Nonequilibrium "Critical" Exponents in the Random-Field Ising Model

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The lower critical dimension for a random-field Ising model cooled from the paramagnetic region is found to be 4, although it is 2 at equilibrium. The coherence length R is proportional to $H^{-\nu_H} f(t)$, where H is the random-field amplitude and t is the time. At low temperature, $\nu_H = 2$ and $f(t) = \ln(t/\tau)$. At the transition temperature T_{c0} of the pure system, $\nu_H \simeq 1$ and $f(t) = 1$. The agreement with experiment is acceptable.

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Two experimental papers on the random-field Ising model (RFIM) have recently appeared in the Physical Review. The value of the lower critical dimension D_c (i.e., the dimension below which no long-range order is possible in the presence of weak random fields) was reported to be $D_c < 3$ in one of these articles,[1] and $D_c > 3$ in the other one.[2]

The statement to be made in this note is that both papers may be right. In thermal equilibrium, $D_c = 2$ as stated in Refs. 3 to 13. On the other hand, it will be argued that $D_c = 4$ in the case of a system cooled from the paramagnetic region, which has not reached thermal equilibrium If the size of such a system is infinite, it can reach thermal equilibrium only after an infinitely long time. This statement is not in contradiction with experimental results.[2,13]

Some theories predict $D_c = 3$.[14-16] They have already been refuted in Refs. 4 to 6, as far as thermal equilibrium is concerned. They are also unreliable for nonequilibrium systems because, when the free energy has several extrema, some of them are weighted by negative "probabilities" as can be seen from Eq. (4) of Ref. 17. The existence of multiple minima of the free energy is a necessary feature of a nonequilibrium system in a metastable state.

In a RFIM, metastable states may be attributed to the formation of ordered domains of opposite order parameter, separated by walls. The domain radius R will be assumed to have a well-defined order of magnitude. For a spatial dimension $D > 2$, the positive (respectively, negative) magnetization region is expected to consist of an infinite domain made of entangled, branched tubes, in addition to finite domains, as in a percolation problem. Both tubes and finite domains have radii of the same order of magnitude R, which go to ∞ during the evolution toward equilibrium according to formula (18) below. The free energy of a domain of radius R may be estimated[3] to be of order

$$W \approx gR^{D-1} - HR^{D/2} m_0, \qquad (1)$$

where g is the surface tension of the wall, $H^2 = \langle H_i^2 \rangle$ is the variance of the random field, and m_0 is the spontaneous magnetization per atom at $H = 0$. The units are such that $m_0 = 1$ at $T = 0$, and the interatomic distance is 1. The first term of (1) is the exchange energy and the second term is the "Zeeman" energy due to random fields. For dimension $D > 2$, and $H \ll g$, the energy (1) is minimum for $R = 0$; therefore $D_c = 2$.[3]

In this Letter, it will be argued that domains may be metastable for $D > 2$ because their walls are pinned by random fields. The effect of these fields is that a piece of wall of size ρ, which otherwise would be flat, is distorted (Fig. 1). Metastability occurs because several deformations are possible,

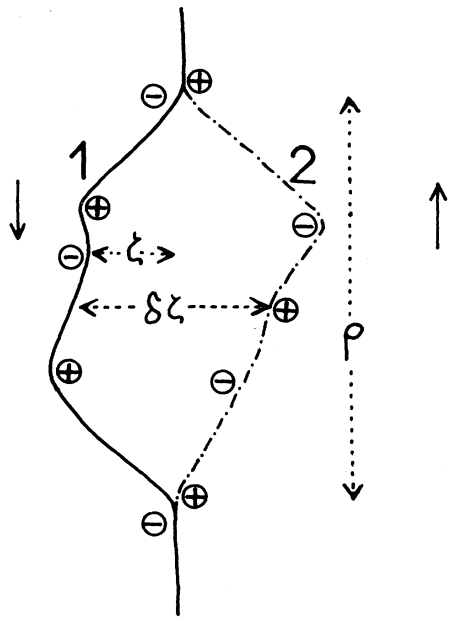

FIG. 1. Two metastable positions of a wall separating an up-spin region (right-hand side) from a down-spin zone. Random fields are suggested by circled plus and minus signs.

