Neutron Phase Shift in a Rotating Two-Crystal Interferometer

D. K. Atwood,[(a)] M. A. Horne,[(b)] C. G. Shull, and J. Arthur[(c)]
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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The phase shift introduced by rotational motion of a two-crystal neutron interferometer has been measured and found to agree with prediction within 0.4%. This agreement is obtained without making the in-crystal phase corrections employed in a recent study of a linearly accelerated three-crystal interferometer.

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The phase shift introduced by rotational motion of an optical interferometer was first demonstrated by Sagnac[1] in 1913 and was observed in the 1925 experiment of Michelson, Gale, and Pearson[2] in which terrestrial rotation was employed. Although the inertial properties of photons and neutrons differ, an analogous effect for neutrons was predicted by Page[3] in 1975. This effect was seen in the gravitation experiment of Colella, Overhauser, and Werner[4] in which terrestrial rotation contributed a small phase shift in a three-crystal neutron interferometer. Later investigation[5] found this to be within 3% of the predicted value.

A related effect caused by linear acceleration of a three-crystal neutron interferometer has been reported recently by Bonse and Wroblewski.[6] Their experiment and the earlier rotation experiment are intimately related since, for each case, the interferometer is at rest in a noninertial frame and neutrons are subject to inertial forces in that frame. Bonse and Wroblewski report phase shifts within 4% of predicted values, after making a 10% correction for in-crystal dynamical diffraction phase shifts. Although a similar correction might improve the agreement with theory in the rotation experiment,[5] Staudenmann *et al.* did not include it in their evaluation. Such inclusion would apparently affect the excellent agreement (1 part in 600) in the gravity experiment.[4]

The present Letter reports more precise measurements of the Page shift using laboratory controlled rotation of a two-crystal neutron interferometer. Measured over a range of rotation speeds 32 times greater than that of the Earth, the shift agrees with the Page formula to within 0.4% without correction for in-crystal effects.

The Page phase shift ϕ_R due to interferometer rotation at angular velocity $\vec{\omega}$ has been derived in a number of ways: by optical analogy,[3] general relativity considerations,[7] dynamical analysis in a noninertial frame,[5] kinematical analysis in an inertial frame,[8] and by analogy with the Bohm-Aharonov phase shift.[9] If centrifugal effects are negligible, the result, to *first order* in ω, is

$$\phi_R = 2m\hbar^{-1}\vec{\omega}\cdot\vec{A}. \tag{1}$$

Here $\vec{A}$ is the area enclosed by the *unperturbed* interferometer beams and any change in the area due to rotation is assumed negligible.

Our experiment was performed with a two-crystal interferometer shown in Fig. 1. A beam of thermal neutrons (of mean wavelength 1.564 Å and angular

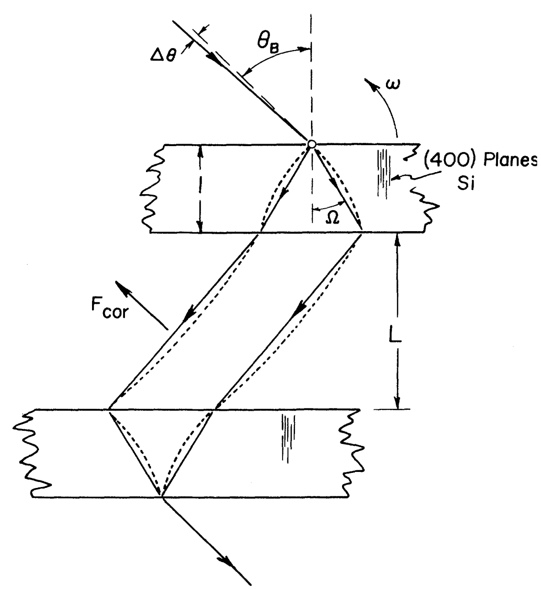

FIG. 1. Typical neutron current flow in a two-crystal interferometer without rotational motion of interferometer (solid lines) and with rotational motion about entrance point (dashed lines). Overlap focusing occurs at an exit point for either case (slightly modified entrance conditions are necessary to maintain the same separation of the illustrated beams) when a suitable wedge is positioned in the gap between the crystals. The trajectory curvature, arising from the Coriolis force on the neutrons, is very much larger (and with both signs) within the crystals than in the free space region.

