dc Acceleration of Charged Particles by an Electrostatic Wave Propagating Obliquely to a Magnetic Field

Ryo Sugihara
Institute of Plasma Physics, Nagoya University, Nagoya, Japan

and

Satoshi Takeuchi, Kazuo Sakai, and Michio Matsumoto
Faculty of Engineering, Yamanashi University, Kofu, Japan
(Received 28 September 1983)

A charged particle trapped in an electrostatic wave is accelerated in the plane perpendicular to the wave vector $\vec{k}$. It is found that there is an optimum angle $\theta = \theta_m$ at which the particle gains a maximum energy which is about four times larger than that at $\theta = \pi/2$, θ being the angle between $\vec{k}$ and the magnetic field.

PACS numbers: 52.35.Mw, 52.35.Fp, 52.65.+z

dc acceleration of a charged particle trapped in an electrostatic wave propagating *perpendicularly* ($\theta = \pi/2$, θ the angle between $\vec{k}$ and $\vec{B}_0$) to a magnetic field $\vec{B}_0$ has been theoretically studied by Sagdeev and Shapiro[1] and Sugihara and Midzuno.[2] Dawson *et al.* investigated this acceleration when the wave electric field E is so strong that the trapping velocity $(eE/mk)^{1/2}$ is comparable to the phase velocity $v_p = \omega/k$. Nishida, Yoshizumi, and Sugihara[4] experimentally proved the existence of this acceleration process.

The mechanism (abbreviated by $\vec{v}_p \times \vec{B}_0$ acceleration) is characterized by the fact that a dc field appearing in the wave frame accelerates a trapped particle, which makes a clear contrast to the usual stochastic accelerations in which the particle diffuses in real and velocity spaces as a result of interactions with random forces. The accelerated particle detraps from the wave potential trough when the Lorentz force becomes comparable to the electrostatic force of the wave and the acceleration ceases.

In this paper we extend the theory to acceleration of a charged particle which is trapped in an electrostatic wave propagating *obliquely* ($\theta \neq \pi/2$) to the magnetic field.[5] We show that there is an optimun angle at which the energy that the particle gains takes a maximum value, about four times larger than that for $\theta = \pi/2$.

The equation of motion for a particle (e.g., electron) in the wave frame is given by

$$\dot{v}_x = -\omega_{cz}(V_y + v_p) + \omega_{cy} v_z, \tag{1}$$

$$\dot{V}_y = -(eE/m)\sin kY + \omega_{cz} v_x, \tag{2}$$

$$\dot{v}_z = -\omega_{cy} v_x. \tag{3}$$

Here the magnetic field is chosen as $\vec{B}_0 = (0, B_y, B_z)$, $(\omega_{cy}, \omega_{cz}) = (B_y, B_z)e/mc$, $Y = y - v_p t$, $V_y = v_y = v_p$, and quantities of real space and velocity space represented by lower case letters refer to the laboratory frame.

We choose two test particles for convenience of discussion. One is an initially, deeply trapped particle (IDTP) whose initial values of v_x and v_z are $v_x(0) = v_z(0) = 0$ and which is most favorably situated at $t = 0$ in real space for the acceleration. This particle is the one which we have mentioned so far just in terms of "trapped particle." The other is an initially, weakly trapped particle (IWTP) which has $v_x(0) = v_z(0) = 0$, too, and would be marginally trapped if there were no magnetic field. We will show some specific features of the set of Eqs. (1)–(3). Let us note that for IDTP, the relation $|V_y| << v_p$ usually holds. Then during the trapping, Eq. (1) is approximated by $\dot{v}_x = -\omega_{cz} v_p + \omega_{cy} v_z$ which is combined with Eq. (3) to yield an equation for the harmonic oscillator in the form

$$\ddot{v}_x = -\omega_{cy}^2 v_x.$$

Under the initial condition $v_x(0) = v_z(0) = 0$, we get

$$v_x = -u \sin\omega_{cy} t, \tag{4}$$

$$v_z = u(1 - \cos\omega_{cy} t), \tag{5}$$

$u \equiv (\omega_{cz}/\omega_{cy}) v_p = (B_z/B_y) v_p$ being the amplitude of v_x. Hence if the relation

$$E > uB_z/c = (v_p/c) B_z^2/B_y \tag{6}$$

holds, IDTP never detraps from the wave potential.

