Diffusion-Limited Aggregation and Two-Fluid Displacements in Porous Media

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Remarkable parallels in the behavior of diffusion-limited aggregation and two-fluid displacements in porous media exist; hence, the former can be used to simulate the latter. Both processes can be described by the application of Laplace's equation with similar boundary conditions. Displacements can be stabilized by reversing the flow direction and interfacial tension can be incorporated to broaden dendrites or fingers. Furthermore, diffusion-limited aggregation can be used to simulate flow in anisotropic or inhomogeneous porous media.

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Recently Witten and Sander[1] have described the movement of a random walker in the context of diffusion-limited aggregation (DLA) so that in the continuum limit

$$\partial u/\partial t=\eta\nabla^2 u, \quad (1)$$

where $u(x,t)$ is the probability that a walker is at the point x at time t, η being a diffusion constant. With a steady flux of walkers from a source that is far away from the aggregate on which the walkers stick, Witten and Sander reduce (1) to Laplace's equation

$$\nabla^2 u=0, \quad (2)$$

with the condition $u=0$ on the boundary of the aggregate. The boundary moves with velocity proportional to ∇u. They briefly remark on the analogy between diffusion-limited aggregation and electrostatics.

This process is also analogous to two-fluid displacements in porous media where one fluid has a much larger viscosity than the other and a sharp transition between the two fluids is assumed to exist. In this limit we may approximate $\phi=0$ in the less viscous fluid, ϕ being the macroscopic velocity potential. For the more viscous fluid, flow is ruled by Darcy's law

$$\vec{v}=(k/\mu)\nabla\phi, \quad (3)$$

where $\vec{v}$ is the macroscopic specific fluid discharge, k is the permeability of the porous medium, and μ the viscosity of the viscous fluid. With the incompressibility condition $\nabla\cdot\vec{v}=0$, this becomes

$$\nabla^2\phi=0. \quad (4)$$

In the absence of surface tension, $\phi=0$ on the boundary in the viscous fluid, and the boundary moves according to (3), analogous to DLA.[2] Witten and Sander also investigated the introduction of a "surface tension" in DLA, albeit in an *ad hoc* form. Meakin[3] also recently presented some interesting simulations of DLA.

Remarkable similarities exist between these DLA simulations and visualizations of viscous fingering in flow through porous media.[4] The introduction of surface tension broadens fingers in both cases.[5] DLA can be stabilized by reversing from growth to decay, just as displacements in porous media can be stabilized by reversing flow direction.[6] By decay, we mean changing the DLA rules so that the walker sticks after crossing the boundary of the aggregate (so that the aggregate is eaten away), rather than sticking outside the existing aggregate. Interestingly, the DLA simulation captures much more of the appearance of displacements in porous media than flows in Hele Shaw cells, which have been traditionally used as analogs for flow in porous media. Both DLA and flow through porous media have directional fluctuations on the microscopic scale that average out in the macroscopic description; the Hele Shaw cell has no such fluctuations. DLA produces objects with nontrivial fractal dimension (1.7 in a plane) and it appears viscous fingering grows an interface with similar nontrivial fractal dimension.

A relevant scaling factor for unstable flows in porous media is the ratio of the width of the porous medium to the width of the fingers. The equivalent scaling factor for the DLA simulations is the number of unit cells across the simulation lattice, the minimum finger width being the size of a unit cell. In a porous medium in whch the finger is the same width as the medium, displacements become stable and approach 100% efficiency. Likewise, for a DLA simulation on a lattice one cell wide, the displacement is stabilized with a displacement efficiency of 100%.

Thus we have a method that allows the simulation of two-fluid displacements in porous media where one of the fluids has negligible viscosity.

