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Wide-Band Bar Detectors of Gravitational Radiation

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"Five-mode" massive aluminum gravitational radiation detectors cooled to millikelvin temperatures are considered for the purpose of approaching quantum sensitivity over a large bandwidth. A five-mode detector with a center frequency at 800 Hz would have a two-sided usable bandwidth of the order of 400 Hz. A five-mode detector with a center frequency at 1660 Hz would have a usable two-sided bandwidth of the order of 1000 Hz.

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In past searches for gravitational radiation performed with massive (> 1000 kg) resonant Weber bar detectors operating at kilohertz frequencies, the effective bandwidths have typically been less than 1 Hz. Such bandwidths could be adequate for the simple detection of the arrival of a pulse. A significant increase in the bandwidth is, however, necessary if it is desired to recover the shape and precise arrival time of pulses originating in astrophysical events.

Massive bar detectors are not inherently limited to a narrow effective bandwidth. However, in past experiments the energy coupling β between the resonant bar antenna and its associated amplifier has been inadequate to permit observation with useful sensitivity over a large bandwidth. Typical values of β have been of the order of 10^{-3} or less.[1,2] A value of the order of unity is required to achieve a bandwidth of the order of the center frequency of the detector.

Electromagnetic[3] and mechanical[4] resonant harmonic oscillators coupled to a bar antenna have been used as "two-mode" systems to achieve effective couplings of the order of unity between the antenna and the amplifier. In such systems the bandwidth has an upper limit approximately equal to the inverse of the time broadening of a short pulse as it propagates from the antenna to the coupled oscillator. The time-broadening effect has been important in two-mode systems because of the large ratio of the mass of the antenna to the effective mass of the coupled oscillator.[5] Bandwidths larger than 25 Hz appear difficult to achieve. The "multimode" gravitational radiation detector[6] has been proposed to circumvent that limitation. In its dynamics, it is a stepwise approximation to an antenna with an exponentially decreasing cross section.[7] In principle, it allows for a coupling of the order of unity and a very significant increase in the bandwidth.

The multimode detector is shown schematically in Fig. 1. It consists of an arbitrary number n of coupled harmonic oscillators of geometrically decreasing masses and similar high mechanical quality factors. The largest one describes the Weber resonant bar of effective mass M_1 which provides the coupling to the gravitational field and operates in its fundamental longitudinal mode of frequency f_0. The frequency of each other oscillator when uncoupled is also f_0. M_1 and the following oscillator masses are related by $M_{i-1}/M_i = \mu$ where $i = 1$ to n and μ is a selected mass

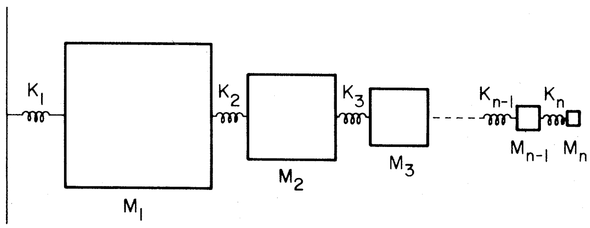

FIG. 1. The n-mode detector consisting of n coupled oscillators of geometrically decreasing masses.

ratio. The mass of the nth and smallest oscillator is chosen to be small enough for the electromechanical transducer to provide an energy coupling of the order of unity between M_n and the sensing amplifier. The system in Fig. 1 exhibits n modes around f_0. The spacing of these modes depends on the ratio μ. If energy is instantaneously deposited in M_1, the modes are excited with specific phases and amplitudes. Beats between them carry the energy into M_n.[8] In such a system, the shortest observation time τ compatible with a system overall effective coupling of the order of unity is equal to the width t_w at half-power points at the nth oscillator of a short pulse initially deposited in M_1. This is given by

$$\tau \simeq t_w \simeq 0.5(f_0\sqrt{\mu})^{-1}. \tag{1}$$

The system two-sided bandwidth is estimated from[9]

$$\Delta f \simeq 1/\tau. \tag{2}$$

In the rest of this paper we analyze the requirements for and the resulting bandwidths of two massive five-mode systems.

Parameters for 1200-kg, 1660-Hz, and 2400-kg, 800-Hz five-mode detectors are given in Table I. As a reference, parameters for a three-mode detector currently under tests at Maryland are given in the first column.[10] 4- to 0.050-K temperatures are assumed. The quality factors shown are the minimum ones needed to reach the quoted sensitivities. Higher Q's have been obtained[11,12] with aluminum alloy 5056. The tuning requirement for the coupled oscillators is[13] $\lesssim 1/\tau$. The lowest amplifier noise temperature quoted corresponds to a one-quantum sensitivity[14-16] at 1660 and 800 Hz. A mode effective noise temperature for pulse detection is calculated from the expression[17-19]

$$T_p \approx T_a \frac{2\tau}{\tau_m} + T_n\left[\frac{2(\lambda+\lambda^{-1})}{\beta_5\tau\omega} + \frac{\beta_5\tau\omega}{2\lambda}\right], \tag{3}$$

where T_a is the detector thermal temperature, τ_m is the mode damping time, T_n and λ are the amplifier noise temperature and reduced impedance, and ω is the mode radial frequency. The first term describes the thermal fluctuations. The energy coupling between the fifth oscillator and the amplifier is selected to equalize the amplifier forward (second term) and backward (third term) noise and is given by

$$\beta_5 \approx \sqrt{2}\,(\pi\tau f_0)^{-1}. \tag{4}$$

TABLE I. Parameters for three-mode and five-mode gravitational radiation detectors.

