Inflation without Tears: A Realistic Cosmological Model

So-Young Pi
Department of Physics, Boston University, Boston, Massachusetts 02115
(Received 10 February 1984)

An inflationary model which satisfies constraints from particle physics and cosmology is presented. The key feature is an SU(5)-singlet field which drives inflation, leads to proper density fluctuations, and solves the strong *CP* problem by the invisible axion mechanism. Thus, for the first time, all reservations about the inflationary universe are removed.

PACS numbers: 98.80.−k, 12.10.En, 14.80.Gt

The inflationary universe[1] provides solutions for the flatness, horizon, and monopole overabundance problems in the standard cosmological model. Furthermore, it has a natural explanation for the origin of initial inhomogeneities, necessary for galaxy formation, which produce roughly scale-invariant density fluctuations $\Delta\rho/\rho$.[2] However, the realization of the inflationary scenario in the standard SU(5) model[3] with a Coleman-Weinberg potential[4] has the serious difficulty that the magnitude of the fluctuations, when they enter the horizon, is about 10^5 times too large. These large fluctuations arise from the large gauge coupling constant α $(=e^2/4\pi)$ in the Coleman-Weinberg potential. $\Delta\rho/\rho$ has been found to be of order $10^3\alpha$ or about 50 [using the running coupling constant of unbroken SU(5) at a scale 10^{10} GeV, $\alpha^2\approx10^{-3}$].[2] According to Harrison and Zeldovich,[5] $\Delta\rho/\rho$ should be $O(10^{-4})$ which would require α of order 10^{-7}. Such an extremely small coupling cannot be expected if the one-loop effective potential involves gauge field contributions. Therefore, one may have to look for a Coleman-Weinberg potential without a gauge coupling constant—for example, a model in which an SU(5)-singlet scalar field drives inflation. Such a model has been recently considered by Shafi and Vilenkin.[6] They have added to the minimal SU(5) model a real singlet scalar field, which drives inflation, and the resultant $\Delta\rho/\rho$ is about 10^4. However, in order to build a realistic model for particle physics and cosmology one needs a justification for this singlet field.

The purpose of this paper is to present a model in which the inflation-driving singlet field is needed for particle physics reasons. The model has the following features: (a) It contains the Peccei-Quinn $U(1)_A$ symmetry which is needed to solve the strong *CP* problem.[7] (b) It possesses a complex singlet field, which contains the axion[8] and which drives inflation. (c) $\Delta\rho/\rho\lesssim10^{-4}$.

The model involves one real **24** representation (Φ), two **5**'s $(H_{1,2})$, and one complex singlet (ϕ). Fermions are in **10** (ψ_L) and **5*** (χ_R) representations. The Lagrangian is invariant under

$$\phi\to e^{i\omega/2}\phi,\quad \psi_L\to e^{-ix_1\omega/2}\psi_L,\quad H_1\to e^{ix_1\omega}H_1,\quad \chi_R\to e^{-i(x_1/2+x_2)\omega}\chi_R,\quad H_2\to e^{ix_2\omega}H_2, \tag{1}$$

where $x_1-x_2=1$. Equation (1) contains a $U(1)_A$ symmetry. I shall assume that the scalar potential has only scale-invariant couplings, as in the Coleman-Weinberg case:

$$V=V_1+V_2, \tag{2a}$$

$$V_1=\tfrac14c(\mathrm{Tr}\Phi^2)^2+\tfrac12b\mathrm{Tr}\Phi^4+\sum_{i=1}^{2}\{\alpha_iH_i^\dagger H_i\mathrm{Tr}\Phi^2+\beta_iH_i^\dagger\Phi^2H_i+\tfrac14\lambda_i(H_i^\dagger H_i)^2\}$$
$$+K(H_1^\dagger H_1)(H_2^\dagger H_2)+\delta(H_1^\dagger H_2)(H_2^\dagger H_1), \tag{2b}$$

$$V_2=\tfrac14f(\phi^\dagger\phi)^2+\tfrac12g\phi^\dagger\phi\mathrm{Tr}\Phi^2+\tfrac12\sum_{i=1}^{2}\gamma_iH_i^\dagger H_i\phi^\dagger\phi+\eta[H_1^\dagger H_2\phi^2+H_2^\dagger H_1\phi^{*2}]. \tag{2c}$$

