Transverse Electromagnetic Waves with Nonzero $\vec{E}\cdot\vec{B}$

In a recent Letter,[1] Khare and Pradhan demonstrate electromagnetic wave equations with nonzero $\vec{E}\cdot\vec{B}$ values and obtain nonzero values of $q=-\int\vec{E}\cdot\vec{B}\,d^4x$ and $s=\int(E^2-B^2)\,d^4x$. The first result is not actually that remarkable. The latter two seem to stem partly from an error in analysis and partly from introducing static uniform components of $\vec{E}$ and $\vec{B}$ in the long-wavelength limit that fill the universe and give the nonzero q and s. The transversality and mutual orthogonality of $\vec{E}$ and $\vec{B}$ for ordinary light waves are usually demonstrated for unidirectional, propagating waves. However, the quantities q and s need not be zero for static fields, and such nonzero values can be introduced artificially as the zero-frequency limit of a spectral representation. There are nevertheless finite-frequency waves for which $\vec{E}\cdot\vec{B}$ is nonzero.[2] Consider a single wave, here labeled (1), to have $E_y^{(1)}=B_z^{(1)}$ and propagation vector $k_x^{(1)}$. Superimposing a second wave labeled (2) propagating in the opposite direction, $k_x^{(1)}=-k_x^{(2)}$, but with $E_z^{(2)}=B_y^{(2)}=E_y^{(1)}$, then gives a standing wave with $\vec{E}\cdot\vec{B}=2(E_y^{(1)})^2[\cos(2kx)+\cos(2\omega t)]$ which has nonzero time-averaged $\vec{E}\cdot\vec{B}$.

To construct solutions of the above type with nonzero q, consider (instead of sine waves) isolated square-wave pulses having the polarization properties above and encountering one another. During the brief moment that the waves overlap, one has finite nonzero $\vec{E}\cdot\vec{B}$, and zero all other times. If one now considers an infinite train of such pulses going in each direction, then $\vec{E}\cdot\vec{B}$ will apparently have a finite time and spatial average. But such wave trains have dc components of E and B. What we are really doing is filling the universe with uniform $\vec{E}$ and $\vec{B}$ fields, in which case (at least for an empty Euclidean universe) there is no prohibition against having nonzero q and s. The condition for nonzero $\vec{E}\cdot\vec{B}$, in the above example, is that the wave forms have nonzero path integrals: $\int_{-\infty}^{+\infty}A(\eta)\,d\eta\neq 0$, where $A(kx\pm kt)$ is the wave form and A can be E, B, or the vector potential. Such waves simply have a static component, as does a single square wave rising from zero initial amplitude. In a spectral representation, the issue is then how the amplitude varies as $k\to 0$.

Khare and Pradhan also take a faulty limit. One sees in their Eqs. (15b) and (15c) that their q and s vary as $\sin 2\alpha$ and $\cos 2\alpha$, where α is the phase of the above standing wave, written as $\sin(Kt+\alpha)$, etc. [their Eqs. (5) and (8)]. If so, these two "gauge and Lorentz-invariant properties" depend on the choice of the zero of the arbitrary time coordinate, and are not even Galilean invariants. In the case of q, for example, this result seems to follow in Eq. (12b) from rewriting $2\sin(Kt+\alpha)\cos(Kt+\alpha)$ as $\sin(2\alpha)\cos(2Kt)$, apparently on the assumption that the time integral over the $\sin(2Kt)$ piece from $-\infty$ to $+\infty$ is symmetric and therefore identically zero. Such a "symmetry" is gauge noninvariant. The average of these two limits is indeterminant, and cannot be constrained to fall at $t=0$ regardless of the (arbitrary) choice of $t=0$! In any case, it seems obvious by inspection that the time average of $\sin(kt+\alpha)\cos(kt+\alpha)$ is zero if k is nonzero. In a separate erratum,[3] the same authors still claim to obtain their original results ["...q, s, and E are all finite and identical to Eqs. 15(a) to 15(c)"] under even more general conditions.

F. Curtis Michel
Department of Space Physics and Astronomy
Rice University, Houston, Texas 77251

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