Calculable Nonperturbative Supersymmetry Breaking

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A weakly coupled four-dimensional model is presented which exhibits dimensional transmutation and spontaneous breaking of gauge, chiral, and super symmetries by instanton effects.

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Recently there has been considerable progress in understanding the dynamics of supersymmetric gauge theories.[1-18] Many of these theories have "flat directions," i.e., directions in which the scalar potential vanishes. Because of the nonrenormalization theorems these flat directions persist to all orders in perturbation theory. In our work on QCD (with one fewer flavor than color, $N_f = N-1$)[12,16] we took advantage of the fact that the gauge group is completely broken along some flat directions to do reliable instanton calculations. These generated a superpotential in the effective Lagrangian. Unfortunately, the resultant scalar potential decreases to zero at infinity[8,12,16]; the theory has no ground state. If a small mass is added, the classical flat directions are lifted and supersymmetric vacua appear at large field strength where the theory is weakly coupled.

Recently[17] we have argued that if any continuous global symmetry is spontaneously broken then either there is a noncompact vacuum degeneracy or no supersymmetric vacuum state. [Massive QCD with $N_f = N-1$ is not an exception to this because no continuous global symmetries are spontaneously broken.] We gave an example of a model where such symmetry breaking could be expected to produce a stable vacuum with broken supersymmetry [SU(5) with one matter field in the $\mathbf{5}^*$ representation and one in the $\mathbf{10}$]. However, the vacuum occurred at small field strength so the theory was strongly coupled. In this paper we present a model in which the vacuum appears at large field strength so the theory is weakly coupled (like QCD) but instanton effects spontaneously break chiral and super symmetries.

Our model is an SU(5) gauge theory with two $\mathbf{5}^*$'s, $\bar{F}_i^\alpha$ ($\alpha = 1,2$; $i = 1, \dots, 5$) and two $\mathbf{10}$'s, T_α^{ij}. We include the most general superpotential allowed by gauge invariance $W = \lambda\epsilon_{\alpha\beta}\bar{F}_i^\alpha \bar{F}_j^\beta T_1^{ij}$. The $\bar{F}$'s transform as a doublet under a global SU(2) symmetry and there are two nonanomalous Abelian symmetries $U_A(1)\times U_R(1)$ (the latter is an R symmetry). The classical potential has no flat directions. To see this observe that the condition for vanishing D term is that the Hermitian 5×5 matrix $D = 2T_\alpha^\dagger T_\alpha - \bar{F}^\alpha \bar{F}^{\alpha\dagger}$ be proportional to unity.[19] By a gauge transformation we can choose $T_1^\dagger T_1$ to be diagonal, with entries $(a,a,b,b,0)$ with $a,b \geqslant 0$. The conditions $\partial W/\partial T_1 = 0$, $\partial W/\partial \bar{F}^\alpha = 0$ imply that the two $\bar{F}$'s are parallel and that $T_1^\dagger T_1 \bar{F} = 0$. It is then straightforward to show that the only solution is $T_\alpha = \bar{F}^\alpha = 0$.

It is useful to consider first the theory with no superpotential ($\lambda = 0$). This theory has flat directions, in which the gauge symmetry is completely broken, for example $\bar{F}_1^1 = T_1^{12} = a$; $\bar{F}_4^2 = T_2^{24} = b$, $T_1^{34} = T_2^{25} = (|a|^2 + |b|^2)^{1/2}$. There are also flat directions with unbroken SU(3) or SU(2) gauge symmetry (for instance, set $b = 0$ above). We may use the techniques of Refs. 8, 12, and 16 to show that a superpotential is generated dynamically which lifts all flat directions. This superpotential must be invariant under the full symmetry of the theory: $\mathrm{SU}(5)\otimes \mathrm{SU}(2)_F \otimes \mathrm{SU}(2)_T \otimes \mathrm{U}(1)_{A'} \otimes \mathrm{U}(1)_{R'}$. The two SU(2)'s act on the $\bar{F}$'s and T's, respectively; the $\bar{F}$ and T superfields have A' charges 3 and -1, R' charges -4 and 1 (the gaugino has R charge 1). Amazingly, there is a unique invariant as in QCD.[8] A multiple of $\epsilon_{\alpha\beta}\bar{F}^\alpha \bar{F}^\beta T_1 T_1 T_1 T_2 T_2 T_2$ is required to make an invariant. The $\bar{F}$'s form a $\mathbf{10}^*$ of SU(5) and T_1^3, T_2^3 form the representation $\mathbf{10}_S^3 = \mathbf{45} + \mathbf{175}''$. The resultant product contains a single SU(5) invariant

