Synchrotron Radiation and Channeling of Ultrarelativistic Particles

J. C. Kimball and N. Cue
Physics Department, State University of New York at Albany, Albany, New York 12222
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A new and relatively simple theory of channeling radiation by ultrarelativistic particles, based on synchrotron radiation, yields promising results. The theory incorporates important quantum electrodynamic corrections, and provides detailed predictions of enhanced radiation.

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The radiation intensity produced by energetic electrons and positrons directed along crystal axes can be much larger than the normal bremsstrahlung radiation produced when the same particles travel in a random direction. Theoretical investigations which treated the radiation classically have successfully explained many aspects of this anomalous "channeling radiation."[1] However, at very high particle and photon energies, quantum electrodynamic (QED) corrections become important. A theory which considers these corrections is presented here.

Our description of channeling radiation views the aligned crystal as presenting a distribution of transverse electric fields.[2] Synchrotronlike radiation then arises from the (nearly) circular path segments traversed by the acclerated channeling particle in these fields. In this approach, all QED effects are easily incorporated in the calculations of the radiation. The synchrotron approximation to channeling radiation is expected to be valid when the incident particle and radiated photon energies are both sufficiently large.

The importance of QED corrections to classical synchrotron radiation is commonly expressed in terms of the dimensionless ratio[3]

$$\chi = \hbar\omega_c/3\epsilon, \qquad (1)$$

where ϵ is the energy of the incident particle, and ω_c is the *classical* synchrotron frequency.[4] When χ is on the order of unity, classical theory improperly predicts that a large fraction of the radiated photons will have more than the total incident energy. Even when $\chi \sim \frac{1}{10}$, QED corrections are significant. For the channeling situation, χ can be expressed in terms of the crystal electric field, E, and the energy of the incident particle:

$$\chi = \gamma |E| a_0^2/137^3 e, \qquad (2)$$

where $\gamma = \epsilon/mc^2$, a_0 is the Bohr radius, and e is the charge of the proton. In real crystals, Ea_0^2/e can exceed 100 near a string of atoms, and present-day accelerators can produce electrons and positrons with γ's in excess of 10^5. Thus χ can be quite large. Earlier, Wedell[5] obtained a more conservative estimate of χ, and concluded that QED corrections were unimportant for incident particle energies less than 300 GeV. Our analysis shows the effects to set in at considerably lower energies.

Basic to our synchrotron-radiation analogy is the emphasis on the instantaneous acceleration of the channeled particle. Velocity correlations associated with noncircular motion are neglected. These correlations occur over relatively long time periods, and give structure to the radiation spectrum at frequencies comparable to the dipole radiation frequency, ω_d. In our case, the spectral density is peaked near the synchrotron frequency, ω_c, and so we expect our results to accurately represent the bulk of the radiation only when $\omega_c > \omega_d$.

The dipole frequency is related to the transverse oscillation frequency, ω_0, of the particle in the channel. Roughly, $\omega_0 = (K/\gamma m)^{1/2}$, where K is an effective spring constant. Associating K with the force divided by the distance, $d = sa_0$, which characterizes the amplitude of oscillation gives

$$\omega_0 \sim (eE/sa_0\gamma m)^{1/2}. \qquad (3)$$

Photons emitted in the forward direction are Doppler shifted by $\sim 2\gamma^2$, so that the resulting photon energy is approximately

$$\hbar\omega_d = 2\epsilon(\chi/137s)^{1/2}. \qquad (4)$$

With use of these expressions for ω_c and ω_d, the condition $\omega_c > \omega_d$ can be written as

$$(137s)\chi > \tfrac{4}{9}. \qquad (5)$$

When the emitted photon energy is greater than half the energy of the incident particle, this validity criterion should be modified to take into account the spatial variation of the field E.

In channeling, the most significant contribution

TABLE I. Estimated $e^{\pm}$ beam energies (in gigaelectronvolts) above which the synchrotron approximation is expected to be increasingly valid.

Crystal	Axis	ϵ_{crit}	Plane	ϵ_{crit}
Diamond	⟨110⟩	3.7	(110)	20.5
Si	⟨110⟩	2.9	(110)	19.0
Ge	⟨110⟩	1.5	(110)	10.9
W	⟨100⟩	0.47	(110)	4.5

to the radiation comes from particles in the largest electric fields, and the maximum field occurs at a distance from the string or plane which is comparable to the root-mean-square thermal vibration amplitude. By associating sa_0 with this distance, we find that $137s$ is generally larger than unity. Thus under the experimental situations where QED is important ($\chi > \frac{1}{10}$) one also expects high-frequency channeling radiation to be adequately described in terms of synchrotron radiation ($137s\chi > \frac{4}{9}$). Estimates of the particle energies necessary for the synchrotron approximation to apply are shown in Table I. Much larger energies are required for planar channeling because the maximum electric fields are nearly an order of magnitude smaller. The numerical results were obtained by use of a superposition of atomic Thomas-Fermi potentials (Moliere approximation) to produce the channeling potential.

