T Dependence of the Conductance in Quasi One-Dimensional Systems

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We present a theory of the temperature-dependent conductance $G(T)$ of one-dimensional systems which depends only on the general principles of quantum resonance. Resonant and nonresonant conductances and temperature parameters T_0 are found to be related in a way which is in qualitative agreement with recent experiments. A new low-temperature regime is predicted in which $G(T)$ should change abruptly from Mott type to T independent or metallic.

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A variety of experiments[1,2] have been devised to examine the temperature dependence of the conductance $G(T)$ of quasi one-dimensional systems. When these experiments are in the strong-localization regime, they have found that at low T, $G(T)$ contains apparently random-peak-and-valley structure as a function of the Fermi energy E_F; this structure is not simply noise, however, since it is reproducible upon temperature cycling. It has become clear that these peaks and valleys are actually detailed quantum signatures of the system, macroscopic manifestations of microscopic structure. Theoretical work has shown[3] that similar behavior for $G(T)$ as a function of E_F is a manifestation of transmission through the sample via resonant tunneling through localized eigenstates, which is responsible for a highly structured $G(E_F)$ at $T=0$. Within a theory which includes only thermal population effects,[4,5] the structure seen in the $T\neq 0$ experiments can be explained in terms of the $T=0$ resonances. This theory can also predict the ensemble average of the temperature dependence, $\langle \ln G\rangle \propto -(T_0-T)^{1/2}$, as seen experimentally in the "valleys" of $G(T)$. This finite-temperature theory is clearly incomplete, however, since it does not take into account the inelastic scattering processes which occur for $T>0$; to provide a realistic understanding of present experiments, this theory must be extended to include such effects.

Here we propose a modification to the previous theory to include thermal scattering which, while somewhat speculative, provides a qualitatively correct description of the existing experimental data. We predict that there should be at least three distinct temperature regimes, and as an example we give explicit numerical estimates of these temperatures for the system of Ref. 1, for which roughly $L_0\sim 4\times 10^3$ Å (the localization length), $T_0\sim 10$ K (defined above), $L\sim 1\times 10^5$ Å (the length of the sample), and $w\sim 3\times 10^2$ Å (the apparent width of the sample).[6] These three regimes are as follows: (1) For $T>T_3$, ordinary Mott variable-range hopping[7] dominates the conduction and no resonances are seen. Below we show that $T_3\approx T_0/4[\ln(L/2L_0)]^2\approx 0.4$ K for Ref. 1; this is approximately the temperature where resonant structure is observed to disappear in the experiment. (2) For $T_2<T<T_3$, inelastic scattering broadens but does not destroy the eigenstates of the system. Conduction proceeds by resonant tunneling through these renormalized levels. Resonant structure is observed in G. The T dependence of the resonance remains Mott type, but with a T_0 different from that in the nonresonant regime. We will demonstrate that $T_2\sim (L_0/2L)^2T_0\sim 4$ mK, below the lowest T studied in Ref. 1, where Mott T dependence and resonant structure was observed from ~ 1 K down to ~ 50 mK. (3) For $T<T_2$ inelastic tic scattering ceases to influence G. A dramatic change in the temperature dependence of G is predicted to result: G should become T *independent* in the valleys, and have a universal *metallic* T

dependence ($G \propto T^{-1}$) at the peaks. The T dependence for the system in Ref. 1 should therefore change sharply to an essentially athermal regime below ~ 4 mK.