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divergence 0.52°) illuminates the first of two crystal plates (silicon) through a $1\times 8\text{-mm}^2$ entrance slit. A single plane-wave component of this beam is diffracted by internal (400) planes if the incident angle is within the Darwin range (0.66 arc sec) of the Bragg angle θ_B. The angular deviation $\Delta\theta$ from the Bragg angle is conveniently specified by a parameter y,

$$y = -(E/V_G)\sin 2\theta_B \Delta\theta, \tag{2}$$

where E is the neutron energy and V_G is the (400) Fourier component of the crystal potential. The diffraction process gives rise to two neutron currents (shown as solid ray lines in the figure) traveling at opposite but symmetrical angles $\pm\Omega$ relative to the lattice planes with Ω determined by y. At the back surface each current releases both forward and Bragg diffracted waves at positions specified by the parameter $\pm\Gamma$ defined as

$$\Gamma = \tan\Omega/\tan\theta_B, \tag{3}$$

where $0<\Gamma<1$. At the second crystal each Bragg plane wave again produces two currents traveling at angles $\pm\Omega$. Two of these four currents meet at an overlap position on the back face of this crystal, thereby completing the interferometer paths associated with the incident plane wave being considered. Other incident plane waves generate current paths with different Γ values and we adopt the model that these act independently, i.e., that the various incident plane waves are incoherent.[10]

To test this description, a wedge of refractive material may be inserted transverse to the beam in the gap between the crystals, thereby introducing a phase difference ϕ_w which is proportional to the separation parameter Γ, i.e.,

$$\phi_w = \Phi_w \Gamma, \tag{4}$$

where Φ_w is the phase difference for edge rays ($\Gamma = 1$). With use of standard wave amplitude expressions, the neutron intensity released at the overlap position may be obtained by integration over the parameter Γ, noting both amplitude and phase, yielding

$$I = I_0[\tfrac{1}{2} + \Phi_w^{-1} J_1(\Phi_w)], \tag{5}$$

where J_1 is the first-order Bessel function. This distribution may be tested with continuous variation of Φ_w by rotating a fixed-angle wedge about an axis parallel to the beam. Figure 2(a) illustrates such a wedge phase scan, using a crystalline silicon wedge of angle 5.94°, as compared to the expected distribution. Although the results are generally consistent with Eq. (5), a significant shift in the central