Our calculations of the channeling radiation intensity are shown in Fig. 1 for thin targets (uniform particle distribution in the transverse plane) and incident particles with energies of 5, 20, or 100 GeV directed along the Si ⟨110⟩ axis. The comparisons of the classical and QED synchrotron formulas shown in this figure indicate that QED effects (such as the quantum recoil during photon emission) are discernible at 5 GeV, and they become increasingly significant with increasing beam energy.

There are no experimental data which can be directly compared with these calculations. A proper comparison of our results with presently available data is possible when corrections for two factors are considered. First, these data are restricted to very low energies (on the scale of Fig. 1) where the synchrotron-radiation analogy must be modified through the introduction of an effective path length. In addition, the targets may not be sufficiently thin to justify the assumption of a uniform distribution of particle flux, and so other possible flux distributions should be considered.

When photon frequencies are sufficiently small,

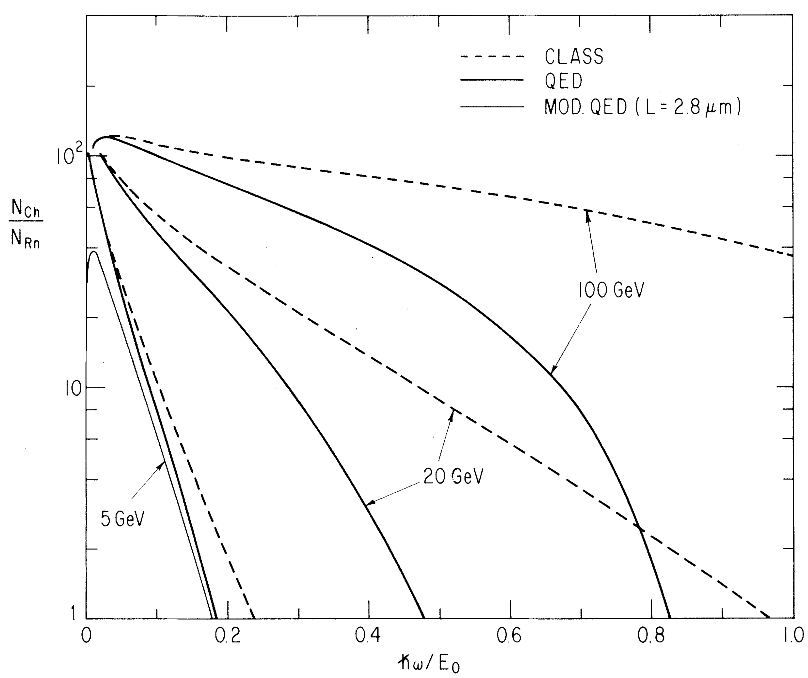

FIG. 1. Radiated photon energy spectra, obtained in the synchrotron approximation, for a uniform flux of electrons or positrons directed along the Si ⟨110⟩ axis. All results are normalized by the corresponding ordinary bremmstrahlung spectrum for screened atoms. Calculations using a finite path length of $L = 2.8$ μm are also shown for 5 GeV.

$\omega \sim \omega_d$, the radiated photon amplitude is determined by the motion of the channeled particle over relatively large distances. Effects associated with noncircular channeling paths, beam angular divergence and misalignment, and crystal imperfections lead to an effective path length, L, which limits the distance over which the acceleration can contribute to the radiation amplitude. The integrals which determine the amplitude of electromagnetic radiation are "cutoff" at $\pm L/2$ rather than extended to infinity. This reduces the low-frequency radiation intensity.

The dominant integral which determines the classical electromagnetic field amplitude in synchrotron radiation is[4]

$$A = (\text{const}) \int_0^\infty t \sin(t^3 \omega \omega_c^2/54\gamma^6 + \omega t/2\gamma^2)\, dt, \quad (6)$$

for radiation in the plane of the acceleration. The introduction of the path length L changes the upper limit of integration from infinity to $t_{\max} = L/2c$. For reasonable values of L ($\sim$ micrometers) the resulting reduction of the integral is well approximated by

$$R = 1 + 0.47(\omega_c/\omega)^{1/3}(\lambda/L)/[(137s)\chi]^{1/2}. \quad (7)$$

The length λ which appears in this expression is $2\pi c/\omega_0$. It is an estimate of the distance traveled by the channeling particle while making one oscillation. Under good channeling conditions, L can be comparable with λ. Squaring R gives the factor which reduces the synchrotron radiation intensity. We assume that this same correction factor applies even when the proper QED synchrotron radiation intensities are used.

The modified calculations, using $L = 2.8$ μm (~ 5 times the minimum λ) for 5-GeV electrons or positrons along the Si $\langle 110 \rangle$ axis, are also displayed in Fig. 1. The decreasing significance of the correction with increasing photon energy is clearly seen.

For relatively thick crystals, the distribution of particle flux is difficult to measure or calculate since it depends on crystal orientation and quality, beam energy and angular divergence, and small-angle multiple scattering. Two extreme cases are the "equilibrium" flux and "uniform" flux. The equilibrium flux results from energy conservation and statistical equilibrium for motion in the transverse plane. This distribution is an idealization which generally overestimates the concentration of particles in the low-energy regions of the channels. The uniform flux distribution applies when the crystal is so thin that significant particle redistribution cannot take place.

