Relaxation of the Cosomological Constant

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(Received 9 February 1984)

We present a classical field theory which has a long epoch resembling the Friedmann universe despite the presence of a large negative cosmological constant. In its most plausible realization the model involves a massive third-rank antisymmetric-tensor gauge field. To be consistent with standard cosmology the mass must be extremely small and the only reasonable mechanism for generating it is a semiclassical tunneling effect.

PACS numbers: 98.80.−k, 03.50.Kk

Several authors[1,2] have suggested that a third-rank antisymmetric-tensor gauge field $A_{\mu\nu\lambda}$ may play a role in solving the problem of the cosmological constant. In attempting to implement this idea one quickly finds that most local Lagrangians involving $A_{\mu\nu\lambda}$ lead to equations of motion whose solution involves an undetermined integration constant. The paradigm for this phenomenon is the free action

$$\frac{1}{2}\int\left[\frac{\epsilon^{\mu\nu\lambda\kappa}}{4!}\partial_\mu A_{\nu\lambda\kappa}\right]^2 \equiv \frac{1}{2}\int F^2. \tag{1}$$

While the integration constant (in this case F) may be chosen to cancel a negative bare cosmological constant, there is no dynamical reason for this to occur.

In this Letter we will explore the consequences of adding a nonlocal term

$$\tfrac{1}{2}\mu^2\int d^4x\, d^4y\, F(x)\partial^{-2}(x-y)F(y) \tag{2}$$

to the action. In "Landau gauge" ($\partial^\mu A_{\mu\nu\lambda}=0$) this is just a mass term $\frac{1}{2}\mu^2 A^2_{\mu\nu\lambda}$. We will find that if μ is extremely small, the solution of the resulting cosmological equations has a long period which resembles what we presently know of the history of the universe.

Equation (2) is the analog of the nonlocal gauge-invariant mass term which is generated for the two-dimensional electromagnetic field in the Schwinger model.[3] Alternatively, we may think of it as arising from a Higgs-Stueckelberg Lagrangian

$$\tfrac{1}{2}\mu^2\left[\partial_{[\mu}\theta_{\nu\lambda]}-A_{\mu\nu\lambda}\right]^2$$

by solving for $\theta_{\nu\lambda}$ in terms of $a_{\mu\nu\lambda}$. In any event, it is technically natural to take $\mu^2=0$.

The generally covariant extension of (1) + (2) is

$$S=\tfrac{1}{2}\int d^4s\, F^2(x)/(-g)^{1/2}+\tfrac{1}{2}\mu^2\int d^4x\, d^4y\, F(x)\Delta^{-1}(x,y)F(y), \tag{3}$$

where $\Delta=\partial_\mu[(-g)^{1/2}g^{\mu\nu}\partial_\nu]$. Note that F is a scalar density.

If the metric has the Robertson-Walker (RW) form and we use Coulomb gauge ($\partial_k A_{klm}=0$), then the equations of motion imply $A_{0km}=0$ and we get an effective action for

$$A(t)\equiv\int dx^k\wedge dx^l\wedge dx^m A_{klm}, \tag{4}$$

$$S=\int dt[(\dot{A}^2/2R^3)-\tfrac{1}{2}\mu^2A^2/R^3]; \tag{5}$$

$R(t)$ is the RW scale factor.

The equations of motion for A and R are ($F\equiv\dot{A}/R^3$, $\mathscr{A}\equiv A/R^3$, and we take flat spatial sections for simplicity)

$$\dot{F}=-\mu^2\mathscr{A}, \tag{6}$$

$$\dot{\mathscr{A}}+3H\mathscr{A}=F, \tag{7}$$

$$\left(\frac{\dot{R}}{R}\right)^2\equiv H^2=\frac{1}{M^2}\left(\frac{F^2}{2}+\frac{\mu^2\mathscr{A}^2}{2}+\rho-\Lambda^4\right). \tag{8}$$

Here $M\sim10^{19}$ GeV is the Planck mass, ρ is the energy density of matter and radiation, and we have chosen a negative cosmological constant, $-\Lambda^4$.[4] Note that if we define $\mathscr{A}\equiv\phi/\mu$, $F=-\mu\phi$, these are the equations for a scalar field of mass μ.[4]

To analyze these equations we define dimensionless variables

$$s=(\Lambda^2/M)t, \tag{9}$$

$$\sigma=\rho/\Lambda^4, \tag{10}$$

$$H=(\Lambda^2/M)h, \tag{11}$$

$$F=[2(H^2M^2+\Lambda^4-\rho)]^{1/2}\cos\theta/2, \tag{12}$$

$$\mathscr{A}=[2\mu^{-2}(H^2M^2+\Lambda^4-\rho)]^{1/2}\sin\theta/2, \tag{13}$$

$$\epsilon=\mu M/\Lambda^2. \tag{14}$$

Then (6)–(8) are equivalent to

$$dh/ds = -3(h^2+1-\sigma)\sin^2\theta/2 - c\sigma/2, \quad (15)$$

$$d\theta/ds = \epsilon - 3h\sin\theta, \quad (16)$$

if $h^2+1 \geqslant \sigma$ initially. (We have assumed that $\dot{\rho} = -c\rho h$, $c > 0$.)

We take a universe which is initially expanding rapidly: $h > 0$ and $|h| \sim O(1)$ or bigger. For $\epsilon << 1$, Eqs. (15) and (16) have approximate fixed lines $\sin\theta = \epsilon/3h$, which are shown in Fig. 1. The solid lines are attractive, the dashed ones, repulsive.