The middle of the three T regions outlined above is particularly interesting since both coherent and incoherent processes contribute to the properties of the conductivity. The behavior of $G(T)$ in the T regime can be explained in the language of collision theory.[8] First, we suppose that resonances are capable of existing in this T region; we will justify this assertion shortly. Resonant and nonresonant conduction processes should be proportional since both arise from the same microscopic disorder in the sample. This proportionality is expressed in the following way: Near a resonant energy E_r the wavefunction amplitude A of a particle which has passed through the resistor has the general resonant form

$$A_r(E) \propto \Delta(2L_0) A_{nr}/(E - E_r + i\delta E). \quad (1)$$

Here A_{nr} is the nonresonant amplitude outside the resonance, δE is the resonance half-width, and $\Delta E(2L_0)$ represents the energy separation between resonant levels which are less than some distance L_0 from a particular position in the sample (at the center, for example). In a purely one-dimensional case, $\Delta E(x) \approx h v_F/x$ (Ref. 4); in the more realistic case[1] in which there are several one-dimensional channels in parallel, $\Delta E(x) \approx 2\pi\hbar^2/wxm^*$. Here v_F is the Fermi velocity and m^* is the effective mass. This formula includes neither a spin nor a valley degeneracy factor because we expect negligible spin-flip and intervalley scattering, so that the system actually consists of several essentially isolated subsystems. This was taken into account in the experimental estimate of L_0. Equation (1) correctly relates the amplitude precisely at the resonance ($E = E_F$) to that far away from it ($|E - E_r| >> \delta E$); however, the explicit Breit-Wigner[8] form which we use for the resonance line shape depends on the details of the system and simply provides a convenient interpolation between the resonant and nonresonant regions. The transmission coefficient is given by the square of the amplitude in Eq. (1), $t_m = |A|^2$ ("m" denotes "monochromatic").

Temperature influences Eq. (1) in two ways. First, it controls the effective resonance width δE. From the uncertainty principle $\delta E \sim h/\tau$, where τ is lifetime of the resonant eigenstate. τ can be expressed as $\tau \sim \tau_1 p$. τ_1 is a "single attempt" time; in the strictly one-dimensional case, $\tau_1 \sim a/v_F$. a is the size of the effective well in which the localized eigenstate sits; in the case of strong localization a is the probability-density localization length L_0. p is the probability of decay on a single attempt. Temperature will influence p much more than it will τ. Thus in one dimension $\delta E(T) \sim h v_F(T)/L_0$, or more generally

$$\delta E(T) \sim \Delta E(2L_0) p(T). \quad (2)$$

There is a second important way in which T will modify the observed resonance. The carriers will have a spread in energy of order kT around the Fermi energy; we will refer to this as the "nonmonochromatic" width E_{nm}, since this width could arise from processes other than temperature. It is always true that $kT >> \delta E$; thus, even if E_F coincides with a resonance, most of the carriers fall outside the resonance width and are not transmitted. The observed transmission includes this nonmonchromatic effect and is given by $t_{nm} \sim (\delta E/E_{nm}) t_m$.

From Eqs. (1) and (2) the monochromatic transmission at resonance ($E = E_r$) is related to the nonresonant transmission t_{nr} by

$$t_m \sim t_{nr}/p^2. \quad (3)$$

The largest possible resonant transmission is obviously $t_m^{max} \approx 1$; by Eq. (3) the decay probability for a strong resonance is thus

$$p^{max}(T) \sim t_{nr}^{1/2}(T). \quad (4)$$

Since t_{nr} is T dependent (as we discuss below), so is p. Using Eq. (2), we can also relate the resonance width to the nonresonant transmission[9]:

$$\delta E \sim \Delta E(2L_o) t_{nr}^{1/2}. \quad (5)$$

(Note that while the intrinsic width of the resonant peak is given by δE, the actual *observed* width cannot be smaller than E_{nm}.)

If we suppose that $E_{nm} \approx kT$, then the nonmonochromatic transmission at resonance (an experimentally accessible quantity) is

$$t_{nm}^{max} \sim [\Delta E(2L_0)/kT] t_{nr}^{1/2}. \quad (6)$$

We may transcribe this directly into a formula relating experimentally measured resonant and nonresonant conductances:

$$G_r \propto [\Delta E(2L_0)/kT] G_{nr}^{1/2}. \quad (7)$$

Although the transition from Eq. (6) to Eq. (7) may appear perfectly straightforward, it actually contains an interesting subtlety. In identifying t with G on the left-hand side of these equations, we make use of Landauer's formula[10] $G \approx t$. This $T = 0$ relation is valid because the physics of conduction through the resonance is dominated by the