Detector parameters	Current	Five-mode	
Antenna mass ($2M_1$) (kg)	1200	1200	2400
Center mode frequency (Hz)	1660	1660	800
Antenna temperature (K)	4	0.05	0.05
Number of modes	3	5	5
Mode quality factor	3×10^{-7}	7.5×10^{6}	2×10^{7}
Amplifier noise temperature (K)	10^{-5}	1.4×10^{-7}	5.7×10^{-8}
Last resonator mass (kg)	0.004	0.1	0.1
Successive mass ratio (kg)	0.0025	0.11	0.095
Second resonator mass (kg)	1.54	68	114
Third resonator mass (kg)		7.7	11.0
Fourth resonator mass (kg)		0.88	1.04
Averaging time t_m (s)	0.0067	0.0010	0.0023
Mode-amplifier coupling	0.04	0.26	0.24
Mode noise temperature (K)	5.6×10^{-5}	7.5×10^{-7}	3.0×10^{-7}
Resolution on h	1.4×10^{-19}	1.6×10^{-20}	7.4×10^{-21}
System bandwidth (Hz)	148	982	433
Resolution on h/Hz$^{1/2}$	1.2×10^{-20}	5.2×10^{-22}	3.5×10^{-22}

The resolution in the measurement of the space-time metric perturbation h over the calculated bandwidth is obtained from

$$(\delta h)^2 = 4k_B T_p / M_1 \omega^2 L^2, \tag{5}$$

where k_B is the Boltzmann constant and L, the length of the bar antenna. Equation (5) applies in the case of optimum orientation of the antenna with respect to the direction and the polarization of the incident pulse.

For the two five-mode systems considered, we have chosen a value of M_5 equal to 0.1 kg. With $\lambda = 1$, the maximum coupling is of the order of 0.25.[20] The usable bandwidths calculated from the corresponding μ ratio, Eqs. (1) and (2), are 433 and 982 Hz for detectors with center frequencies at 800 and 1660 Hz.

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[1] A bandwidth of the order of 800 Hz has been reported for a 300-kg antenna by R. W. P. Drever, J. Hough, R. Bland, and G. W. Lesnoff, Nature (London) 246, 340 (1973); J. Hough, J. R. Pugh, R. Bland, and R. W. P. Drever, Nature (London) 254, 498 (1975). The techniques used have not been extended to massive antennas ($m > 1000$ kg).

[2] With parametric up-conversion, the coupling is ω_p / ω_s times larger than the one achieved with a dc biased capacitance transducer. ω_p and ω_s are the pump and the signal frequencies. In practical cases, however, that potential gain is partially cancelled out by the need to use a small capacitance to reach a high pump frequency. Up-converter systems have been extensively discussed by R. Giffard and H. J. Paik, private communication; W. Hamilton and D. Darling, IEEE Trans. Magn. 17, 853 (1981); D. G. Blair, in *Gravitational Radiation, Collapsed Objects and Exact Solutions*, edited by C. Edwards, Lecture Notes in Physics Vol. 124 (Springer-Verlag, Berlin, 1980), p. 314; W. W. Johnson and M. F. Bocko, Phys. Rev. Lett. 47, 11 (1981).

[3] J. Weber, Phys. Rev. Lett. 17, 1228 (1966).

[4] H. J. Paik, J. Appl. Phys. 47, 1168 (1976).

[5] This large ratio has been necessary to maintain a large coupling between the resonator and the amplifier.

[6] J.-P. Richard, in *Proceedings of the Second Marcel Grossmann Meeting on General Relativity, Trieste, 1979*, edited by R. Ruffini (North-Holland, Amsterdam, 1982).

[7] P. S. Aplin, "Long Bar Antennae for Gravitational Radiation," H. H. Wills Physics Laboratory, University of Bristol, Report (unpublished).

[8] Detailed analytical analyses have been made for two-mode and three-mode systems. Computer simulations of the response to short pulses have been carried out for systems with up to five modes.

[9] That value of the bandwidth is close to the spacing of the lowest and highest modes. For the two-mode and the three-mode cases, it is approximately equal to the value obtained on the basis of an optimum-filtering analysis by R. F. Michelson and R. C. Taber, J. Appl. Phys. 52, 4313 (1981), and in a private communication by the same authors.

[10] W. S. Davis *et al.*, in Proceedings of the Third Marcel Grossmann Meeting on General Relativity, Shanghai, China, August 30–September 4, 1982 (to be published).

[11] T. Suzuki, K. Tsubono, and H. Hirakawa, Phys. Lett. 67A, 2 (1978).

[12] Achieving the required Q's with five-mode systems may require carving the antenna M_1 and some of the larger resonators out of a single cylindrical bar. Such a scheme is discussed by J.-P. Richard, University of Maryland Technical Report No. PP-84-10, 1983 (unpublished).

[13] Such a tuning requirement is relatively easy to satisfy in actual systems.

[14] J. Weber, Phys. Rev. 90, 977 (1953).

[15] H. Heffner, Proc. IRE 50, 1604 (1962).

[16] C. M. Caves, Phys. Rev. D 26, 1817 (1982).

[17] R. P. Giffard, Phys. Rev. D 14, 2478 (1976).

[18] J.-P. Richard, Acta Astron. 5, 63 (1978).

[19] The results derived in the two previous references apply directly to systems with a passive transducer. In other cases, equivalent amplifier and transducer parameters must be used in Eq. (3).

[20] A possible transducer-amplifier combination is the inductance-modulation scheme described in Ref. 4 followed by a SQUID (superconducting quantum interference device) amplifier.