each of them corresponding to a relative minimum of the energy, or (at $T \neq 0$) of the Landau free-energy functional. In order to evaluate the typical height and thickness of the energy barriers, consider two neighboring metastable positions 1 and 2 (Fig. 1) which correspond to the same position of the edge, and to average displacements ζ and $\zeta_2 = \zeta + \delta\zeta$, respectively, of the other points. An argument *à la* Imry and Ma[3] suggests that the Zeeman energy barrier between 1 and 2 is of order $H(\rho^{D-1}\delta\zeta)^{1/2}m_0$, where $\rho^{D-1}\delta\zeta$ is the number of spins to be reversed when moving the wall from 1 to 2. However, the Zeeman energy should not be considered if it is much weaker than the exchange energy, which is of order $g\rho^{D-1}(\zeta/\rho)^2$. Thus, the maximum energy barrier ω is obtained for $\delta\zeta \approx \zeta$ and the Zeeman energy $H(\rho^{D-1}\zeta)^{1/2}m_0$ comparable with the exchange energy $g\rho^{D-3}\zeta^2$. The results are

$$\omega \approx H(H/g)^{1/3}\rho^{(D+1)/3}m_0^{4/3}, \tag{2}$$

$$\zeta \approx (Hm_0/g)^{2/3}\rho^{(5-D)/3}. \tag{3}$$

There are also lower-energy barriers, some of which correspond to much more distorted walls, i.e., larger ζ.[18] I choose to consider only the highest barriers, which satisfy (2) and (3). The motivation is the following. At a given time t, only a particular order of magnitude[19] $\rho(t)$ of the radius should be considered. At smaller scale, there is thermal equilibrium, and larger scales need not be considered because they correspond to domains larger than the typical size R at time t. The particular size $\rho(t)$ is characterized by the fact that the *highest* potential barriers have not yet been overcome, but the lower barriers are already irrelevant.[18] Formulas (2) and (3) have already been obtained with a different meaning in Refs. 4 to 7. They can be applied to a piece of radius ρ of a sphere or cylinder of radius R, provided ζ is less than the height $R - (R^2-\rho^2)^{1/2} \simeq \rho^2/2R$; otherwise surface tension dominates random-field effects. Using (3), one obtains the condition

$$\rho \lesssim (m_0H/g)^{2/(D+1)}R^{3/(D+1)}. \tag{4}$$

The above formulas can only be applied if the wall thickness λ is narrow enough, namely,

$$\lambda << \zeta << \rho < R. \tag{5}$$

The above formulas are sufficient to evaluate the domain size at low temperature, given by formula (18) below. However, before doing that, it is relevant to elucidate whether metastable domains are formed when cooling through the critical region. The first task is to rewrite (2) and (5) in this region, with use of the relations[20]

$$\lambda \sim m_0^{-\nu/\beta}, \tag{6}$$

$$g \sim Jm_0^{(D-1)\nu/\beta} \tag{7}$$

which are correct for $D \leq 4$ in zero field, and will be assumed to hold in a weak random field as well. J is the absolute value of the exchange integral, assumed to couple nearest neighbors only. Relation (3) implies $D \geq 2$ because ζ should be smaller than ρ. Inserting (7) into (1), the domain energy is found to be positive if $Jm_0^{(D-1)(\nu/\beta)-1} \times R^{(D-2)/2} > H$ for all radii R larger than the wall thickness (6). This implies that the ordered state is stable with respect to domain formation if

$$m_0 \gtrsim (H/J)^{2\beta/\gamma} = (H/J)^{(D-2+\eta)/(2-\eta)}. \tag{8}$$

At equilibrium, the transition occurs when this condition fails to be satisfied, i.e., at a temperature T_c related to the zero-field value by

$$(T_{c0} - T_c)/T_{c0} \sim (H/J)^{2/\gamma}. \tag{9}$$

At sufficiently lower temperatures, domains of sufficiently large size may be metastable. The domain magnetization $\pm m_0$ is not very different from the zero-field value. Inserting (7) into (2)–(4), one finds the barrier thickness ζ and the height ω associated with a piece of wall of size ρ, and the maximum value of ρ for a domain radius of order R:

$$\omega^3 \approx H^4J^{-1}\rho^{D+1}m_0^{2(D-3+2\eta)/(D-2+\eta)}, \tag{10}$$

$$\zeta^3 \approx H^2J^{-2}\rho^{5-D}m_0^{-2(D-\eta)/(D-2+\eta)}, \tag{11}$$

$$\rho^{D+1} \lesssim H^2J^{-2}R^3m_0^{-2(D-\eta)/(D-2+\eta)}. \tag{12}$$

Relations (6), (11), and (8) imply that conditions (5) are satisfied if

$$\rho^{5-D} >> J^2H^{-2}m_0^{2(D-3-\eta)/(D-2+\eta)}. \tag{13}$$

Relations (12) and (13) imply that metastable domains cannot have a radius smaller than some bound $R_m(T)$. R_m decreases with increasing T, and its minimum value, reached near T_c, is, according to (8),

$$R_{m0} \approx (J/H)^{2/(2-\eta)}. \tag{14}$$

It will now be seen that this value is also the maximum value of the correlation length $1/\kappa$ *above* T_c, beyond which linear-response theory cannot be applied. The linear response to the Fourier component H_k is

