Parity-Nonconserving Spin Rotation in Weak Neutron-Deuteron Scattering

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Parity nonconservation in weak *nd* scattering is studied at threshold where neutron spin rotation might occur. The standard (Faddeev) three-body theory with an *s*-wave separable potential is employed together with modifications introduced to account for the weak *NN* interaction, for which I take the one-pion-exchange isovector term. I find $d\phi/dz = 6.13\times 10^{-9}$ rad/cm and estimate the errors resulting from the adoption of the pertinent approximations.

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For about three years, the neutron spin-rotation experiments performed at Grenoble[1] have been studied by many authors.[2] The central motivation in most of these works is the hope to get an enhanced effect due to neutron resonance so that invariance under parity is violated almost "strongly." However, neutron resonance is in general a complex many-particle phenomena and that makes the interpretation difficult. The study of parity nonconservation (PNC) in nuclei should be motivated by the hope to understand the weak *NN* interaction since we still do not have a fundamental theory for that. The *existence* of a flavor-conserving, nonleptonic PNC weak interaction has already been established beyond doubt. (Unlike the case of *T*-invariance violation.)

I think that an understanding of weak *Nd* scattering is crucial in this context. The possibility of reaching *pp*, *np*, and *nn* scattering turns this reaction into an indispensable probe regarding the isotopic structure of the weak interaction. Several authors[3] studied weak *Nd* scattering and calculated asymmetries in the total cross section. However, it appears that an experiment of neutron spin rotation on deuterium is feasible and hence theoretical estimates seem timely.

In this Letter, the results of pertinent calculations are presented. I consider then a system of three nucleons interacting both strongly and weakly. It is now believed that at low energy, the mechanism leading to weak *NN* interaction arises from the exchange of mesons. Thus, the weak *NN* interaction graphs are similar to the strong-interaction ones, except that one vertex contains a weak coupling constant and the interaction is odd under parity. The evaluation of the weak coupling constants from basic principles is a fundamental problem which has received much attention recently. The motivation for this (and others') study is to shed some light on it.

In order to solve the three-nucleon problem with strong and weak *NN* interactions I use a combination of Alt-Grassberger-Sandhas equations[4] and two potential formulas.[5] In fact, I do not solve the ensuing three-body equations but use threshold approximations to evaluate the amplitudes by quadratures. I also assume that at threshold, the main contribution to the weak *NN* interaction arises from the weak one-pion exchange (OPE) which is known to contribute only to the isovector potential. Addition of other terms can also be studied. Finally, I need to know the analytic form of the weak *NN* potentials in momentum space. These are taken from Eqs. (3.1) of Lassey and McKellar's work.[6]

Let me denote by $\langle\lambda_1\lambda_2\vec{k}|W|\lambda_1\lambda_2\vec{k}\rangle$ the weak part of the forward elastic *nd* amplitude in the helicity representation,[7] where $\lambda_1=0,\ \pm 1$ and $\lambda_2 = \pm\frac{1}{2}$ are the helicities of the deuteron and neutron, respectively. For polarized targets the neutron spin-rotation angle per unit length [in a target whose density is ρ (atoms/cm^3)] is given by

$$d\phi(\lambda_1)/dz = -(2\pi/k)\rho\,\mathrm{Re}[\langle\lambda_1\tfrac{1}{2}\vec{k}|W|\lambda_1\tfrac{1}{2}\vec{k}\rangle - \langle\lambda_1-\tfrac{1}{2}\vec{k}|W|\lambda_1-\tfrac{1}{2}\vec{k}\rangle] \tag{1}$$

in the limit $k\to 0$. It is useful to express the partial-wave helicity amplitudes $\langle\lambda_1\lambda_2k|W^J|\lambda_1\lambda_2k\rangle$ in terms of those in the *LSJ* representation

$$\langle LSk|W^J|L'S'k\rangle = W_{LS,L'S'}(k,k)$$

in which channel orbital angular momenta (L,L') and spins (S,S') are specified, since it is in these later amplitudes that PNC is explicitly manifested by $|L-L'|=1$. (The total angular momentum J will not be specified except when necessary.) As I have shown[5] the amplitudes W (in matrix form) are given as a distortion of weak driving terms Z^w by strong amplitudes X. In the separable approximation for the strong NN interaction (which I adopt throughout) the amplitudes X satisfy the usual three-body equations $X=Z+Z\tau X$ with strong driving terms Z and pair spectator propagators τ. Thus, formally