zero-temperature quantum mechanics of tunneling. On the other hand, the conductance far away from resonance proceeds via thermally assisted hopping, according to the ordinary Mott picture. Thus the statement that $G \propto t$ on the right-hand side of (6) and (7) represents the usual identification of particle flux with conductance which is made in the variable-range hopping analysis.[7] So the sample simultaneously exhibits the extremes of quantum mechanical and nonresonant behavior. Within the classical (i.e., hopping) regime, we expect that the conductance will be given by the usual one-dimensional Mott formula[11]:

$$G_{nr} \propto \exp[-(T_0/T)^{1/2}], \quad T_0 \sim \Delta E(2L_0). \tag{8}$$

Through this equation both p [Eq. (4)] and δE [Eq. (5)] acquire a T dependence, as claimed above.

We may now specify the temperature range in which this sort of behavior should be observed. It is clear that the entire resonant picture will break down, and resonant structure will disappear altogether, when the eigenstate width exceeds the spacing $\Delta E(L)$ between the energy levels of the sample: $\delta E > \Delta E(L)$. Thus from Eqs. (5) and (8) we obtain an estimate for the temperature T_3 at which resonances disappear: $T_3 \sim T_0/4[\ln(L/2L_0)]^2$. For the parameters given above from Ref. 1, this gives $T_3 \approx 0.4$ K, which is near the temperature where resonant structure disappears in the experiment. For $T > T_3$ conductance should proceed purely according to multiple hopping.[12]

When $T < T_3$, resonant structure will begin to appear; that is, once a carrier enters a resonant level, it is likely to remain in it throughout its passage through the sample. This is so because quantum mechanical tunneling times are short,[13] while hopping times are exponentially long.[7] G_{nr}, however, will continue to obey the Mott law,[14] Eq. (8). The T dependence of the highest resonant conductance [i.e., of the maximum $G(T)$ over the whole range of E_F] is given by Eq. (7) and is predicted to be

$$G^{\max} \propto \exp[-\tfrac{1}{2}(T_0/T)^{1/2}]. \tag{9}$$

Thus both "peak" and "valley" conductances [Eqs. (9) and (8)] should be linear on a $\ln G$ versus $T^{1/2}$ plot, although the effective T_0 as deduced from the slope of these plots should vary as much as a factor of 4 between the highest peak and a neighboring valley; this fourfold variation has been found experimentally.[1] When all of the resonant peaks, not just the highest ones, are considered, the theory predicts a distribution of ratios $T_0^{\text{valley}}/T_0^{\text{peak}}$ between 4 and 1 such that the random variable $(T_0^{\text{valley}}/T_0^{\text{peak}})^{1/2}$ is uniformly distributed between 2 and 1. Experiment[1] is consistent with this prediction. We have shown the expected behavior of the conductances at peaks and valleys in Fig. 1 for the various T ranges described in the paper.

As the figure shows, another conductance regime is predicted at temperatures below a temperature T_2. This break point occurs when the nonresonant conductance of Eq. (8) reaches the ultimate low-temperature conductance:

$$\ln G_{nr}^0 \sim 2L/L_0. \tag{10}$$

G_{nr}^0 is determined by ordinary (i.e., nonresonant) tunneling through the sample[4] and G_{nr} cannot be smaller than this intrinsic $T = 0$ conductance. Thus G_{nr} turns over sharply at T_2, whch from (10) and (8) is given by $T_2 \sim T_0(L_0/2L)^2 \sim 4$ mK for the experimental system of Ref. 1. While this T_2 is well below the present experimental range ($\gtrsim 50$ mK, limited by the electron temperature), it is perhaps not inaccessibly low, and T_2 may easily differ by a factor of 10 from sample to sample because of variations of T_0, L, and L_0. It should be well worth studying because of the predicted abrupt change in the valley conductance. By Eq. (7), we expect that the resonance conductance will also change abruptly to $G_r \sim T^{-1}$, a universal, weakly metallic dependence. This will occur whenever an eigenlevel is thermally accessible to a carrier. Since the "nonmonchromaticity" of the carriers is $\sim kT$, the probability P that any given sample is in the resonant regime is $P \propto T$. Consequently, over an ensemble of samples the average conductance $\langle G \rangle \sim PG^{\max}(T) \sim \Delta E(2L_o)\exp(-L/L_0)$ is *T independent.* $\langle G \rangle$ may represent, for example, the

