Quantum-Slip Effect on the Viscosity of Superfluid ^{3}He-B at Very Low Temperatures

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We demonstrate that the flow of excitations in superfluid ^{3}He in restricted geometries is strongly influenced by Andreev scattering at very low temperatures. The effective viscosity is calculated in a simple model and shown to agree with the one observed in a torsional-oscillator experiment.

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The shear viscosity of a normal Fermi liquid is known to increase as $1/T^2$, when the temperature T is lowered, as a result of the increase in the quasiparticle mean free path. In the pseudoisotropic B phase of superfluid ^{3}He one expects[1] the viscosity at low temperatures to approach a constant value. This result reflects the competition between the rapid increase in mean free path and the rapid decrease in the number of excitations as the temperature is lowered. Despite the general nature of this prediction, experiments[2] have yielded an effective viscosity that tends towards zero with decreasing temperature. The explanation of this discrepancy is believed to be associated with mean-free-path effects which eventually become dominant at low temperatures where the bulk mean free path may become larger than characteristic dimensions in the experiments. In this temperature region a conventional hydrodynamic analysis of an experiment yields an effective viscosity which may differ by an order of magnitude from the true one. Mean-free-path effects are governed by the nature of the scattering of excitations at the confining walls. At low temperatures and for a microscopically rough surface one may expect elastic scattering with a broad angular distribution to be dominant. However, by assuming purely diffuse scattering one may account for only part of the discrepancy,[3] and quantitative agreement cannot be obtained at low temperatures.

In this Letter we shall discuss a new mechanism for interaction of excitations in superfluid ^{3}He with a boundary, which is associated with the change of the order parameter near a wall. The particular scattering process responsible for the increase in fluid slip is the so-called Andreev reflection[4] well known from metallic superconductors. This reflection process reverses the group velocity of a quasiparticle, but leaves its momentum nearly unchanged. A microscopic treatment of Andreev scattering for the case of a specularly scattering surface has been given recently by Kieselmann and Rainer.[5] It has also been considered by Greaves and Leggett[6] in the context of quasiparticle ballistics in the A phase. Here we show that this scattering process increases the flow of excitations between parallel plates which determines the damping in a torsional-oscillator experiment.[3] We refer to this as a quantum-slip effect. We shall see that inclusion of Andreev scattering may explain the long-standing discrepancy between the measured and the calculated shear viscosity.

There exists by now a large body of experimental work on the flow of quantum liquids in a region where the mean free path of the excitations is comparable to or larger than characteristic dimensions in the experiment.[7-11] To account in a first approximation for the effects of boundaries under such circumstances Højgaard Jensen *et al.*[3] introduced a slip boundary condition relating the velocity at a wall to the velocity gradient near the wall, $u_0 = \zeta(\partial u/\partial z)_0$, and calculated the temperature-dependent slip length ζ in the superfluid.

The slip length is generally of the same order of magnitude as the bulk mean free path except when the scattering is nearly specular and the slip length therefore very long. When the bulk mean free path is larger than the characteristic dimensions in the experiment, this slip concept loses its meaning. In this regime the flow must be determined from solutions to a kinetic equation that takes the appropriate boundary conditions into account for the geometry in question. For Poiseuille flow between parallel plates this was done in Ref. 3 for diffuse scattering by use of variational methods, which were shown recently by Einzel *et al.*[12] to be accurate to better than 2% in the slip regime. The Poiseuille flow determines the damping of the torsional oscillators used in Refs. 2 and 8.

As recently shown by us[12] the variational method employed in Ref. 3 can be applied to more general scattering laws than the diffuse one. Here we shall use the method to investigate the effects of Andreev reflection.

The magnitude of the quantum-slip effect compared to the usual one depends on the degree of suppression of the gap parameter at the wall. Though this is not known in detail yet, the gap suppression has been determined for the case of specular scattering near T_c by Ambegaokar, de Gennes, and Rainer[13] and by Buchholtz and Zwicknagel.[14] In the specular case one finds that the gap component parallel to the surface normal is completely suppressed, while the perpendicular components are slightly increased. The diffuse case was also investigated by Ambegaokar, de Gennes, and Rainer[13] who found that all components were affected, though by different amounts.