$$W=(1+X\tau)Z^{w}(1+\tau X). \qquad (2)$$

The weak driving terms Z^w (the central quantities throughout) are composed of weak nucleon exchange graphs [Fig. 1(a)] which come from P-wave components in the deuteron wave function, and intermediate weak NN scattering [Fig. 1(b)]. The weak dnp vertices γ^w (triangles) are expressible in terms of the strong ones γ^s (circles), the weak NN interaction w (squares), the NN free propagator $g_0(e)$, and odd-l strong amplitudes $t^s(\text{odd})$, namely $\gamma^w=[1+t^s(\text{odd})g_0]wg_0\gamma^s$. Here, however, I restrict myself only to the $^3S_1$ strong NN interaction (adopted also in previous works because of the enormous complications otherwise). Hence, only the term $wg_0\gamma^s$ survives and I may write

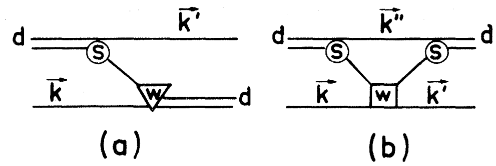

FIG. 1. Born terms for weak Nd scattering. (a) Weak nucleon exchange graph consisting of three-nucleon propagation between strong (S) and weak (W) $d\rightarrow n+p$ vertices. The weak vertex is related to the basic weak NN interaction w according to Eq. (3). (b) Intermediate weak NN scattering. A deuteron "dissociates" strongly into $n+p$, followed by weak NN scattering with the third nucleon and a subsequent strong "recombination" of the deuteron.

$$\gamma^{w}_{jl's's}(q',e)=\int_0^\infty q^2\,dq\; w^{j\mu}_{l's';ls}(q',q)(q^2-e)^{-1}\gamma^{s}_{jls}(q), \qquad (3)$$

in which $l'=j=s=1$ and $l=0$. The partial-wave w's (with $\mu=t_z$, t being the two nucleons' isospin) have been calculated in Sec. 3 of Ref. 6. Evidently, $s=1$ requires $t=0$. Since I concentrate only on the isovector part of the weak force, I have $t'=1$ and hence $s'=1$.

The main results could be summarized as follows: (1) At threshold, the weak nucleon exchange graphs Z^{w1} [Fig. 1(a)] can be calculated analytically to first order in k. (2) An approximate evaluation of the distortion effect at threshold is given in terms of doublet and quartet nd scattering lengths. (3) Intermediate weak NN scattering graphs Z^{w2} [Fig. 1(b)] are complicated and are not evaluated here. I reason, however, that at threshold they are smaller than Z^{w1}. As a consequence of the above three results I have a reasonable approximation for the weak partial amplitudes in the LS representation (namely $W_{LS,L'S'}$) for $k\rightarrow 0$. [Clearly, only $L=0$, $L'=1$ are of interest here, which imply $J=S=\frac{1}{2},\frac{3}{2}$. The on-shell condition $k^2=\frac{4}{3}(E+\alpha^2)$ means that the total three-body energy E equals $-\alpha^2$, the deuteron binding energy.] Hence, by transforming to helicity amplitudes one can calculate $d\phi/dz$. I shall now briefly elaborate on points (1)–(3) and then present the numerical results.

In Eq. (3) I use as usual $\gamma^s_{jls}(q)=N/(q^2+\beta^2)$ and set $e=-\alpha^2$. The isovector part of the weak NN interaction arising from OPE corresponds to $i=1$ in Eq. (3.13a) and Table 3 of Ref. 6 with $m_1=m_\pi$. Apart from geometrical factors, Γ and Ω, and weak coupling V_1 one has to integrate the term in the curly brackets therein multiplied by $[(q^2+\alpha^2)(q^2+\beta^2)]^{-1}$. In the limit $q'\rightarrow 0$ the result is then

$$\gamma^{w}_{jl's}(q')=V_1\Omega_1\Gamma^{j\Delta t}_{l's';ls}NI(\alpha,\beta,m_\pi)q', \qquad (4)$$

where $j=l'=s'=\Delta t=s=1$, $l=0$, and

$$I=(2\pi^2)^{-1}\int dq\, q^2[(q^2+m_\pi^2)(q^2+\beta^2)(q^2+\alpha^2)]^{-1}[1-\tfrac{2}{3}q^2/(q^2+m^2)].$$