$$m_k = \chi_k H_k, \tag{15}$$

where

$$\chi_k \approx J^{-1}(\kappa^2+k^2)^{-1+\eta/2}. \quad (16)$$

The mean square magnetization per site is

$$m^2 = N^{-1}H^2\sum_k \chi_k^2 \approx H^2 J^{-2}\kappa^{D-4+2}. \quad (17)$$

In a uniform field, the response m per spin is linear if $m^{2-\eta} \ll m^{\delta}$. This condition also holds for a staggered field of wave vector $k \lesssim \kappa$. It holds for random fields too, because the sum (17) is dominated by Fourier components with $k \lesssim \kappa$. Replacing m by the square root of (17), one obtains the condition $1/\kappa \lesssim R_{m0}$, where R_{m0} is given by (14).

The existence of nonlinear (saturation) effects is a necessary, though not sufficient condition for the appearance of potential barriers and metastability. Relation (14) has been obtained above T_{c0} as a condition for incipient saturation, and below T_c as a lower bound of the metastable domain size. This suggests that in the intermediate range, if jumps over barriers are ignored, the coherence length does not vary much, and therefore metastable states do form during the cooling process. As an illustration and support of this statement, one can consider a negative-field volume of radius about R_{m0}, embedded in a positive-field sample. Figure 2 shows a guess for the free energy as a function of the local magnetization m. Slightly above T_{c0}, when (15) applies, there is a single minimum (curve a) for $m < 0$. Slightly below T_c, where relation (14) has been derived, this minimum is still there (curve c), but the absolute minimum corresponds to $m > 0$. Curve b is a guessed interpolation near T_{c0}, showing the birth of a potential barrier. At lower temperature (curve d) the potential barrier disappears. However, at larger scale, barriers are still present and small domains just merge into bigger domains.

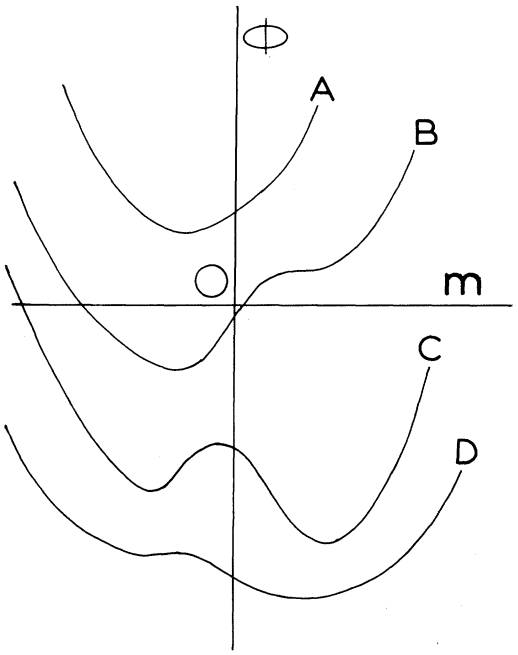

FIG. 2. Genesis of a metastable state. Free energy of a space region with negative average field embedded in a volume with positive field, as a function of the local magnetization. Decreasing heights correspond to decreasing temperatures.

The time dependence will now be considered. The basic assumption[21] is that the height of the barriers remaining after a waiting time t is given by the Arrhenius law $\omega = T\ln(t/\tau)$, where τ is a microscopic, constant time. Using (10) and (12), one finds the typical domain radius

$$R \approx H^{-2}JTm_0^{2(1-\eta)/3(D-2+\eta)}\ln(t/\tau). \quad (18)$$

On the other hand, at T_{c0}, the correlation length is given by (14). The time does not appear there, and this indicates that the system is in equilibrium. Relations (18) and (14) both have the form $R \sim H^{-\nu_H}$, where $\nu_H = 2$ at low temperature, and $\nu_H = 2/(2-\eta)$ at T_{c0}. The experimental results at low temperature are[2,13] $\nu_H = 3.6 \pm 0.3$ for $Co_{0.35}Zn_{0.65}F_2$, $\nu_H = 2.17 \pm 0.3$ for $Co_{0.26}Zn_{0.74}F_2$, and $\nu_H = 2.1 \pm 0.2$ for $Fe_{0.35}Zn_{0.65}F_2$. The agreement with (18) is good in the last two cases. At T_{c0}, the experimental result[13] for $Fe_{0.35}Zn_{0.65}F_2$ is $\nu_H = 0.8 \pm 0.1$, somewhat below the theoretical result (14).

Materials studied experimentally are Fishman-Aharony systems,[22] more complicated than the proper RFIM considered in this note. A more detailed analysis is necessary for a conclusive comparison between theory and experiment. From the present work, one can at least conclude that there is no reason to be pessimistic.

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