For a first estimate we shall approximate the order parameter $d_{\alpha i}$ of the Balian-Werthamer state by $d_{\alpha i}(\vec{r}) = R_{\alpha i}\tilde{\Delta}(z)\Delta$, where $R_{\alpha i}$ is an $\vec{r}$-independent orthogonal matrix and $\tilde{\Delta}(z)$ a function of the distance z from the boundary situated at $z=0$, while Δ is the bulk energy gap. The function $\tilde{\Delta}(z)$ varies on the scale of the coherence length and tends to unity far from the wall.

The interaction of the Bogoliubov quasiparticles with the wall induced by the gap suppression is characterized by the reflection coefficient R, defined as the ratio of the reflected and incident currents. The current is proportional to $(|u|^2 - |v|^2)$, where u and v are the usual quasiparticle amplitudes. These are determined as solutions of the Bogoliubov–de Gennes equations, which may be written in the form[4]

$$-i(dv/dz) = ku - \tilde{\Delta}(z)v,$$
$$i(dv/dz) = kv - \tilde{\Delta}(z)u. \qquad (1)$$

The dimensionless variable z is here the distance from the wall in units of $\hbar v_F/\Delta$, and k is a dimensionless energy variable, $k = E/\Delta$. Here $E = (\Delta^2 + \xi^2)^{1/2}$ denotes the bulk excitation energy for a Bogoliubov quasiparticle with ξ being the normal-state excitation energy measured from the Fermi surface. Equations (1) describe the situation at normal incidence. When the momentum of the incoming quasiparticle makes an angle θ with the surface normal, Eqs. (1) are unchanged, but the spatial derivative d/dz has to be replaced by $\cos\theta(d/dz)$. As a model for the gap variation we take

$$\tilde{\Delta}(z) = \begin{cases} p + (1-p)\sin(\pi z/2z_1); & 0 < z < z_1, \\ 1; & z > z_1. \end{cases} \qquad (2)$$

The boundary condition at the diffusely scattering wall is simulated by assuming $\tilde{\Delta}(z) = 0$ for $z < 0$. The constant z_1 is of order unity, corresponding to a suppression of the order parameter over a region of the size of the coherence lenght $\hbar v_F/\Delta$. The parameter p is the ratio of the gap at the wall and that in the interior and is expected to be less than unity. We treat both p and z_1 as independent parameters in the following calculations.

The reflection coefficient R for Andreev scattering depends on the energy variable E as well as the angle θ. For a step-function variation, $\tilde{\Delta}(z) = 0$ for $0 < z < z_1$, $\tilde{\Delta}(z) = 1$ for $z > z_1$, one finds[4] the reflection coefficient $R^{\text{step}} = (E - |\xi|)/(E + |\xi|)$. At the gap edge $(\xi = 0)$ the reflection is complete $(R = 1)$. Generally, R decreases faster with increasing energy for a larger value of the gap at the wall and/or a larger width z_1.

Next, we shall indicate how one calculates the flow of the excitation gas taking into account the Andreev scattering at the walls. Let us consider the problem of shear flow with the velocity $u(z)$ in the half-space $z > 0$, the velocity being parallel to the infinite plane wall at $z = 0$. We introduce distribution functions $f_{\vec{p}}^{>}(z)$ for quasiparticles moving away from the wall (velocity $v_z > 0$) and $f_{\vec{p}}^{<}(z)$ for quasiparticles moving towards the wall $(v_z < 0)$. The boundary conditions on $f_{\vec{p}}$ may be expressed as follows:

$$|v_z| f_{\vec{p}}^{>}(0) = \sum_{\substack{\vec{p}' \\ v_z' < 0}} (\vec{p}|w|\vec{p}')|v_z'| f_{\vec{p}'}^{<}(0), \qquad (3)$$

where the transition probability $(\vec{p}|w|\vec{p}')$ is the probability that a particle hitting the wall with momentum $\vec{p}'$ leaves it in the momentum state $\vec{p}$. To proceed further we shall assume that $(\vec{p}|w|\vec{p}')$ is the sum of two terms, one describing the diffuse scattering by the wall and one proportional to $R\,\delta(\hat{p} - \hat{p}')\delta(\xi_p + \xi_p')$, which accounts for the Andreev scattering.