$$\Delta = \epsilon_{\alpha\beta}\epsilon_{abcde}\epsilon_{ijklm}\bar{F}_r^\alpha \bar{F}_s^\beta T_1^{ri} T_1^{bc} T_1^{de} T_2^{sa} T_2^{jk} T_2^{lm}.$$

It can be seen to be an $\mathrm{SU}(2)_T$ singlet by observing that $\epsilon_{\alpha\beta}\bar{F}^\alpha \bar{F}^\beta T_1 T_1 T_1 T_1 T_2 T_2$ contains no SU(5) invariants. One can also show that there is a unique

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$SU(5)\otimes SU(2)_F$ invariant in $(\bar F^\alpha \bar F^\beta T_1^3 T_2^3)^n$, namely Δ^n. To see this note that $SU(2)_F$ invariance requires $(\bar F^1 \bar F^2)^n$ to form the Young tableaux of Fig. 1(a), and T_1^{3n} can only form the tableaux of Fig. 1(b). There is a unique singlet in the product as indicated in Fig. 1(c). Finally, R' invariance dictates a unique power of Δ in the superpotential: $W_{\rm eff} = k\Lambda^{11}/\Delta$, where Λ is the scale of the gauge theory and k is a constant of order one (the power of Λ is determined by dimensional analysis).

We can show that $W_{\rm eff}$ is generated precisely as in QCD[12,16]: We perform a constrained instanton calculation[20] in a vacuum in which the gauge symmetry is completely broken and the effective coupling is small. Two necessary requirements for a nonzero effect are met: (1) The Atiyah-Singer index theorem applied to the R symmetry respected by the scalar expectation values (under which all matter-field fermions have opposite charge to that of the gluinos) requires only two zero modes. (2) If the scalars are of order v, the one-instanton contribution to W behaves as $v^3\exp[-8\pi^2/g^2(v)] \sim \Lambda^{11}/v^8$.

The coefficient, k, can be related directly to that in QCD[12,16] by the following observation. Since there is a single undetermined constant it is enough to study any convenient point in field space. We choose the flat direction given above in the limit $a \gg b \gg \Lambda$. At energies much below a but much above b, the SU(5) gauge symmetry is broken to SU(2). The spectrum at these energies consists (apart from gauge multiplets) of two SU(2) doublets, Q, $\bar Q$, and five weakly coupled singlets. The effective theory at this scale is identical (apart from the presence of the singlets and various higher dimension operators scaled by $1/a$) to supersymmetric QCD with two colors and one flavor. $b=0$ is the symmetric QCD vacuum, and varying b corresponds to traversing the flat direction in QCD. As shown in Refs. 12 and 16, SU(2) instantons generate an effective superpotential in this theory, $W_{\rm eff} \sim \Lambda_{\rm QCD}^5/(\bar Q Q)$, where $\Lambda_{\rm QCD}$ is the QCD scale. This is related to the SU(5) scale by the one-loop β function: $W_{\rm eff} \sim \Lambda^{11}/(a^6 \bar Q Q)$, which gives the leading behavior of Λ^{11}/Δ for $b \ll a$. Note that $W_{\rm eff}$ blows up along "ridges" of incompletely broken gauge symmetry. This reflects the fact that the theory is strongly coupled near these ridges and our approximations break down. We argued in Refs. 12 and 16 that QCD does not have a zero-energy state at zero field strength, so that the potential energy density $\sim \Lambda_{\rm QCD}^4$. (This also follows from the arguments in Ref. 15.) This implies that at large field strength, along a ridge, the potential-energy density is set by the effective QCD scale $E \sim \Lambda^{44/5}/a^{24/5}$.