(The coupling constant η has been taken real without loss of generality because any phase may be absorbed into the complex scalar fields.) V_1 contains only SU(5)-nonsinglet fields and the couplings in V_1 are of order e^2 (except that β_i can be smaller to make colored Higgs triplets light). V_2 involves the singlet field ϕ and the couplings in V_2 are small: $g, \gamma_i, \eta \lesssim O(10^{-6})$ and $f \lesssim \max(g^2, \gamma_i^2, \eta^2)$. Assuming such small couplings in the scalar potential is not necessarily unnatural. V has the tree-level minimum at $\langle\Phi\rangle = \langle H_{1,2}\rangle = \langle\phi\rangle = 0$. The radiative correction to V_1 is negligible, but V_2 gets a large one-loop correction:

$$V_2^{(1)} = (1/64\pi^2)[24g^2 + 10(\tfrac{1}{4}\gamma_1^2 + \tfrac{1}{4}\gamma_2^2 + \eta^2)]|\phi|^4 \ln|\phi|^2/\mu^2 + \text{const}, \tag{3}$$

where μ is an arbitrary renormalization point. Notice that once ϕ develops a nonvanishing expectation value, Φ and $H_{1,2}$ get effective mass terms through their couplings to ϕ. For $g < 0$, and $15c + 7b > 0$, Φ develops an expectation value

$$\langle\Phi\rangle = [v/(15)^{1/2}]\,\mathrm{diag}(1,1,1,-\tfrac{3}{2},-\tfrac{3}{2}), \tag{4a}$$

$$v^2 \equiv [30/(15c+7b)]|g||\phi|^2. \tag{4b}$$

Ignoring $H_{1,2}$ one can write the potential V as

$$V = \frac{15c+7b}{240}v^4 - \frac{1}{4}|g||\phi|^2 v^2 + \frac{1}{4}B\left[|\phi|^4\left(\ln\frac{|\phi|^2}{M^2} + \frac{15g^2}{4B(15c+7b)}\right) + \frac{1}{2}(M^4 - \phi^4)\right], \tag{5}$$

where $B = (1/16\pi^2)[24g^2 + 10(\tfrac{1}{4}\gamma_1^2 + \tfrac{1}{4}\gamma_2^2 + \eta^2)] \simeq 10^{-12}$. The one-loop correction of Eq. (3) has been included in Eq. (5). M has been chosen such that the minimum of V occurs at $\langle\phi\rangle = M$ and $v = \{[30/(15c+7b)]|g|\}^{1/2}M$. For $b, c \sim O(e^2)$ and $|g| \simeq 10^{-6}$, M has to be of order 10^{18} GeV, in order to have $v \simeq 10^{15}$ GeV $\approx M_X$. For a finite range of parameters and $\beta_i, \eta < 0$, $H_{1,2}$ develop nonvanishing vacuum expectation values

$$\langle H_{1,2}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\0\\0\\h_{1,2}\end{pmatrix}. \tag{6}$$

In order to have small $h_{1,2} \approx O(M_W \approx 100 \text{ GeV})$, I require the usual hierarchy condition

$$(\alpha_i + \tfrac{3}{10}\beta_i)v^2 + \gamma_i M^2 + 2\eta M^2 \frac{h_j}{h_i} \approx O(M_W^2), \quad i \neq j = 1,2. \tag{7}$$

(The vacuum expectation values $\langle\phi\rangle = Me^{i\theta_M}$ and $\langle H_{1,2}\rangle = (1/\sqrt{2})h_{1,2}e^{i\theta_{1,2}}$ are in general complex, but the phases are not determined by the potential except that $\theta_1 - \theta_2 = 2\theta_M$. I have chosen $\theta_1 = \theta_2 = \theta_M$ to be zero at the true minimum where $\theta_{\mathrm{QCD}} = 0$.) In this model, the axion field is $a \approx \mathrm{Im}\phi$, ignoring a small admixture of $H_{1,2}$, and its mass is $m_a \approx f_\pi m_\pi/M \approx 10^{-11}$ eV. The real part of ϕ, which I shall denote by σ, is a heavy particle with mass $m_\sigma \simeq (2B)^{1/2}M \simeq 10^{12}$ GeV and is responsible for inflation. The present model is essentially an SU(5) version of the Dine-Fischler-Srednicki "invisible axion" model.[9]

