Interface Motion and Nonequilibrium Properties of the Random-Field Ising Model

R. Bruinsma
IBM T. J. Watson Research Center, Yorktown Heights, New York 10598

and

G. Aeppli
AT & T Bell Laboratories, Murray Hill, New Jersey 07974
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The dynamics of a continuum interface model in a random medium are studied and the results applied to the random-field Ising model. We find that if the dimensionality $d < 5$, the interface will move only if a force beyond a finite depinning threshold $F_c \sim h^{4/(5-d)}$ is applied, where h is the random field strength. Thus, when the random-field Ising model is quenched to low temperatures, there is a critical value $R_c \sim 1/h^{4/(5-d)}$ for the average radius of curvature R of the domain walls. If $R > R_c$, the domain structure is frozen. If $R < R_c$, the domain structure evolves until $R \sim R_c$.

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It is well known that random magnetic systems relax slowly, if at all, towards equilibrium. The most common examples of such systems are spin-glasses, which display unusual hysteresis when the temperature and applied fields are varied.[1] Recently, similar phenomena[2,3] have been observed for diluted Ising antiferromagnets in uniform fields.[2-4] If a field is applied after cooling below the Néel temperature T_N, long-range antiferromagnetic (AFM) order is maintained. On the other hand, if the sample is cooled through T_N in a field of the same strength, AFM order is never established.[3]

Diluted Ising antiferromagnets in uniform fields are believed to behave like pure Ising ferromagnets in site-random fields.[5] Domain-wall arguments[6] suggest that for low temperatures, small random fields destroy long-range ferromagnetic order only when the spatial dimensionality $d \leq 2$. This result is in seeming contradiction to the neutron-scattering data obtained upon cooling antiferromagnets through T_N in a field.[3,4]

In this Letter, we consider the nonequilibrium behavior of the random-field Ising model in a uniform field conjugate to the order parameter. We find depinning phenomena analogous, but not identical, to those associated with type-II superconductors and charge-density-wave compounds. Our study leads to the conclusion that for $d < 5$, the random field prevents the growth of an ordered state after a quench to low temperatures from the paramagnetic regime.

The standard theory for the growth of a stable phase after a quench through a second-order transition is due to Lifshitz, and has been refined by many authors since.[7] In this theory, one studies the motion of a *single* wall between domains of opposite spin polarity. The mean radius of curvature R for such a wall is then identified with the correlation length ξ for the *entire* many-wall system. Generally, R grows with time t according to a $t^{1/2}$ law. Computer simulations and more elaborate theory bear out the conclusion that ξ obeys the same simple growth law.[7]

We now generalize the Lifshitz theory to take account of random fields. An interface between domains of opposite spin polarity is defined by a profile $y = f(\vec{x},t)$ where $\vec{x}$ belongs to a $(d-1)$-dimensional hypercube with side length L. As usual, overhangs are neglected, so that $f(\vec{x},t)$ is a single-valued function. The energy of the interface, when treated as an elastic membrane,[8] is

$$E(f) = \tfrac{1}{2}J\int d^{d-1}x\,|\nabla f|^2 + 2\int d^{d-1}x \int f(x)\,dz\,h(\vec{x},z). \quad (1)$$

In (1), J is the exchange constant. The random-field variables $h(\vec{x},z)$ at each site $(\vec{x},z)$ are independently selected from a Gaussian distribution with zero mean and mean square Δ. In the presence of a uniform driving force F per unit area, the equation of motion for the interface is

$$\partial f/\partial t = F - \delta E(f)/\delta f = F + J\nabla^2 f - 2h(\vec{x}, f(\vec{x},t)). \quad (2)$$

In simple ferromagnets, F is a uniform field applied in addition to the random field. More generally, F

corresponds to the free-energy difference between the two phases bounding the interface.

Equations (1) and (2) define a continuum interface model, and the *quantitative* conclusions of this paper are valid only insofar as such a model is valid. The continuum description is certainly correct for magnets where the lattice constant is substantially smaller than the interfacial width determined by magnetic anisotropy and exchange energies. Furthermore, at a *qualitative* level, the nonequilibrium behavior of the random-field Ising model, like its equilibrium properties,[8] should be independent of whether a discrete or continuum description is used.