For a large set of initial conditions with $h > 0$, the flow is drawn in to the line marked with an arrow in a time of order 1 (in units of M/Λ^2). h then evolves more or less along this line. If $\sigma << 1$, the motion is very slow until h decreases to $O(\epsilon)$. Then h speeds up, crosses 0 (the universe begins to contract), and reaches negative values of order 1 in a time of order 1. The universe has entered on a period of explosive contraction.

In terms of the scalar field ϕ, this long period of slowly changing expansion rate is easy to understand. When $h \sim O(1)$, ϕ experiences frictional forces of order 1 and (if $\epsilon << 1$) a very small restoring force. ϕ quickly goes down almost to zero (the fixed line) and stays there until the restoring force can compete with the friction $h \sim O(\epsilon)$.

The above scenario has a long period which resembles our universe if $\epsilon, \sigma << 1$. As long as we are near the fixed line, then in the regime $\epsilon << h << 1$ we have

$$dh/ds = -\tfrac{1}{2}c\sigma - \epsilon^2/12h^2. \quad (17)$$

This is the equation of the usual Friedmann models

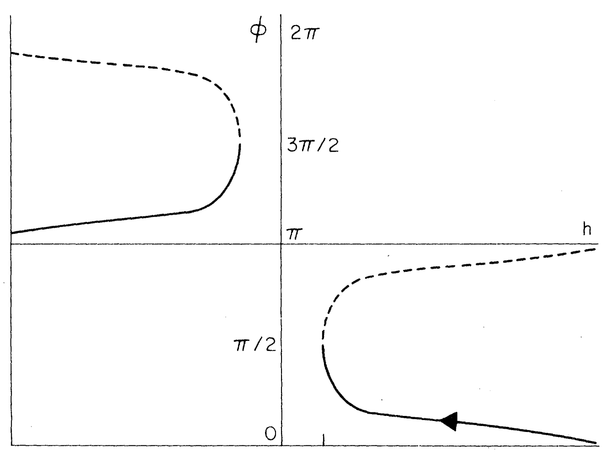

FIG. 1. Approximate fixed lines of Eqs. (15) and (16).

if

$$\epsilon^2/h^2 << \sigma \quad (18)$$

and $h = \sqrt{\sigma}$. The strongest bound is at the present time since h and σ have been decreasing. This gives

$$\mu < 10^{-120}M^3/\Lambda^2 \quad (19)$$

so that $\epsilon/h_{\text{now}} << 1$, as assumed. This means that the Friedmann equations have been valid for a long time and will continue to be so for a long time to come. Note, however, that the present era is in no way special. Prior to the Friedmann era the universe went through a long period of exponential expansion during which the expansion rate changed even more slowly than it does today.

The present analysis should be thought of as provisional. We have introduced the matter-energy density ρ in a phenomenological way, imagining it to have been created at a finite time in the past in some cosmological phase transition, rather than continuing to increase indefinitely as $t \to -\infty$. The real implications of our scenario for cosmological phase transitions, the baryon asymmetry, etc., have not yet been thought through. We hope to report on these questions in a future publication.

The extremely small value of μ which is necessary to make our model work cannot be explained in terms of ratios of any of the scales characterizing ordinary particle physics. Even if we consider supersymmetric models, in which Λ can be as small as 10^3 GeV, we still have $\mu < 10^{-65}$ GeV. The only plausible reason for the appearance of such a small number is a semiclassical tunneling factor.

For example, it is easy to construct extensions of the standard $SU(3) \otimes SU(2) \otimes U(1)$ model in which there are global symmetries spontaneously broken at some large scale f and explicitly broken by SU(2) instantons. The corresponding Goldstone bosons get a mass

$$m \sim (m_W^2/f)\exp\left(-\frac{\pi}{\alpha}\sin^2\theta_W\right)$$

which can satisfy the bound (19) if α, $\sin^2\theta_W$, and/or the instanton action are allowed to differ from their true values by factors of order 1.

Unfortunately, even if we interpret our equations as referring to a scalar field, it cannot be a Goldstone field for a compact symmetry. Such a field lives on a compact manifold. If its potential is a tiny symmetry-breaking effect, it can never grow large enough to cancel the cosmological constant.

We do not know enough about theories in which the field $A_{\mu\nu\lambda}$ appears to know whether a mass term can be induced by semiclassical effects. The requisite tunneling configurations live on three surfaces in Euclidean four-space, and their existence and stability depend on the short-distance physics. Until such time as a theory can be found in which a convincing semiclassical calculation can be done, our "solution" of the problem of the cosmological constant will only have replaced it with the mystery of why μ is so small. Nonetheless, since all perturbative contributions to μ in four dimensions can be eliminated by symmetry arguments, I believe that some progress has been made.

Conversations with E. Witten and J. Polchinski are gratefully acknowledged. I also wish to express my thanks to the Institute for Advanced Study for its hospitality while this work was in progress. This work was supported in part by the Israel Commission for Basic Research.

[a]On leave from Tel-Aviv University, Ramat-Aviv, Israel.

[1]E. Witten, "Fermion Quantum Numbers in Kaluza-Klein Theories (to be published).

[2]S. Hawking, "Quantum Cosmology" (to be published)

[3]J. Schwinger, Phys. Rev. **128**, 2425 (1962).

[4]The possibility that a negative cosmological constant could be cancelled by a scalar field with $\dot{\phi} \neq 0$ was considered in unpublished work by the author and S. Raby and independently by J. Breit, S. Gupta, and A. Zaks. The cosmological equations in such a model do have a solution in which the scalar field energy exactly cancels any negative cosmological constant. However, this solution is unstable to arbitrary perturbations. If we start with a small expansion rate h, the universe passes quickly into a contracting phase. From this point of view the present model solves the problem by making the scalar-field potential very flat in the region where it cancels the cosmological constant. However, for the antisymmetric-tensor field the flatness is technically natural.