The form $\gamma^w=\text{const}\,q'$ is of course expected from P-wave vertices at threshold. Now I can obtain the dynamical part of the weak nucleon exchange graph Z^{w1} [Fig. 1(a)] whose formal expression is $\gamma^s(\vec{q})D(\vec{k},\vec{k}',E)\gamma^w(\vec{q}')$. It is given in detail by Eq. (5.2) of Ref. 5. Again, I can exploit the threshold conditions $k,k'\rightarrow 0$, $q=|(\vec{k}+\frac{1}{2}\vec{k}')|\rightarrow 0$ and set

$$\gamma^s_{jls}(q)=N(q^2+\beta^2)^{-1}\approx N\beta^{-2}, \qquad (5a)$$

$$D(k,k',x,E)=(E-k^2-k'^2-kk'x)^{-1}\approx E^{-1}=-\alpha^{-2} \qquad (5b)$$

in the pertinent equation. Notice that the threshold behavior $Z^{w1}\approx\text{const}\,k$ is implied by the k dependence of A^{IJ} in Eq. (5.1) of Ref. 5. In this equation, the sum is restricted by $L=0$, $L'=1$, $j=j'=1$, $l=0$, $l'=1$, $I=\frac{1}{2}$, $t=0$, $t'=1$, $N=0$, and $J=S$. The final form of the weak nucleon exchange graph of Fig. 1(a) is then

$$Z^{w1}_{L=0,S;L'=1,S';J=S}(k)=C_1(S,S')(f_\pi g_\pi/4\pi\sqrt{2})(N^2/\alpha^2\beta^2)I(\alpha,\beta,m_\pi)k, \tag{6}$$

where $C_1(S,S')$ is a geometrical factor which is easily determined for the four cases, $S,S'=\frac{1}{2},\frac{3}{2}$. Clearly, $\dim(Z)=[L]$.

To assess the effect of distortion, I notice first that in the present approximation the strong amplitudes $X_{LS;L'S'}(k,k')$ are diagonal in L and S. Secondly I intend to put $X_{LS;LS}$ on shell and use $X_{LS;LS}(k,k)\sim k^{2L}$. Hence only $L=0$ strong amplitudes survive at threshold. The amplitude which should be added to Eq. (6) is then given by

$$X_{0S;0S}(k,k)\int\tau(-\alpha^2-\tfrac{3}{4}k''^2)Z^{w1}_{0S;1S'}(k'',k)k''^2\,dk''.$$

I replace the strong amplitude by the corresponding scattering length a_S and in Eqs. (5) I set $k'\to 0$, $k\to k''\neq 0$. The approximately distorted first graph is then given by

$$\bar{Z}^{w1}_{0S;1S'}=\left[1+a_S\alpha^2\beta^2\int_0^\infty dq\,q^2\frac{\tau(-\alpha^2-\frac{3}{4}q^2)}{(q^2+\beta^2)(q^2+\alpha^2)}\right]Z^{w1}_{0S;1S'}. \tag{7}$$

Both the doublet and quartet scattering lengths are positive while $\tau(x)<0$ for $x<-\alpha^2$. Thus, somewhat unfortunately, the distortion acts in an opposite direction to the Born term.

Consider now Z^{w2} of Fig. 1(b). At threshold, it can be approximated by an expression analogous to (6):

$$Z^{w2}_{L=0,S;L'=1,S'}(k)=C_2(S,S')(f_\pi g_\pi/4\pi\sqrt{2})Yk, \tag{8}$$

where $C_2(S,S')$ is a (formidable) geometrical factor and

$$Y=N^2(2\pi^2)^{-1}\int_0^\infty q^2\,dq\,[(q^2+\alpha^2)(q^2+\beta^2)]^{-2}[x^{-1}\ln(1+x)-1],$$

where $x=4q^2/m_\pi^2$. Evaluation of Y then gives $|Z^{w2}/Z^{w1}|\approx 0.184|C_2|/|C_1|$. In the present context I believe an omission of Z^{w2} is bearable. (The constants used above are given below.)