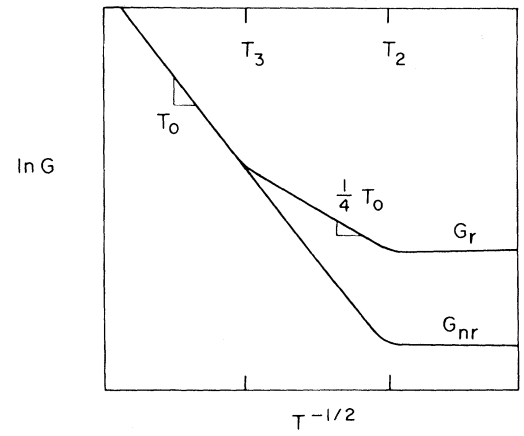

FIG. 1. The general behavior of $G(T)$ for a valley G_{nr}, and for a high peak G_r, for the three different T regimes discussed in the text. We schematically indicate the effective T_0 deduced from Eqs. (8) and (9).

conductance of a quasi one-dimensional sample consisting of many one-dimensional channels conducting in parallel.

To conclude, we have presented a theory which provides a qualitatively accurate description of existing experimental observations of conductance in strongly localized quasi one-dimensional systems. While somewhat speculative, this theory is based on the physically reasonable concept that the behavior of G is determined by eigenstate tunneling effects so long as the with of the levels is less than the energy separating them. We find that the distribution of temperature parameters T_0 between peak and valley is correctly predicted by the theory. Also, we find that for sufficiently low T, $G(T)$ should change over abruptly to an essentially athermal behavior.

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[6]These estimates are quite rough, and the actual values of L_0 and w may be significantly different than those assumed here.

[7]N. F. Mott and E. A. Davis, *Electronic Processes in Non-Crystalline Materials* (Oxford Univ. Press, Oxford, 1971), p. 40.

[8]L. D. Landau and I. M. Lifshitz, *Quantum Mehcanics: Non-relativistic theory* (translated by J. B. Sykes and J. S. Bell (Addison-Wesley, Reading, Mass., 1965), 2nd ed., p. 514. We are grateful to M. Büttiker, who first drew our attention to the Breit-Wigner formula.

[9]Most resonances are not perfectly transmitting, so that $t_m < 1$; thus Eq. (5) is actually satisfied as an inequality: $\delta E^h > \Delta E(2L_0)t_{nr}$.

[10]R. Landauer, Philos. Mag. **21**, 863 (1970).

[11]For very large L, conduction is percolation limited in one dimension and $\ln G(T) \propto T^{-1}$ [see, e.g., J. Kürkijarvi, Phys. Rev. B **8**, 922 (1973)]. For the experiments which have been performed up until now, L is small enough that Eq. (9) is correct.

[12]This is not to say, however, that the one-dimensional Mott law [Eq. (8)] will necessarily apply. It is quite possible that variations in the density of states or of the localizaton length with energy will cause the conductance to deviate from the $T^{1/2}$ law, and these effects should be looked for experimentally; this is discussed in detail in M. Ya. Azbel and D. P. DiVincenzo, to be published.

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[14]It is interesting to note that the identical temperature dependence in this temperature range is also given by a theory which includes only thermal population effects. If these effects are important, they will manifest themselves experimentally in giant nonthermodynamic fluctuations (Ref. 5).