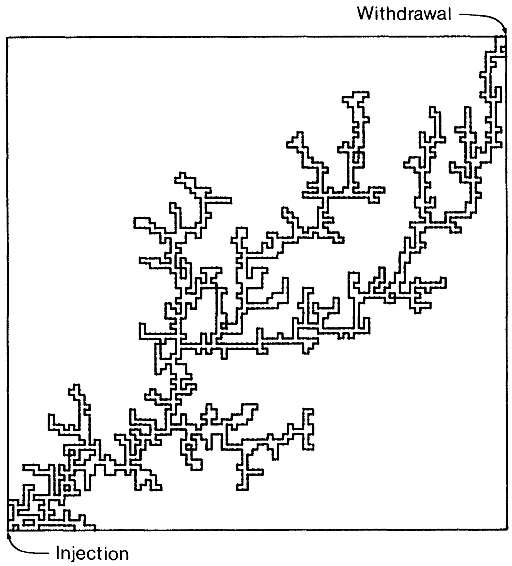

FIG. 1. An example of the interface resulting from a DLA simulation on a 100× 100 square lattice representing one quarter of a five spot.

The advantages of the method are that it requires little computational capacity, complex heterogeneous and isotropic media can be treated, and the unstable flows involving viscous fingering can also be studied. The extension to anisotropic media is achieved by biasing the direction in which the random walker moves. The use of a lattice for the framework on which the random walker moves corresponds to the use of a lattice in microscopic studies of flow and displacements through porous media.[7,8]

As a first example of the method, we consider a problem relevant to oil recovery involving a pattern known as a "five spot". For unstable displacements, the simulation is conducted so that the initial aggregate begins at the injection well, walkers being released from the withdrawal well. Periodic boundary conditions can be simulated either by reflecting the walkers at the boundaries or by cyclically transmitting them from one boundary to another. The former approach has been adopted for the examples presented in this paper as experiments in laboratory models have always resorted to solid boundaries when representing periodic well patterns. The result of an unstable DLA simulation on a 100× 100 square lattice, representing one quarter of a five spot, is presented in Fig. 1. Conversely, a stable displacement (anti-DLA) is simulated by having the initial aggregate filling the lattice and releasing walkers from the injection well. In Fig. 2 an experiment by Habermann[9] in a consolidated sandpack is compared to the anit-DLA simulation. Although the agreement is generally good, there is a discrepancy in the top right-hand corner. This is accounted for by Habermann's experiment having a viscosity ratio of displaced to displacing fluids of 0.151, whereas the simulation has assumed a ratio of 0. Elementary considerations show that this discrepancy would be removed had the experiments been undertaken at a smaller viscosity ratio. This does, however, illustrate the major drawback of the simulation method: that it may be difficult to treat flows where one of the fluids does not have negligible viscosity without invoking complex mechanisms that will destroy the simplicity of the approach.

As a second example, we consider the problem of predicting displacement efficiencies in underground flows where viscous fingering occurs, such as in carbon dioxide flooding of oil reservoirs. Experi-

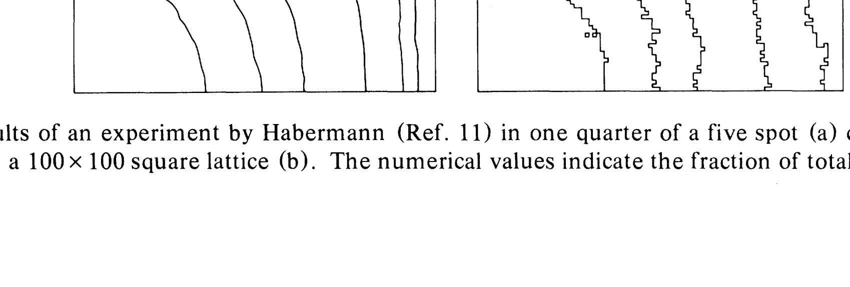

FIG. 2. The results of an experiment by Habermann (Ref. 11) in one quarter of a five spot (a) compared to an anti-DLA simulation on a 100× 100 square lattice (b). The numerical values indicate the fraction of total area displaced.