Invisible axions not only solve the strong *CP* problem but also are cosmologically interesting as candidates for the dark matter in galactic halos.[10] However, it has been pointed out[11] that in order for the axion energy density not to exceed the critical energy density of the universe, the $U(1)_A$-symmetry-breaking scale M must be less than 10^{12} GeV *if* one assumes (a) the initial amplitude of the axion field oscillation to be of order 1, and (b) the ratio of axion number density to entropy density, n_a/s, to be constant. In the present model the $U(1)_A$ symmetry breaking occurs at $M = 10^{18}$ GeV. I shall argue later that this large scale poses no serious problems in the new inflationary scenario which relies on a "one bubble universe."

Now I shall discuss the early universe phase transition in this model. The high-temperature effective potential[12] is given by

$$V_{\mathrm{eff}}(T) = V(T=0) + F_\Phi T^2 \mathrm{Tr}\Phi^2 + \sum_{i=1}^{2} F_i T^2 H_i^\dagger H_i + F_\phi T^2 \phi^\dagger \phi, \tag{8}$$

where F_Φ and $F_{1,2}$ are of order e^2 and positive, and F_ϕ is of order $\gamma_i \simeq 10^{-6}$ and positive for $\gamma_1 + \gamma_2 > \frac{12}{5}|g|$. $SU(5) \otimes U(1)_A$ symmetry is unbroken at high temperatures and the minimum of $V_{\mathrm{eff}}(T)$ is at $\langle\Phi\rangle = \langle H_{1,2}\rangle = \langle\phi\rangle = 0$.

As the temperature decreases the standard new inflationary picture[1] develops: The region which evolves into the observed universe cools into the false vacuum ($\phi \approx 0$), and is soon accurately described as a de Sitter space, with a Robertson-Walker metric, $ds^2 = dt^2 - R^2(t)d\vec{x}^2$, where $R(t)$

$=e^{\chi t}$ and $\chi=[(8\pi/3)GV(0)]^{1/2}\simeq 10^{11}$ GeV. At a temperature near the Hawking temperature $T_H=\chi/2\pi\simeq 10^{10}$ GeV, the false vacuum is destabilized by quantum fluctuations with the behavior[13] $|\phi(t)|^2=(\chi^2/4\pi^2)\chi t$. This is obtained for a free massless scalar field, but presumably it is valid for an interacting field with very small coupling as in this model. At later times, the evolution of ϕ can be described by the classical equation of motion. For $|\phi| \ll M$, we can approximate $V(\phi)\simeq V(0)-\frac{1}{4}\Lambda|\phi|^4$, where $\Lambda\equiv B|\ln|\phi|^2/M^2|$. (I use the flat-space potential, but expect that gravitational corrections, which include Hawking-temperature effects among others, would not significantly change the results.) If we take Λ to be constant, which is about $30B$ during inflation, the homogeneous solution $\phi_0(t)$ of the classical equation of motion is given by $|\phi_0(t)|\approx(3\chi^2/2\Lambda)/\chi(t_f-t)$ for $\chi(t_f-t)\gg 1$. One expects that any single fluctuation region which grows large enough to be described by the above classical equation will expand and evolve to a region which resembles our observed universe. The length scale of a single fluctuation region is of order χ^{-1} which is the Hubble radius in de Sitter space. By taking into account both the quantum and the classical behavior of ϕ, one finds the "rollover" time $\chi\tau$ to be of order $\pi^2/\Lambda^{1/2}\simeq 10^6$.

$U(1)_A$ symmetry is spontaneously broken for nonvanishing $\phi=|\phi|e^{i\theta_M}$ with θ_M fixed in each fluctuation region. $SU(5)\to SU(3)\otimes SU(2)\otimes U(1)$ symmetry breaking occurs through the coupling $-\frac{1}{2}|g||\phi|^2\mathrm{Tr}\Phi^2$ as soon as $|\phi|^2$ develops a nonvanishing expectation value. The orientation of the Φ field is fixed in each fluctuation region and primordial monopole and domain-wall production is strongly suppressed.