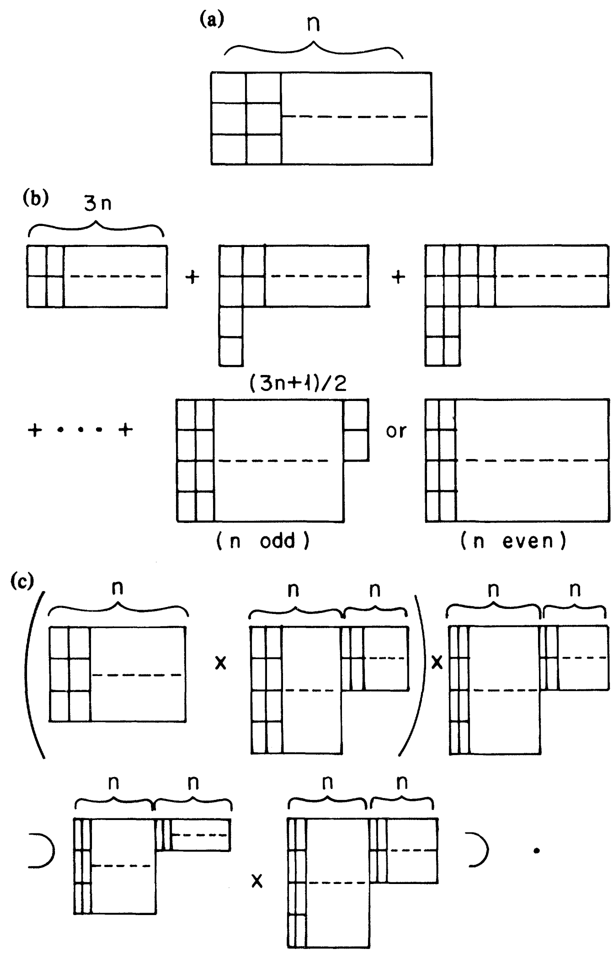

FIG. 1. (a) The unique SU(5) representation in the SU(2)-invariant product of $2n$ $\bar F$'s. (b) All representations in the symmetric product of $3n$ T's. (c) The unique singlet in the product $(\epsilon_{\alpha\beta}\bar F^\alpha \bar F^\beta)^n (T_1)^{3n}(T_2)^{3n}$.

Finally, we argue as in Ref. 17 that this theory does not have a supersymmetric minimum near the origin in field space. If none of the chiral symmetries are broken, six 't Hooft anomaly equations[21] must be satisfied. There is also a Witten anomaly[22] condition; there must be an odd number of $SU(2)_F$ multiplets with $I_F = 2n + 1/2$ (n integer). If supersymmetry is unbroken the massless composite fermions must belong to chiral multiplets (of spin zero or one-half). It follows that fermions of integer I_T have odd R' charge. Since the R'^3 anomaly is -1226 there must be an even number (greater than zero) of integer $SU(2)_T$ isospin multiplets. There are no solutions with fewer than eight chiral multiplets; the simplest case not ruled out has eight chiral superfields in four $SU(2)_T \otimes SU(2)_F$ multiplets. While it is possible that the theory has such a complex low-energy structure, it seems more likely that some of the chiral symmetries (in particular the R' invariance) are broken. By the arguments in Ref. 17 if any continuous symmetries are spontaneously broken then either there is no zero-energy state or else a noncompact set of them. The latter is not possible since the potential energy is nonzero at large field strength, so we conclude that this theory, like QCD, has no ground state.

The connection between breaking of R symmetry and supersymmetry is strengthened by an anomaly recently discovered by Konishi.[13] He showed that

$$\{\bar{Q}_{\dot\alpha}, \bar\psi^{\dot\alpha}\phi\}/2\sqrt{2} = -\frac{\partial W}{\partial\phi}\phi + \frac{g^2}{32\pi^2}\lambda\lambda.$$

Here $\bar{Q}_{\dot\alpha}$ is a supersymmetry charge, (ψ, ϕ) form a gauge-variant chiral multiplet. W is the superpotential and the λ the gluino. (Reference 13 only discussed this anomaly in QCD, but it obviously generalizes to any supersymmetric gauge theory. It is consistent both qualitatively and quantitatively with all our nonperturbative results on QCD.[16]) The first term above is canonical, the second anomalous. If supersymmetry is spontaneously broken then the same methods used to prove the Goldstone theorem in supersymmetry imply that