It is important to know the dependence of the maximum velocity that the IDTP gains on the angle θ. Suppose that E and $B_0 \equiv (B_y^2 + B_z^2)^{1/2}$ are fixed. At some angle $\theta = \theta_m$, the equality $E = (v_p/c) B_z^2/$

B_y or

$$v_E = v_p \sin^2\theta_m / \cos\theta_m \tag{7}$$

is assumed to hold, and we assume $v_E > v_p$, where $v_E = cE/B_0$. We here define maximum values of v_x and v_z of IDTP by v_x^m and v_z^m, respectively. Hence, when $0 < \theta < \theta_m$, v_x^m and v_z^m are given by u and $2u$ as seen in Eqs. (4) and (5). These features are shown in Fig. 1. The agreement between the analytical curves and the numerical ones is surprisingly good.

When $\pi/2 > \theta > \theta_m$, IDTP will detrap after an appreciable acceleration since the condition (6) does not hold. The v_x^m will be approximated by $v_{Ez} \equiv v_E/\sin\theta$ from Eq. (2) and the numerical analysis of the orbit of IDTP confirms the inference as seen in Fig. 1. The v_z^m is given by

$$v_z^m = u[1 - \sin^2\theta\{1 - (v_{Ez}/u)^2\}^{1/2} + \cos^2\theta\{1 + (v_{Ez}/v_p)^2\}^{1/2}], \tag{8}$$

which is the projection of a spiral motion along $\vec{B}_0$ with the detrapping velocity as the initial velocity. Notice that Eq. (8) becomes $v_z^m = u(1 + \cos\theta_m)$ at $\theta = \theta_m$. This indicates a jump in v_z^m at $\theta = \theta_m$ as denoted by PA in Fig. 1. The numerical calculation follows this curve quite well.

We also compute the orbit of IWTP and give the results in Fig. 1. The calculation indicates that there is a critical angle $\theta = \theta_c$ for IWTP too, and for $\theta < \theta_c$ the particle never detraps while for $\pi/2 > \theta > \theta_c$ it detraps after a weak acceleration. We schematically depicit these features in Fig. 2 where θ is between θ_c and θ_m.

We now understand that the maximum, denoted by v^m, of the maximum velocities of IDTP is obtained at $\theta = \theta_m$, which is rewritten as $v^m = 2v_E/\sin\theta_m$ by use of Eq. (7). We note that at $\theta = \pi/2$ we have $v_z^m = 0$ and $v_x^m \approx v_E$ at the detrapping point, which is the reproduction of the already known results.[2,3]

Solving Eq. (7) with respect to θ_m, we have

$$\sin\theta_m = [2/\{1 + (1 + 4v_p^2/v_E^2)^{1/2}\}]^{1/2}.$$

Then we see that the maximum energy of IDTP is

$$m(v^m)^2/2 = 2mv_E^2/\sin^2\theta_m = mv_E^2[1 + (1 + 4v_p^2/v_E^2)^{1/2}],$$

which is about four times larger compared with the energy, $mv_E^2/2$, at $\theta = \pi/2$. Note that $\pi/2 - \theta_m = v_p/v_E$ holds in the limit of small v_p/v_E.