Next I discuss the reheating process.[14] When the value of $|\phi|$ approaches M, ϕ field starts to oscillate about the minimum of the potential. The amplitude of this oscillation is damped by the decay of the heavy particle σ into those light particles contained in $H_{1,2}$ whose mass m satisfies

$$2m < m_\sigma \simeq (2B)^{1/2}M \simeq 10^{12}\ \mathrm{GeV}. \tag{9}$$

In the present model one combination of $H_{1,2}$ has a light SU(2) doublet with mass of order $M_W\simeq 100$ GeV, and a heavy colored triplet with mass of order $|\beta_i|^{1/2}v$; both masses can satisfy Eq. (9). The doublet and triplet in the other combination of $H_{1,2}$ are very heavy with mass of order $|\eta|^{1/2}M$, which does not satisfy Eq. (9). As in the model of Ref. 6, the decay of σ into the light doublet is strongly suppressed as a result of the hierarchy condition, Eq. (7). Reheating occurs slowly, mainly through σ's decay into the heavy colored triplet with mass $m_{\tilde H}\simeq\beta_i^{1/2}v$. This requires, from Eq. (9), $|\beta_i|\lesssim 10^{-6}$. I obtain the reheating temperature, $T_r\sim 10^9$ GeV. When the heavy colored triplets are produced, they are out of thermal equilibrium, and immediately decay into quarks and leptons, creating a baryon asymmetry[15] given by $n_B/s\simeq\epsilon T_r/m_{\tilde H}$, where ϵ is the *CP* violating parameter arising from loop diagrams of the heavy triplet systems. In order to produce a large enough baryon asymmetry we need to add an additional Higgs field H_3 in **5** representation, which transforms as either H_1 or H_2 under $U(1)_A$.[16] Then ϵ can be as large as 10^{-4}, producing $n_B/s\lesssim 10^{-7}$.

The density fluctuations, when they come within the Hubble radius, are given by[17]

$$\frac{\Delta\rho}{\rho}=\left(\frac{4\Lambda}{3\pi^3}\right)^{1/2}\ln^{3/2}(\chi l), \tag{10}$$

where l is the coordinate distance. For typical galactic scales $\Delta\rho/\rho\sim 10^{-4}$. Slow reheating does not affect Eq. (10).[18] A more recent bound[19] $\Delta\rho/\rho\lesssim 2.5\times 10^{-5}$, which applies to an axion-dominated inflationary universe, can easily be satisfied by adjusting the parameters in the present model.

Finally, I discuss the problem associated with axion energy density. $U(1)_A$ symmetry breaking occurs at the initial stage of inflation through nonvanishing value $\langle\phi\rangle=|\phi|e^{i\theta_M}$. The axion expectation value is given by $\langle a\rangle=\theta_M|\phi|$. Initially, the value of θ_M is arbitrarily chosen, but much later, when $T\lesssim 1$ GeV, QCD instanton effects give rise to a potential for the axion field with a minimum for which there is no *CP* violation. (I have chosen $\theta_M=\langle a\rangle=0$ at the true minimum where $\theta_{\mathrm{QCD}}=0$.) When instanton effects come into play, the axion field starts to oscillate with initial amplitude $A_{\mathrm{initial}}=\theta_M$. The requirement that at present the axion energy density $\rho_a\sim m_a^2M^2A^2$ be less than the critical energy density sets the present value of the amplitude A_{present} less than 10^{-21}. With n_a/s constant, the relation between the initial and present amplitudes is[20]

$$A_{\mathrm{present}}\lesssim 10^{-21}\left(\frac{M}{10^{12}}\right)^{7/12}A_{\mathrm{initial}}. \tag{11}$$

This gives a constraint on M *if* the initial amplitude is assumed to be of order 1: M must be less than 10^{12} GeV. The assumption $A_{\mathrm{initial}}=\theta_M\simeq O(1)$ clearly is natural for the *noninflationary* universe, where fluctuations in θ_M occur and can be large.

However, in the *new inflationary* universe, any fluctuation region (or bubble) which grows large enough to roll down the potential can expand to our observed universe, and θ_M is fixed in each such fluctuation region. Thus for us, the constraint in Eq. (11), with $M=10^{18}$ GeV, simply implies that only bubbles with $\theta_M \lesssim 10^{-3}$ can evolve to our universe. Put differently, only one out of a thousand bubbles can become our universe. I emphasize that we are not fine tuning the parameter; rather we are determining which bubbles can evolve to our universe.