It is convenient to study the following, lattice-regulated, version of Eq. (2)[9]:

$$\partial f/\partial t = F + (j/b_0^2)\sum_{\mu=1}^{d-1}\Delta_\mu\Delta_\mu f + 2(\Delta^{1/2}/b_0^{(d-1)/2})\epsilon(\vec{x}_k,f). \tag{3}$$

The continuum has been replaced by a $(d-1)$-dimensional grid of points $\vec{x}_k$ with lattice constant b_0, Δ_μ denotes the lattice difference operator, and $\epsilon(\vec{x}_k,z)$ is a random variable selected from a Gaussian distribution with unit variance. We are interested in the solutions of Eq. (3) for long times t, given that at $t=0$, the interface profile is $f(\vec{x},t=0)=0$. Characterizing these solutions is easiest in the (i) weak-coupling ($J=0$) and (ii) strong-coupling ($J=\infty$) limits. In case (i), (3) becomes a set of decoupled, one-dimensional equations of motion,[10] one for each $\vec{x}_k$. The qualitative nature of the solutions depends strongly on the magnitude of the driving force. For any F, we know that for a finite fraction p of the sites $(\vec{x}_k,z)$, $F-h(\vec{x}_k,z)<0$. The mean distance $\langle f\rangle$ that the wall will travel as $t\to\infty$ is then the mean distance between such "trapping" or pinning sites:

$$\lim_{t\to\infty}\langle f\rangle = p^{-1}-1. \tag{4}$$

Note that if $p<\frac{1}{2}$, which occurs when $F\lesssim\Delta^{1/2}$, the interface is expected to move less than one lattice unit. Thus, Eq. (4) describes two regimes. In the first, where $F\lesssim\Delta^{1/2}$, the typical interface is pinned at its initial location. In the second, $\Delta^{1/2}\lesssim F$, and the interface ordinarily moves a finite distance before it is pinned.

We turn now to case (ii), the strong-coupling ($J=\infty$) limit, where the interface remains flat throughout its motion. Again, the equation of motion is the one-dimensional version of (3). We thus expect the same pinning phenomena as described above for $J=0$. However, the relevant random-field variables are the *averages* of $(L/b_0)^{d-1}$ random variables $h(\vec{x},z)$. The strong-pinning regime where $\langle f\rangle\simeq 0$ is therefore bounded by $F_c=\Delta^{1/2}L^{(1-d)/2}$ rather than by $F_c=\Delta^{1/2}$. Note that in the thermodynamic ($L\to\infty$) limit, F_c vanishes.

The formal identity between the weak- and strong-coupling cases suggests that it is possible to interpolate between them. The essential idea is to identify a length scale l_0 such that for distances larger than l_0, the system behaves as if $J=0$, while for distances small compared to l_0, the behavior is that of the rigid interface. To implement this, we note first that because (3) is linear in F, J/b_0^2, and $\Delta^{1/2}/b_0^{(d-1)/2}$, the depinning force $F_c(\Delta,J)$ must satisfy

$$F_c(\Delta,J) = (\Delta^{1/2}/b_0^{(d-1)/2})F((J/\Delta^{1/2})b_0^{(d-5)/2}). \tag{5}$$

$F(x)$ is an undetermined function, which approaches a finite value in the weak-coupling ($x\to 0$) limit, and converges to zero as $x\to\infty$.

From Eq. (3), we see that the interface width, which represents the typical deviation of $f(\vec{x}_k,t)$ from its mean value established over an area l^{d-1}, is bounded by

$$w_l \sim (\Delta^{1/2}/J)\,l^{(5-d)/2}. \tag{6}$$

Thus, for $d>5$, the interface becomes smoother with rescaling, while for $d<5$, it becomes coarser. In other words, for $d>5$, we are always in the strong-coupling regime, and an infinite interface moves rigidly for all $F>F_c=0$. For $d<5$, the interface is essentially flat ($w_l\lesssim 1$) for length scales less than l_0, where

$$l_0 \sim (\Delta^{1/2}/J)^{2/(d-5)}. \tag{7}$$

This means[11] that for $b<l_0$, we are allowed to replace $f(\vec{x},t)$ in Eq. (3) by its local average over areas b^{d-1}. The difference equation (3) can then be studied on a grid with lattice constant b instead of b_0. Now, if b is taken to be a fixed fraction of l_0, the scaling form (5) yields

$$F_c \sim \Delta^{1/2}(\Delta^{1/2}/J)^{(d-1)/(5-d)}. \tag{8}$$

Figure 1 illustrates the physical origin of the nonzero F_c for $d<5$. The random field acting on the wall [as, for example, used in Eq. (3)] is defined as the random field at the site to the right of