The acceleration time or the detrapping time τ_a for $\theta = \pi/2$ is known to be $\tau_a = v_E(\omega_c v_p)^{-1}$. In the present case, the time t_a needed for IDTP to reach v_z^m at θ_m, or v^m, is, from (5),

$$t_a = \pi/\omega_{cy}. \tag{9}$$

Bearing in mind that at $\theta = \theta_m$ the relation $(cE/B_z)/v_p\omega_{cz} = \tau_a/\sin^2\theta_m = 1/\omega_{cy}$ holds and usually $B_z \sim B_0 >> B_y$, we have

$$t_a \approx \pi\tau_a.$$

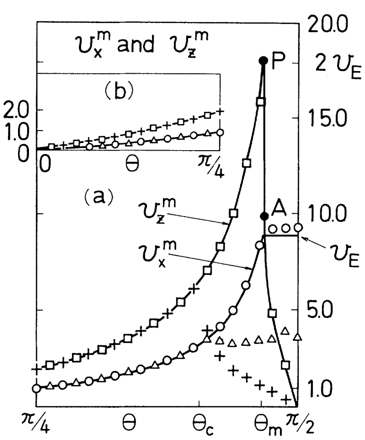

FIG. 1. Maximum values of v_x and v_z of IDTP and IWTP. $v_E/v_p = 8.89$. Solid lines, analytic curves for IDTP; circles, numerical values of v_x^m for IDTP; squares, numerical values of v_z^m for IDTP; triangles, numerical values of max(v_x) for IWTP; crosses, numerical values of max(v_z) for IWTP. (a) $0 < \theta < \pi/4$. (b) $\pi/4 < \theta < \pi/2$.

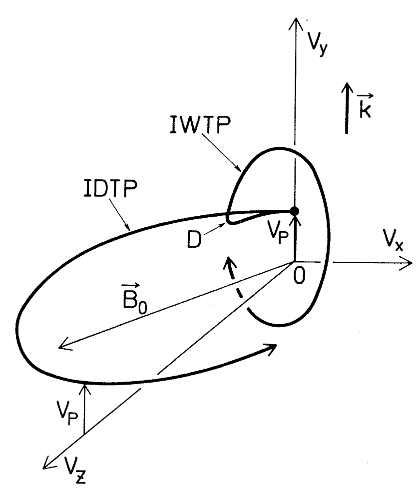

FIG. 2. Schematical drawing of the particle orbits. $\theta_m > \theta > \theta_c$. IDTP circulates in the v_x-v_z plane while IWTP detraps at the point D and then gyrates around the magnetic field $\vec{B}_0$. The unit of the vertical coordinate is v_p.

We shall discuss the present process from another viewpoint. It is rather easy to get the energy conservation relation,

$$\tfrac{1}{2}m[(v_y-v_p)^2+(v_z-B_zv_p/B_y)^2+v_x^2] \\ -(eE/k)\cos(ky-\omega t)=K,$$

where use has been made of the momentum conservation relations along z and x directions and K is a constant. Think of a case where $v_y=v_p$ and $ky-\omega t=0$ always hold or the particle is trapped in the wave. Using the initial condition $v_x(0)=v_z(0)$, we have

$$[(v_z-B_zv_p/B_y)^2+v_x^2]=(B_zv_p/B_y)^2. \tag{10}$$

There is nothing but the orbit of IDTP depicted in Fig. 2.

We will show characteristic features of this acceleration by means of a particle simulation. The simulation was performed by using a $2\frac{1}{2}$-dimensional fully electromagnetic particle code.[6] All quantities are assumed to be uniform in the z direction. A homogeneous magnetic field $\vec{B}_0=(0,B_0\cos\theta,B_0\sin\theta)$ is imposed. An electric field propagating obliquely to $\vec{B}_0$ is excited by an external current $\vec{j}=(0,j_0\sin[ky-\omega t],0)$ with a frequency ω larger than the electron plasma frequency ω_{pe}, where j_0 and k are constants. The initial velocity distribution of electrons is Maxwellian and the thermal velocity $v_e=(T/m)^{1/2}$ is $0.07c$, c being the light velocity. The number of accelerated particles is relatively small compared with that when $\theta=\pi/2$.[2,3] Hence, in order to increase the number of trapped particles, on the tail of the distribution we add a bump, the velocity of which is adjusted to v_p. The phases, in real space, of the particles composing the bump are chosen to be slightly different from each other.