When $R = 0$ we thus recover the results of Ref. 3, which were based on diffuse scattering. It is straightforward to include the Andreev scattering in the general variational expressions for the slip length ζ discussed in Ref. 12. We shall now indicate how to treat the more general case of Poiseuille flow in a parallel-plate geometry for arbitrary Knudsen numbers in the presence of Andreev scattering.

Following Ref. 3 one may derive an integral equation for the velocity field of Poiseuille flow between infinitely extended parallel plates at $z = \pm d/2$ incorporating the boundary condition (3). The kernel of this integral equation is given by

$$H(z,z') = \delta(z-z') - \frac{1}{\rho_n^0} \sum_{\substack{\vec{p} \\ v_z > 0}} \left(-\frac{\partial f^0}{\partial E} \right) p_x^2 \kappa_p^{-1} \{ \exp(-\kappa_{\vec{p}}|z-z'|) + 2\tilde{R}_{\vec{p}}(1-\tilde{R}_{\vec{p}})^{-1} \cosh(\kappa_{\vec{p}} z)\cosh(\kappa_{\vec{p}} z') \}, \tag{4}$$

where $\rho_n^0 = \sum_{\vec{p}} p_x^2(-\partial f^0/\partial E)$ is the density of the normal component in the absence of Fermi-liquid effects and $\tilde{R}_{\vec{p}} = R\exp(-\kappa_{\vec{p}} d)$ describes the Andreev reflection effect. $\kappa_{\vec{p}} = (v_{\vec{p}z}\tau)^{-1}$ is a $\vec{p}$-dependent wave number and $f^0(E)$ is the Fermi function. Since H is a positive semidefinite operator, the usual variational principle yields a lower bound on the average velocity field, which may be conveniently expressed in terms of the effective viscosity $\eta_{\rm eff}$ as

$$\frac{\eta}{\eta_{\rm eff}} \geqslant A \left\{ \frac{1}{d} \left[\int_{-d/2}^{d/2} dz\, \phi(z) \right]^2 \left[\int_{-d/2}^{d/2} dz \int_{-d/2}^{d/2} dz'\, \phi(z) H(z,z') \phi(z') \right]^{-1} - 1 - \lambda_2 B \right\}. \tag{5}$$

Here

$$B = \tfrac{5}{12} d^2/v_{\rm F}^2\tau^2, \quad A = B^{-1}y/(1-\lambda_2 y), \quad y = v_{\rm F}^{-2} \sum_{\vec{p}} v_{\vec{p}}^2(-\partial f^0/\partial E) / \sum_{\vec{p}}(-\partial f^0/\partial E);$$

τ is the quasiparticle lifetime.

We have evaluated and optimized the bound Eq. (5) using the single parameter trial function $\phi(z) = C - z^2$, which accounts for a parabolic as well as a flat velocity profile in the hydrodynamic and Knudsen regimes, respectively. In Fig. 1 the best lower bound on $\eta/\eta_{\rm eff}$ determined in this way, multiplied with the inverse Knudsen number $d/l_\eta = (d/v_{\rm F}\tau) \times (1-\lambda_2 y)/\sqrt{y}$, is plotted as a function of temperature for a plate separation of $d = 94$ μm and parameters appropriate for 30 bars pressure. For this we have used the values $\tau_N^0 T^2 = 0.26$ μsec (mK)^2 for the normal-state relaxation time τ_N^0 and $\lambda_2 = 0.74$ for the collision parameter[3] λ_2, in accordance with the recent work by Pfitzner and Wölfle,[15] as well as the explicit form for $\tau(T)$ and an approximate gap parameter $\Delta(T)$ discussed by Einzel.[16] The full curves are obtained for diffuse scattering (1), and for diffuse and Andreev scattering (2–4) for various gap profiles. As expected, variations of the gap function as described by the parameters z_1 and p in (2) are seen to produce sizable changes. The dashed curves represent the corresponding slip approximations $\eta/\eta_{\rm eff} = 1 + 6\zeta/d$ which are seen to differ substantially from the full lines for $T/T_c \leqslant 0.4$. In the extreme Knudsen limit ($l_n/d \to \infty$), $\eta/\eta_{\rm eff}$ is seen to diverge logarithmically.