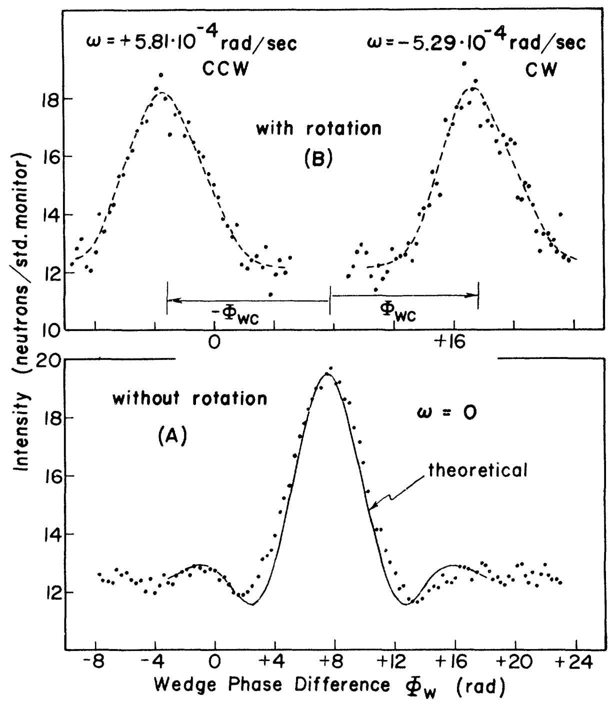

FIG. 2. Neutron intensity from interferometer as a function of phase difference introduced by a wedge inserted between crystals: (a) with no rotational motion and (b) with rotational motion of interferometer. Shifts caused by rotational motion arise from the Coriolis force acting on the neutron. The solid curve in (a) is that expected from Eq. (5) as normalized to the central and wing observed intensities.

peak phase position along with peak broadening and loss of contrast in the outer fringes is to be noted. The former arises from an *intrinsic phase gradient* within the interferometer system (which includes that caused by unvarying terrestrial rotation) and the latter from the presence of subtle residual vibration effects. This systematic effect is taken into account in all subsequent results.

In the absence of interferometer rotation, we note that the unperturbed currents leaving the first crystal at positions $\pm\Gamma$ enclose an area $\vec{A}_0\Gamma$ where $\vec{A}_0$ is the area enclosed by the outermost currents, $\Gamma = 1$. Thus the rotationally induced phase shift in *first order* is

$$\phi_R = 2m\hbar^{-1}\Gamma\vec{\omega}\cdot\vec{A}_0 = \Phi_r\Gamma, \tag{6}$$

with the assumption that Eq. (1) is valid for an interferometer utilizing in-crystal currents. This assumption is supported by a first-order dynamical diffraction calculation which yields Eq. (6) directly. However, the adequacy of a first-order equation may itself be questioned in view of the trajectory curvature caused by the Coriolis force and the potentially nonnegligible effect on the area. This curvature, depicted in Fig. 1, is greatly enhanced *(five orders of magnitude)* in the crystals relative to that in free space leading to the concept of an anomalous *effective mass* of the neutrons[11] which may be of either sign. The details of these hyperbolic paths may be evaluated by adapting the trajectory equations of Werner[12] to the Coriolis force.

It may be shown, however, that the large trajectory perturbations in the crystals have little effect on the area. Considering perturbed currents that reach the same exit points $\pm\Gamma$ on the first crystal (these will be generated by a slightly tipped incident ray) we note there will be a steady evolution of the y value that continues through the interferometer gap. Thus the radiation incident on the second crystal is shifted by

$$\Delta y_R = \hbar G \omega V_G^{-1}(l+L) \tag{7}$$

from the value incident on the first crystal for all rays. Here, G is the magnitude of the reciprocal lattice vector. Because of this y shift the currents in the second crystal do not meet at the exit surface and hence the enclosed area is not defined. Suppose, however, that a compensating wedge is present in the interferometer chosen such that

$$\Delta y_{wc} + \Delta y_R = 0. \tag{8}$$

Now the currents do meet as shown in Fig. 1 and, moreover, the enclosed area is still $A_0\Gamma$.

The wedge introduced to refocus the currents in the rotating interferometer should also compensate for the phase difference due to rotation. In general, a wedge that produces an angle bending characterized by Δy_w also produces a phase difference ϕ_w given by

$$\phi_w = 4mV_G l(\tan\theta_B)\Gamma\Delta y_w/G\hbar^2. \tag{9}$$

When Eqs. (7) and (8) are inserted into Eq. (9), the negative of the phase shift given in Eq. (6) is obtained.