Having obtained the partial-wave weak amplitudes in the LS representation, I can transform now to the helicity representation. The normalization is such that

$$\langle\lambda_1\lambda_2|T^J|\lambda_1\lambda_2\rangle=(1-S^J_{\lambda_1\lambda_2;\lambda_1\lambda_2})/(2ik)$$

so that the total amplitude in the forward direction is

$$f_{\lambda_1\lambda_2;\lambda_1\lambda_2}=-\textstyle\sum_J(2J+1)T^J$$

while the total cross section is $(4\pi/k)\mathrm{Im}f$. Expressing $\langle\lambda_1\lambda_2|T^J|\lambda_1\lambda_2\rangle$ in terms of $\bar{Z}^w_{LS;L'S'}$, both $\bar{Z}^w_{0S;1S'}$ and $\bar{Z}^w_{1S';0S}$ are counted so that the resultant helicity amplitudes will of course be time-reversal invariant. Let me denote Re[$\cdots$] in Eq. (1) by Δ_{λ_1} with $\lambda_1=0,\pm 1$. It has been pointed out by Stodolsky[8] that for an unpolarized target, one should take the average $(\sum\Delta_{\lambda_1})/3$. It is easy to see that if parity is conserved then $\Delta_1=-\Delta_{-1}$ and $\Delta_0=0$. On the other hand for the PNC interaction $\Delta_1=\Delta_{-1}$ and $\Delta_0\neq 0$. Since I am concerned with amplitudes, there is no interference between strong and weak parts. Notice also that a spin-rotation experiment must be performed on a completely unpolarized target.

I can now briefly present the numerical results. The constants related to the strongly interacting three-nucleon system are $\alpha=0.2303$ fm^{-1}, $\beta=1.2412$ fm^{-1}, and $N=0.2942$ fm$^{-5/2}$. The weak coupling constant is $G_\pi=f_\pi g_\pi/(4\pi\sqrt{2})$ with $g^2/4\pi=14$ and $f_\pi=4.56\times10^{-7}$ as suggested by Desplanques, Donoghue, and Holstein.[9] To include distortion I took τ in the pole approximation and found that the coefficient of a_S in Eq. (7) is -0.4068 fm^{-1}. With $^2a=0.65$ fm, $^4a=6.35$ fm for the doublet and quartet nd scattering lengths, the distortion factor is 0.7356 for $\frac{1}{2}\to S'$ and -1.5832 for $\frac{3}{2}\to S'$. Thus in units of $G_\pi=3.4034\times10^{-7}$ I found $\Delta_0=(-7.2275\ \mathrm{fm}^2)k$, $\Delta_1=\Delta_{-1}=(-4.9878\ \mathrm{fm}^2)k$ with the average $(-5.7344\ \mathrm{fm}^2)k$. The matter density of liquid deuterium is $\rho_{D_2}(19\ \mathrm{K})\approx 0.5\times10^{23}$ atoms/cm^3. This gives

$$d\phi/dz\approx 6.1313\times10^{-9}\ \mathrm{rad/cm}.$$

In conclusion, I gave preliminary estimates for neutron spin rotation on deuterium. The theoretical basis is Faddeev-Alt-Grassberger-Sandhas three-body theory which I have modified to include weak NN interaction.[5] The starting equations (before use of two potential formulas) take into account the antisymmetrization, and hence the Pauli principle is respected. The threshold nature of the process enables me to circumvent the enormous computational work required otherwise. As it turns out, the result is helped by the fact that the on-shell nucleon exchange graph is proportional (at threshold) to $1/\alpha^2$, and also by the large value of ${}^4a$. As a result of the low capture cross section of neutrons by deuterium ($\sigma_c = 5.21 \times 10^{-4}$ b at $E_n = 20$ meV) the pertinent experiment is feasible.[10]

Possible sources of errors are the following: (i) Omission of heavier meson exchanges in the weak NN force. The most important contribution comes from isoscalar ρ exchange. In weak n-p scattering I estimated it to be about 25% of the OPE terms. (ii) Neglect of Z^{w2} [Fig. 1(b)], discussed already in the text. (iii) Constraints on the form of the strong NN interaction. Comparison between the works of Ref. 3 indicates that (at least above threshold) the results are sensitive (but not dramatically) to the details of the strong NN interaction.

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