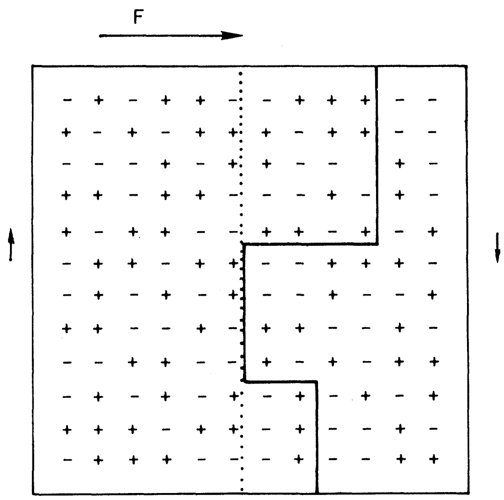

FIG. 1. Effect of roughness on interface motion. The plus and minus signs refer to the polarity of the local random fields. A force F per unit area is acting on the interface. All spins to the left of the interface are "up" and all spins to the right are "down." For the flat interface (dotted line), the excess of pinning sites, where the local fields are negative, is less than for the rough interface (solid line).

the wall. If we attempt to move a flat wall, the full driving force FL^{d-1} is available to overcome the net interfacial random field, which is of order $\Delta^{1/2}L^{(d-1)/2}$. On the other hand, if the interface is allowed to roughen as it moves, it will move even while some of its segments cannot advance because of the associated gain in energy. Eventually, all of the segments will cease to move. The entire wall is then pinned, and can only be dislodged by a force whose density remains finite when $L \rightarrow \infty$.

Now we apply the result (8) to the problem of an Ising ferromagnet quenched to a low temperature in a random magnetic field. As noted above, this problem is generally[7] reduced to that of the collapse of a single antiphase droplet with mean radius of curvature R. The driving force per unit area is[7] σ/R, where σ is the surface tension of the pure Ising model. If $\sigma/R > F_c(J,\Delta)$, the droplet will collapse; if $\sigma/R < F_c(J,\Delta)$, the droplet remains "frozen." The critical radius of curvature R_c is

$$R_c \sim \sigma/\Delta^{2/(5-d)}. \tag{9}$$

Let $\overline{R}$ be the average radius of curvature of all the domain boundaries in a sample. If we prepare the random-field Ising model in a domain-wall state with $\overline{R} = R_0$ and $R_0 > R_c$, then $\overline{R}$ will remain close to R_0. However, if R_0 is less than R_c, $\overline{R}$ will grow with time, by droplet collapse and interface erosion, until it reaches R_c. The identification of F_c with σ/R_c is self-consistent as long as the roughness w_l on the scale $l = R_c$ is considerably smaller than R_c. This translates to the condition $R_c^{(1-d)/4} \ll 1$, which is satisfied for $d > 1$.

The theory we have described has the following implications for dilute antiferromagnets in uniform fields H: (1) If the system (with $H \neq 0$) is quenched to low temperatures from a state with correlation length $R = R_0 < R_c$, R can only grow, and it will grow until it reaches R_c. (2) Once the system has been prepared as in (1), changing H or σ (by changing T) will only lead to an increase in the correlation length. In particular, if H is reduced, R will decrease to a value which it will maintain even if H is subsequently increased again. (3) If $R_0 \ll R_c$, the interfaces will be smooth on the scale R_c, and the neutron-scattering profiles $S(Q)$ will essentially be the squared Fourier transforms of spheres.[12] Thus, for large Q, $S(Q) \sim Q^{-4}$ if $d=3$,[13] and $S(Q) \sim Q^{-3}$ if $d=2$. (4) For $d=3$, $R_c \sim H^{-2}$, while if $d=2$, $R_c \sim H^{-4/3}$. (1)–(3) have all been observed in neutron-scattering experiments[3,14] and recent computer simulations.[15] Furthermore, in some three-dimensional systems,[3] the inverse correlation length κ indeed increases according to an H^2 law, in agreement with (4). In others,[4] κ increases less rapidly, i.e., according to a power law H^ν where $\nu > 2$. For the two-dimensional magnet $RbCo_xMg_{1-x}F_4$, experiment[14] gives $\kappa \sim H^2$, in disagreement with (4). At the fields over which this law was established, the temperature T also enters in a nontrivial way. However, for the lowest fields considered, T does enter in the simplest possible manner: κ is the sum of a field-independent thermal spin-flip probability and a temperature-independent function of H. We consequently expect that higher-resolution measurements at small fields will reveal the $\kappa \sim H^{4/3}$ behavior given by our theory.

By studying the motion of interfaces in the random-field Ising model, we gain understanding of experiments performed on random antiferromagnets in uniform fields. The same theory also has consequences for other problems. In particular, it can account for hysteretic behavior near certain first-order transitions,[16] and depinning phenomena in reentrant spin-glasses.[17]

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communicating unpublished results. After completing the work described in this manuscript, we were informed that Villain[18] has carried out a similar study.

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