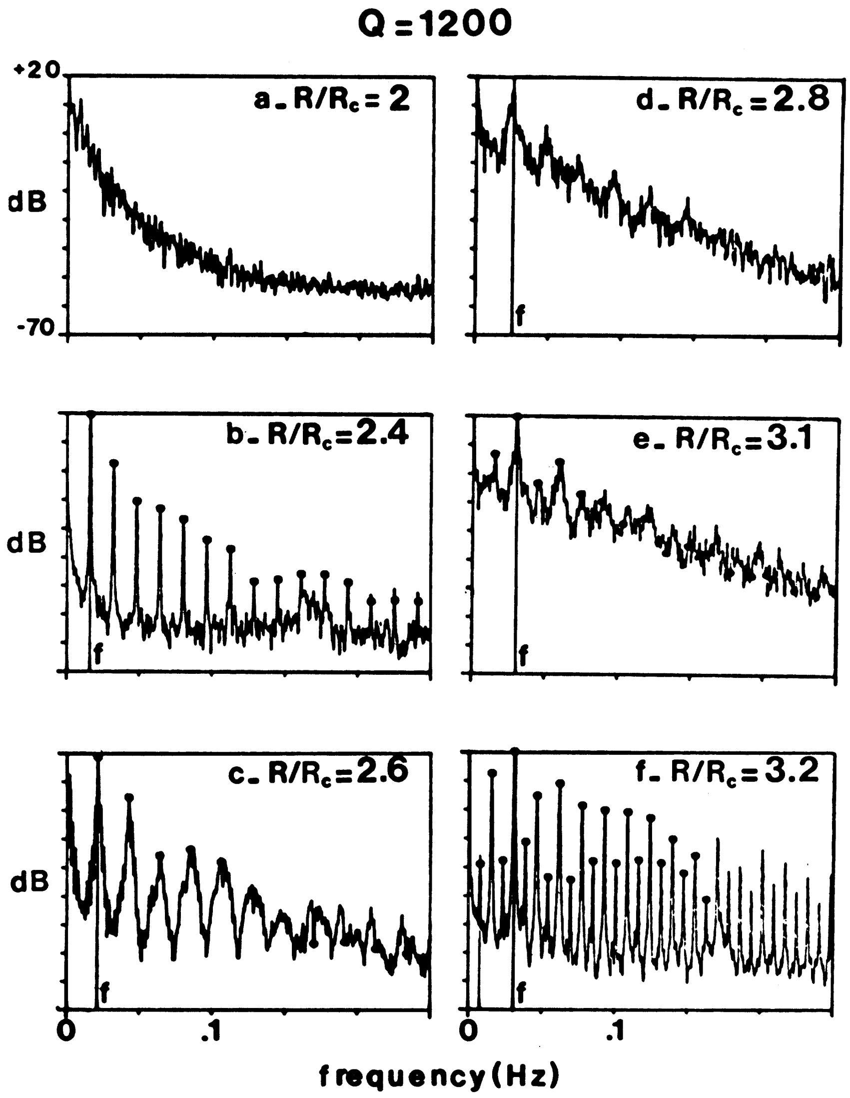

FIG. 4. Frequency spectrum of the temperature for $Q=1200$. During the alternation of chaotic (a),(d), and periodic (b),(f) regimes, the limit cycle of frequency f is preserved and undergoes subharmonic bifurcations $f/2$ and $f/4$.

Then a sharp transition to chaos occurs for $R=R_1(Q)$, and Fourier analysis shows a broadband frequency spectrum [Fig. 4(a)]. However, as the Rayleigh number is increased, the noise decreases and concomitantly a peak at frequency $f \simeq 0.1K/d^2$ emerges [Fig. 4(b)]. It corresponds to the transverse oscillation of the rolls. Then a new transition to chaos occurs after the appearance of a second mode of frequency much smaller than the first one [Figs. 4(c) and 4(d)]. A further increase in Rayleigh number shows laminarization again, but this time concomitant with the appearance of the frequency $f/2$ and then $f/4$ [Figs. 4(e) and 4(f)]. These two relaminarizations correspond to a decrease of the convective heat flux with increasing R (see Fig. 2, $Q=1200$) and therefore to structural changes in the spatial pattern.

In this experiment the magnetic field is a control parameter for the number of degrees of freedom of the system, and determines the nature of the transition to chaos as the Rayleigh number is increased. In the range $0<Q<5000$, the transition to time dependence always involves three-dimensional patterns. This is known for higher Prandtl number fluids and is associated with the skewed-varicose instability,[7] but somewhat surprising with mercury when a large magnetic field two-dimensionalizes the motions. We have observed two kinds of periodic states, rather similar to those recently reported in experiments with liquid helium.[8,9] The diagram of Fig. 1 can be understood in terms of a phase diagram for our physical system, where each curve represents a transition where spatial or temporal order is lost. The curve $R_1(Q)$ corresponds to the onset of time dependence. For all parameter space above $Q_0(R)$, this time dependence involves the interaction of only a few spatial modes, and the transition to chaos has the same characteristics as in low-dimensional systems. For $Q<Q_0(R)$ and $R<R_2(Q)$ the transition to chaos occurs concomitantly with structural changes of the flow manifested in the Nusselt number measurements. For $R>R_2(Q)$ the spatial order is completely lost. This definition of the "turbulent" state frontier of Fig. 1 is somewhat arbitrary, and spatial Fourier spectra of velocity or temperature are necessary for a more quantitative study of this regime.

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