$$\frac{f_s}{2\sqrt{2}}\langle\dot\alpha|\bar\psi^{\dot\alpha}\phi|0\rangle = -\left\langle 0\left|\frac{\partial W}{\partial\phi}\phi\right|0\right\rangle + \frac{g^2}{32\pi^2}\langle 0|\lambda\lambda|0\rangle,$$

where $\langle\dot\alpha|$ is the supersymmetric partner of the Goldstone boson (goldstino) and f_s is the supersymmetry-breaking scale;

$$\langle 0|J^\mu_\alpha|\dot\alpha\rangle = \sigma^\mu_{\alpha\dot\alpha} f_s,$$

where J^μ_α is the supersymmetry current. Thus in any theory in which W is independent of some nonsinglet chiral superfield, $\langle 0|\lambda\lambda|0\rangle$ is an order parameter for supersymmetry breaking [as is $\langle 0|(\partial W/\partial\phi)\phi|0\rangle$ for all other scalar fields ϕ in the theory]. Thus in the model of this paper and the one generation model of Ref. 17 if $\langle 0|\lambda\lambda|0\rangle$ is nonzero, supersymmetry is broken. But $\langle 0|\lambda\lambda|0\rangle$ is also a natural order parameter for breaking of R symmetry and this condensation is suggested by "most attractive channel" arguments.[23] It may be possible to argue directly that $\langle 0|\lambda\lambda|0\rangle$ is nonzero by a generalization of the methods used in Refs. 11 and 15.[24]

We now turn to the theory with a superpotential. If $\lambda \ll 1$ the effective superpotential is the sum of the tree level part $\lambda T\bar{F}\bar{F}$ and the nonperturbative part $k\Lambda^{11}/\Delta$. Near the origin the potential energy is only slightly perturbed but at infinity the flat directions must be lifted since the classical potential has this property, and the nonperturbative effects become negligible. Thus a minimum must appear (at large field strength as $\lambda \to 0$). Since chiral symmetries are spontaneously broken by scalar expectation values, this vacuum state must break supersymmetry. This can easily be seen by examining the explicit form of W_{eff}: $\partial W_{\mathrm{eff}}/\partial T_2^{ij} = -(k/\Delta^2) \times \partial\Delta/\partial T_2^{ij}$. This quantity is nonzero at finite field strength since $T_2^{ij}\partial W_{\mathrm{eff}}/\partial T_2^{ij} = -3k/\Delta \neq 0$.

We have not explicitly minimized the potential but most features of the theory can be deduced indirectly. By a simple scaling argument the scalar expectation values are $\langle\phi\rangle \sim \Lambda/\lambda^{1/11}$. This sets the scale of gauge symmetry breaking and the mass scale of the heavy gauge multiplets, $g\langle\phi\rangle$. The vacuum energy density is $\sim\langle\phi\rangle^4\lambda^2$. There are six light Weyl fermions and twelve light (real) scalars, with masses $\sim\lambda\langle\phi\rangle$ or else zero. The global symmetries are broken to at most a U(1). This can be seen by noting that $\Delta = \epsilon_{\alpha\beta}J_1^\alpha J_2^\beta$, where $J_1^\alpha = \epsilon_{abcde}\bar{F}_f^\alpha T_1^{ab}T_1^{cd}T_2^{ef}$, and J_2^α is obtained by interchanging T_1 and T_2. These objects are color singlets, $SU(2)_F$ doublets and have charges $A = \mp 1$. (Here we have assigned charges $\frac{3}{5}$, $-\frac{6}{5}$, and $-\frac{4}{5}$ to $\bar{F}^\alpha$, T_1, T_2. This normalization forces all color singlets to have integer charge.) Since Δ is nonzero at the minimum at least (J_1^1, J_2^2) or (J_1^2, J_2^1) are nonzero. If only two of these quantities are nonzero the global symmetry is broken to $Q \equiv I_z \pm A/2$. If more than two are nonzero the symmetry is completely broken. Thus there are either four or five Goldstone bosons. If Q is unbroken there is an 't Hooft anomaly condition, tr $Q^3 = \pm 1$. Thus there must be a massless fermion of charge ± 1 (the charges of J_2^1, J_2^2, respectively) as well as the massless goldstino.

In conclusion, by choosing $\lambda \ll 1$ we have obtained a supersymmetry-breaking vacuum at large field strength where the theory is weakly coupled and thus its properties are calculable.

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