Comparisons with essentially all the experimental results so far obtained,[6,7] which satisfy the applicability requirement specified in Table I, are shown in Figs. 2 and 3. Theoretical curves based on both the uniform (UN) and the equilibrium (EQ) flux distributions are compared with each experiment. Since the experimental data are restricted to energies which are sensitive to the path length L, the theory contains one free parameter. The other undetermined quantity in the theory is the flux distribution. As is seen in Fig. 2, a comparison of the theory for the two extreme cases of the uniform and the equilibrium flux shows that this distribution has an effect on the shape of the radiation spectrum.

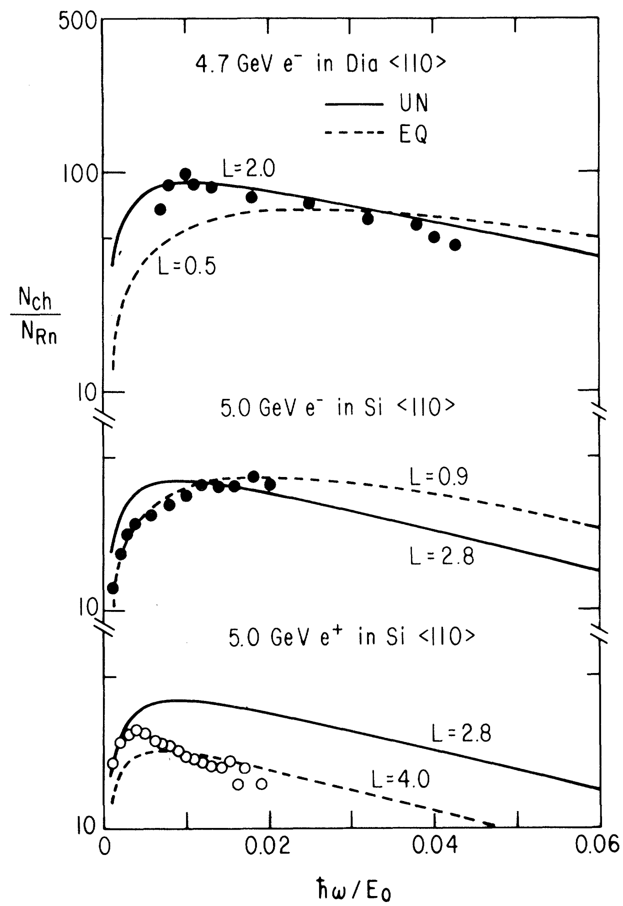

FIG. 2. Comparisons of the theory corrected for finite path length (L in micrometers) with experiments on silicon (Ref. 6) and diamond (Ref. 7) in the axial direction. The solid and dashed curves are based on the uniform (UN) and equilibrium (EQ) particle flux distributions.

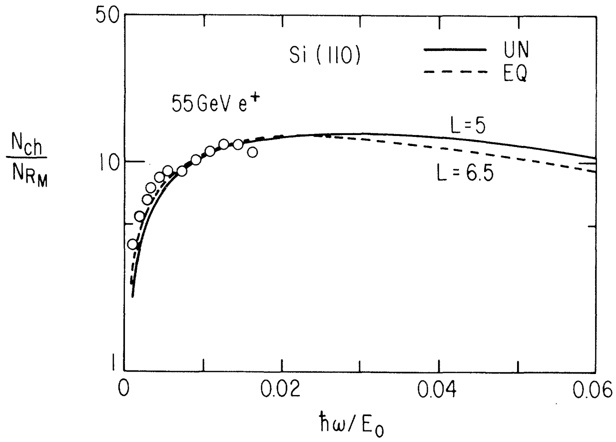

FIG. 3. A comparison similar to that shown in Fig. 2 for planar-channeling data on silicon (Ref. 6).

It is tempting to conclude from the spectral-shape comparisons that our calculations show a nearly uniform flux for the diamond experiment, and a nearly equilibrium flux for the silicon experiment. This conclusion is probably not justified because the assumption $\omega_c > \omega_d$ is but marginally satisfied, and the data may contain vestiges of dipole resonance peaks at low photon energy. However, it is clear that only modest improvements in the data would enable the determination of the effective path length, L, and the channeling flux distribution.

In summary, the theory presented here provides the spectrum of very-high-energy channeling radiation with a simple physical interpretation. The direct connection of the crystal field strengths and flux distributions to the radiation intensities gives the experiments new significance. At sufficiently high energies, the radiation produced by electrons and positrons is predicted to differ only because of differing flux distributions. Thus measurements of the radiation from these particles provide direct information on the flux distributions. In particular, for uniform flux, the theory predicts identical radiation spectra for electrons and positrons. Quantum electrodynamic corrections are uniquely important in channeling radiation because the effective fields and forces experienced by channeling particles are larger than in any other terrestrial system. Only in astrophysics[8] do comparable effects occur. Although few would question the validity of the QED expressions for synchrotron radiation, it is worth noting that channeling radiation measurements provide a unique opportunity to verify these important nonclassical results.

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