Moreover, the assumption of constant n_a/s can be changed, if there is entropy production at a later stage. Entropy production must occur after $T \simeq 1$ GeV, but before nucleosynthesis at $T \simeq 1$ MeV. In the present model, with an additional Higgs field H_3, entropy production by an amount $s_f/s_i \simeq 10^{3-4}$ can still give $n_B/s_f \sim 10^{-(10\text{-}11)}$ at $T \simeq 1$ MeV, and the constraint on the initial amplitude could become $\theta_M \lesssim 10^{-1}$—one out of ten bubbles can produce our observed universe. Possible sources of such entropy production at 1 MeV $\lesssim T \lesssim 1$ GeV have been recently discussed by Steinhardt and Turner.[21]

Thus I have produced a natural model which can satisfy all cosmological constraints in the context of the new inflationary scenario and also solves the strong *CP* problem by invisible axions. The novel observation is that a Peccei-Quinn singlet drives the inflation, giving rise to proper density fluctuations.

I would like to thank L. F. Abbott, A. Guth, R. Jackiw, and A. Vilenkin for many useful discussions. I acknowledge the hospitality of the MIT Center for Theoretical Physics where part of this work was carried out. This work was supported in part through funds provided by the Department of Energy under the Outstanding Junior Investigator Program.

[1]A. H. Guth, Phys. Rev. D **23**, 347 (1981); A. D. Linde, Phys. Lett. **108B**, 389 (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).

[2]A. H. Guth and S.-Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982); S. W. Hawking, Phys. Lett. **115B**, 295 (1982); A. Starobinsky, Phys. Lett. **117B**, 175 (1982); J. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. D **28**, 679 (1983).

[3]H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[4]S. Coleman and E. J. Weinberg, Phys. Rev. D **7**, 1888 (1973).

[5]E. R. Harrison, Phys. Rev. D **1**, 2726 (1970); Ya. B. Zeldovich, Mon. Not. Roy. Astron. Soc. **160**, 1P (1972).

[6]Q. Shafi and A. Vilenkin, Phys. Rev. Lett. **52**, 691 (1984).

[7]R. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977).

[8]S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).

[9]M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **104B**, 199 (1981); J. E. Kim, Phys. Rev. Lett. **43**, 103 (1979).

[10]J. Ipser and P. Sikivie, Phys. Rev. Lett. **50**, 925 (1983); P. J. E. Peebles, to be published.

[11]J. Preskill, M. B. Wise, and F. Wilczek, Phys. Lett. **120B**, 127 (1983); L. F. Abbott and P. Sikivie, Phys. Lett. **120B**, 133 (1983); M. Dine and W. Fischler, Phys. Lett. **120B**, 137 (1983).

[12]L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974); S. Weinberg, Phys. Rev. D **9**, 3357 (1974).

[13]A. D. Linde, Phys. Lett. **116B**, 335 (1982); A. Vilenkin, Nucl. Phys. **B226**, 527 (1983); J. Bardeen, private communication; A. H. Guth and S.-Y. Pi, to be published.

[14]L. F. Abbott, E. Farhi, and M. Wise, Phys. Lett. **117B**, 29 (1982).

[15]For a review, for example, see E. W. Kolb and M. S. Turner, in *Annual Review of Nuclear and Particle Science, Vol. 33* (Annual Reviews, Palo Alto, 1983).

[16]R. Barbieri, R. N. Mohapatra, D. V. Nanopoulos, and D. Wyler, Phys. Lett. **107B**, 80 (1981); J. Ellis, M. K. Gaillard, D. V. Nanopoulos, and S. Rudaz, Phys. Lett. **106B**, 298 (1981).

[17]Guth and Pi, Ref. 2.

[18]This was pointed out to the author by W. Press; see also R. Brandenberger, Harvard University Report No. HUTMP-82/B122, 1982 (to be published).

[19]P. J. E. Peebles, Astrophys. J. **263**, L1 (1982); L. F. Abbott and M. B. Wise, to be published. In these papers, the bound is given by using a variable ϵ which is about 0.1 times my $\Delta\rho/\rho$, i.e., $\epsilon = 0.1\Delta\rho/\rho \lesssim 2.5\times 10^{-6}$.

[20]Abbott and Sikivie, Ref. 11.

[21]P. J. Steinhardt and M. S. Turner, Phys. Lett. **129B**, 51 (1983).