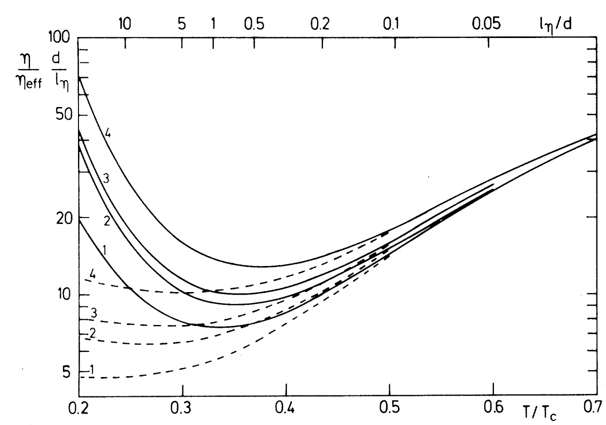

FIG. 1. Inverse effective viscosity of the Balian-Werthamer state scaled with $\eta d/l_\eta$ vs T/T_c for $d = 94$ μm. Full curves are obtained from Eq. (5) for diffuse scattering (curve 1), diffuse and Andreev scattering with $z_1 = 1$, $p = 0.5$ (curve 2); $z_1 = 1$, $p = 0$ (curve 3), and the step function (curve 4). Dashed lines (1–4) are the corresponding slip approximations.

In Fig. 2 we apply our theory to the experimental $\eta_{\rm eff}$ data obtained in Ref. 2 in order to extract bulk viscosity values. Multiplying the experimental data for $\eta_{\rm eff}(T)/\eta_{\rm eff}(T_c)$ by the correction factor $[\eta_{\rm eff}(T_c)/\eta(T_c)]\eta(T)/\eta_{\rm eff}(T)$ for various choices of the parameters p and z_1 one finds the solid curves. This should be compared to the theoretical result for the bulk viscosity $\eta(T)/\eta(T_c)$ obtained by Einzel[16] as indicated by the dash-dotted line in Fig. 2. For $p = 0.3$ and $z_1 = 1$ the agreement is seen to be excellent: The "viscosity droop effect" is

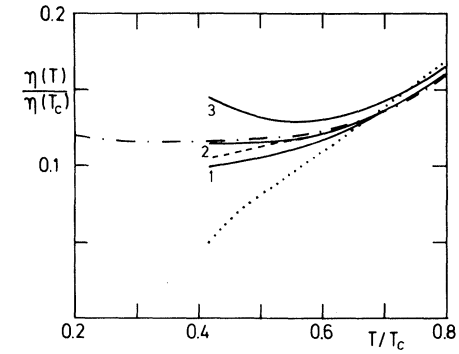

FIG. 2. Viscocity of ^{3}He-B at 30 bars vs T/T_c. Points, effective viscosity $\eta_{\rm eff}(T)/\eta_{\rm eff}(T_c)$ from Ref. 2. Dash-dotted line, bulk theory (Ref. 16). Full lines, data points (Ref. 2) corrected for mean-free-path effects with use of Eq. (5): diffuse scattering, curve 1; diffuse and Andreev scattering ($z_1 = 1$, $p = 0.3$), curve 2; step profile, curve 3. Dashed line, slip approximation of curve 2.

completely removed. It is evident that the step function ($p = 0$, $z_1 = 0$) yields too large a correction, whereas in the absence of Andreev scattering ($p = 1$) the droop remains. It should be emphasized that the droop cannot be removed either by accounting for Andreev scattering within the slip approximation $\eta/\eta_{\rm eff} = 1 + 6\zeta/d$ only.

We stress that this agreement is based on a crude model for the gap variation near a wall and does not allow us to draw conclusions about the actual degree of gap suppression at the wall. Also the assumption of diffuse scattering at the walls may be subject to improvement as very recent experimental results[8] for normal ^{3}He at very low pressure seem to indicate. It is clear, however, that the mechanism of Andreev scattering is an important one and may resolve the long-standing discrepancy between calculated and experimental viscosities.

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