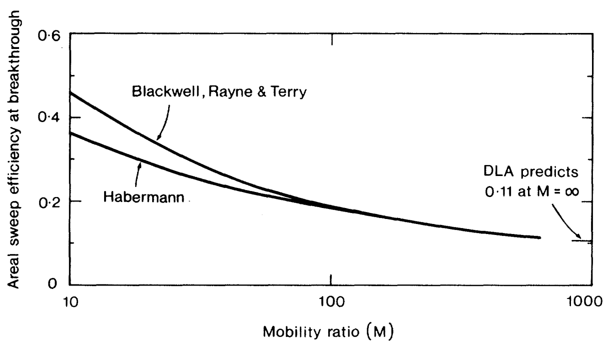

FIG. 3. DLA prediction of areal sweep efficiency at breakthrough on a 100× 100 square lattice compared with experimentally determined values at finite viscosity ratios.

mental values from the literature[9,10] of areal sweep efficiency at breakthrough are plotted in Fig. 3 as a function of mobility ratio (mobility ratio is equivalent to viscosity ratio as used above). DLA simulations with nearest-neighbor sticking (as shown in Fig. 1) predict that, on a 100× 100 square lattice, recovery at breakthrough in a five spot is 0.11 of the total area, with a standard deviation of 0.01 averaged over twenty simulations. This value is slightly dependent on the size of the lattice, but varies little for larger than 50× 50 (e.g., recovery at breakthrough for a 150× 150 square lattice is 0.10, with a standard deviation of 0.01 averaged over twenty simulations). This range of lattice dimensions scales, as far as we can tell, with the number of fingers experienced across the experiments used for comparison. Similar values have also been obtained for simulations on triangular and hexagonal lattices. Blackwell, Rayne, and Terry[10] also included sweep efficiency data for smaller experiments (with fewer fingers appearing) and the recovery improved much as the DLA simulations would predict.

Flow through heterogeneous media is simulated by changes in lattice size, to account for boundary conditions at the changes in permeability: Continuity in pressure and the normal component of velocity must be maintained everywhere across the permeability changes.[11] This is satisfied when the permeability is proportional to the size of the local lattice unit. Sticking probabilities at the moving interface also have to be altered when the sizes of walkers are changed so that a fixed number of walkers impacting on the aggregate causes a consistent increase in the area of the aggregate. With these considerations, as a third example of the method, a simulation with an inner region of half the permea-

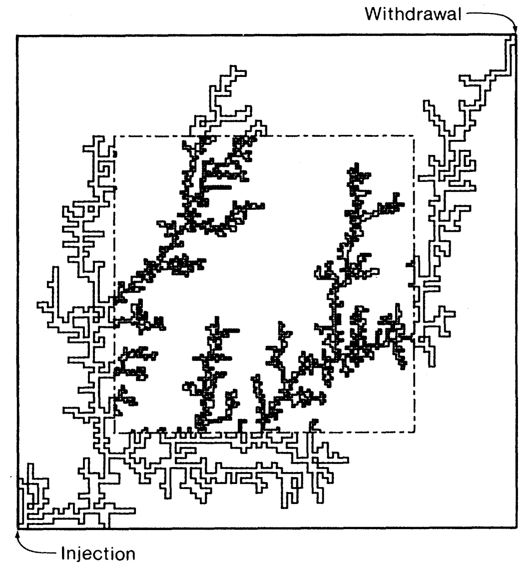

FIG. 4. An example of the interface resulting from a DLA simulation of fingering in one quarter of a five spot in which there is an inner region with half the permeability of the outer region.

bility of the outer region is shown in Fig. 4. Although a universal rule for finger widths is yet to be found, there is experimental evidence[12] for planar immiscible displacements to indicate that finger width is proportional to (permeability)$^{1/2}$; hence the ratio of finger widths should be 1.4 instead of the 2.0 used in the simulation. For lattices much greater in size than the finger widths, sweep efficiency is independent of the finger widths, so this discrepancy should have negligible effect. The heterogeneity indicated in Fig. 4, averaged over twenty simulations, resulted in no significant modification on the recovery at breakthrough, which remained at around 11% of the total area. It is straightforward, however, to contrive heterogeneities which do have a significant effect.

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