The experiment was performed by rotating the interferometer at a fixed rotational speed about a vertical axis located at the neutron entrance point on the first crystal. Wedge scans were performed simultaneously with the rotational motion and are illustrated in Fig. 2(b). The shift in central-peak phase from that with no rotational motion served to identify the compensation phase Φ_{wc}. A computer-controlled stepping motor with suitable gearing and step smoothing was used to drive a micrometer which contacted a moment arm affixed to the spectrometer turntable. A further stage of vibration isolation separated the interferometer crystal from the turntable. The rotational motion was cyclic and limited to a range of 0.5°, thereby maintaining uniform illumination on the crystal at the Bragg angle. Phased counting intervals over a smaller angle range permitted study for both clockwise and counterclockwise motion at preselected constant angular speeds. Calibration wedge scans with no rotation were obtained before and after each motional run.

Consistent results for the wedge scans required stringent control of temperature stability on a millikelvin scale. Peak broadening in the motional wedge scans was found to increase with angular speed and was associated with increased angular vibration. Calculations of this "ac Page effect," based upon direct measurements of the vibration frequency spectrum, gave broadening effects consistent with the observations. It is to be emphasized that the determination of Φ_{wc} is not affected by this contrast reduction.

The results of the experimental study are displayed in Fig. 3 where the rotational phase difference as derived from the wedge compensation value is graphed versus rotational angular speed. A linear

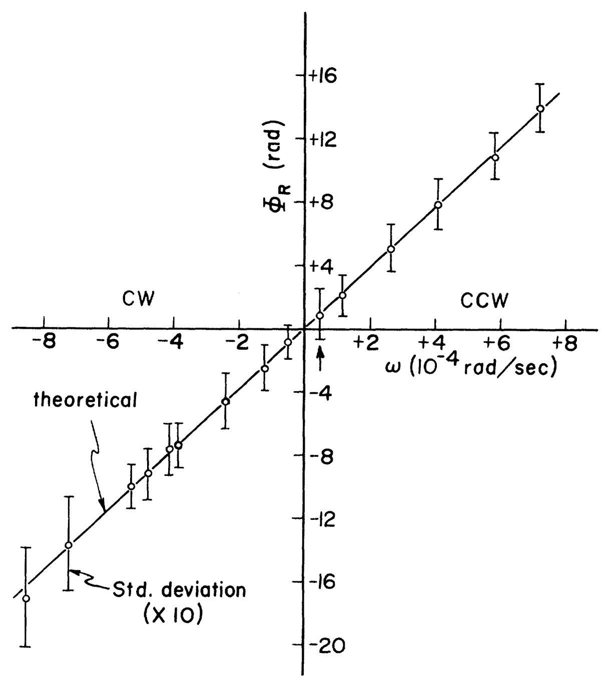

FIG. 3. Interferometer phase difference induced by rotation as a function of rotational angular speed. *For clarity, the standard-deviation brackets of the measured points are presented with tenfold amplification.* The small arrow indicates the effective terrestrial rotation speed at Cambridge, Massachusetts and the solid line is the expected linear dependence of Eq. (6).

regression fit to the data gives

$$\Phi_R = 19\,091(88)\omega - 0.044(0.035)$$

for comparison with the numerical evaluation of Eq. (6) for our interferometer ($A_0 = 6.036$ cm^2, $l = 0.9186$ cm, and $L = 3.747$ cm),

$$\Phi_R = 19\,172\omega,$$

with the phase being expressed in radians and angular speed in radians per second. Agreement with theory is about 0.4% and within standard deviation.

The absence here of a Bonse-type correction, i.e., first order in the perturbing force, is a general feature of the two-crystal interferometer valid for the gravity as well as the Coriolis case. Experiment improvements will ultimately encounter phase shifts that are second order in the perturbation as represented by path integrals along first-order trajectories and these will require careful treatment in the crystals because of the anomalous effective mass of the neutrons. Experiments demonstrating the effects of applied magnetic forces are currently in progress.

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[(a)]Present address: AT&T Bell Laboratories, Murray Hill, N. J. 07974.

[(b)]Permanent address: Stonehill College, North Easton, Mass. 02356.

[(c)]Present address: Oak Ridge National Laboratory, Oak Ridge, Tenn. 37830.

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