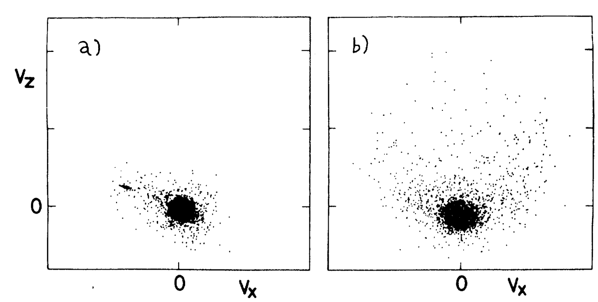

FIG. 3. Distribution of accelerated particles. $\theta=66°<\theta_m$, $v_e=0.07c$, $v_p=0.149c$, $u=0.335c$. (a) $t \ll t_a$. (b) $t \gg t_a$. The envelope of the distribution corresponds to the orbit of IDTP in Fig. 2. The large-amplitude wave excited pushes the main distribution downward.

Figure 3 shows the temporal variations of the velocity distribution in the v_x-v_z plane. The angle is now 66 deg and the wave electric field is somewhat larger than the one at which value the angle θ would become the critical angle θ_m; note that θ_m is a function of E. Actually in this case the critical angle is inferred to be around 77 deg from the measured electric field. The time in Fig. 3(a) is much less than t_a and the acceleration has just started. We see a small high-density islet at approximately 10 o'clock from the large blob. This is the bump accelerated from its initial velocity $(0,v_p,0)$ and the orbits of particles composing the islet will be equivalent to that of IDTP in Fig. 2. The distribution in Fig. 3(b) is obtained at $t \gg t_a$. We see that the envelope of the distribution makes a clear correspondence with the orbit of IDTP in Fig. 2. Inside the envelope, initially, weakly trapped particles are distributed.

The converse process of deceleration is worthy of discussion. Think of the motions of particles when $\theta=\pi/2$ (see Ref. 3) since the essential points of the discussion are the same in the case of $\theta\neq\pi/2$. If we change the sign of time t in the equation governing the motion of a gyrating, high-energy particle which has initially been deeply trapped, the particle will trace back its history, be retrapped, and be decelerated. This implies that some gyrating particles in the real world can be trapped and be decelerated. We notice, however, that since the conditions for a high-energy particle to be trapped are very subtle, we may rarely see such particles in particle simulations.

In conclusion, we have shown that the angle θ between $\vec{k}$ and $\vec{B}_0$ has an optimum value for the $\vec{v}_p\times\vec{B}_0$ acceleration of IDTP for a fixed wave amplitude E and a fixed, applied magnetic field $\vec{B}_0$. At this angle θ_m, the maximum energy that the particle gains is about four times larger compared with that at $\theta=\pi/2$.

The authors deeply appreciate the discussions with Dr. J. I. Sakai of Toyama University. This work was supported in part by a Grant in Aid for Scientific Research from the Ministry of Education, Science, and Culture.

[1]R. Z. Sagdeev and V. P. Shapiro, Pis'ma Zh. Eksp. Teor. Fiz. **17**, 389 (1973) [JETP Lett. **17**, 279 (1973)].

[2]R. Sugihara and Y. Midzuno, J. Phys. Soc. Jpn. **47**, 1290 (1979).

[3]J. M. Dawson, V. K. Decyk, R. W. Huff, I. Jechart, T. Katsouleas, J. N. Leboeuf, and R. M. Martinez, Phys. Rev. Lett. **50**, 1455 (1983).

[4]Y. Nishida, M. Yoshizumi, and R. Sugihara, Institute of Plasma Physics Report No. IPPJ-648, 1983 (unpublished).

[5]R. Sugihara and Y. Midzuno, Insitute of Plasma Physics Report No. IPPJ-263, 1976 (unpublished); this report treated this problem for the first time, but the treatment is incomplete.

[6]O. Buneman, C. W. Barnes, J. C. Green, and D. E. Nielson, J. Comput. Phys. **38**, 1 (1980).