Quantum Oscillations and the Aharonov-Bohm Effect for Parallel Resistors

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The transmission coefficient between two terminals of a one-dimensional ring with arbitrary scatterers is calculated exactly as a function of enclosed magnetic flux, φ. At low temperatures, where the inelastic diffusion length is larger than the size of the ring, its conductance follows from the Landauer formula. Oscillations of the conductance as a function of the characteristics of the scatterers and of φ (with a period $\varphi_0 = hc/e$) are found. The oscillations persist even when the elastic scattering is strong.

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It is known that quantum interference effects are present in superconducting devices as well as in very small pure metallic elements. In particular, in the presence of magnetic flux, Aharonov-Bohm-type oscillations[1] may occur in doubly connected systems.[2-6] For a normal metal the period of these oscillations is $\varphi_0 = hc/e$. It has been believed, though, that such effects should vanish once the elastic mean free path of the electron is smaller than or of the order of the system's size. Recently, it was pointed out[7] that in dirty metals another type of oscillation, of period $\varphi_0/2$, may occur.[8] Later, it was argued[9] that thermodynamic properties of a normal ring may have a periodicity φ_0 in an external magnetic flux also in the case where the *elastic* mean free path is small. An ac-Josephson-type effect follows. This is related to current oscillations in the presence of a time-dependent flux which induces a dc voltage. Such oscillations may also occur in systems exhibiting the quantized Hall effect.[10]

In the present Letter we discuss the transport properties of a one-dimensional disordered metallic ring. We derive an *exact expression* for the transmission, T, through such a device as a function of the transmission and reflection coefficients of each of its channels. With use of the Landauer formula,[11] T may be related to the conductance of this ring. We find a very rich behavior as a function of these coefficients. Some of these results may also be relevant to optical and microwave systems. We next extend the treatment to include the effects of a magnetic flux through the center of the ring. *In this case the conductance exhibits oscillations with a period* φ_0. All these quantum interference effects, both in the presence and the absence of the magnetic flux, exist also in the limit of strong scattering. Even in this limit there are situations where T oscillates between 1 (total transmission) and 0 (total reflection). Thus, the oscillations are not destroyed, in principle, by elastic scattering—in the presence of which eigenstates with well defined (space-dependent) phases still exist. Obviously a strong enough inelastic scattering will, however, wash out these effects. This should occur once the inelastic diffusion length becomes smaller than the dimensions of the ring, and it suggests the possibility of very interesting and peculiar temperature dependences of the conductances of such rings. Our treatment is valid for the case of each branch being a single channel; estimates for the effects of many channels are also made, and the relation with the results of Refs. 7 and 8 is discussed.

The geometry of our system is described in Fig. 1. Each branch of the ring is described[11,12] schematically as a single scatterer connected to

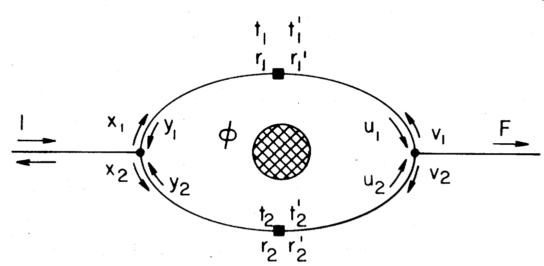

FIG. 1. Schematic picture of the system. The arrows denote the various transmitted and reflected amplitudes, defined close to the junctions. The phases accumulated through the channels are absorbed in the scattering coefficients (r_i, r_i', t_i, and t_i').

an ideal, *mathematically one-dimensional,* channel. All phases and scattering effects along the channels are absorbed in the parameters describing each scatterer. These parameters are t_i ($i=1,2$) and t_i', the transmission amplitudes from the left and from the right, respectively, and r_i (r_i'), the reflection amplitudes on the left (right) of the scatterer. Notice that time-reversal and current-conservation requirements,[12] which imply $t_i=t_i'$ and

$$-t_i/t_i'^* = r_i/r_i'^* \tag{1}$$

(the asterisk denotes complex conjugation), are satisfied also when the phases of each path are absorbed in t_i, etc. Moreover, when magnetic flux φ is applied through the center of the ring, a possible gauge transformation for the transmission and reflection amplitudes yields $t_1 \to t_1 e^{-i\theta}$, $t_1' \to t_1 e^{+i\theta}$, $t_2 \to t_2 e^{+i\theta}$, $t_2' \to t_2 e^{-i\theta}$, $r_i \to r_i$, $r_i' \to r_i'$ ($\theta \equiv \pi\varphi/\varphi_0$), and the transformed t's and r's still satisfy Eq. (1).

Following Shapiro[13] we describe each junction by a 3×3 scattering matrix S,

$$S=\begin{pmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & -1/2 & 1/2 \end{pmatrix}, \tag{2}$$

where each diagonal element, S_{ii} ($i=1,2,3$), denotes the reflection amplitude of the ith channel, and off-diagonal elements S_{ij} ($i\neq j$) are the transmission amplitudes from channel i to j. In Fig. 1 channel 1 of the left-hand-side junction is chosen to be that of the incoming amplitude (unity) whereas channel 1 of the right-hand-side junction is that of the outgoing amplitude (F). We do not expect our results to depend qualitatively on the choice of the junction's scattering matrix.

Writing down the linear relationships among the various amplitudes at the junctions and scatterers and using sum and difference variables (e.g., $x_1 \pm x_2$, etc.), we find after some algebra that the total transmission amplitude of the ring is given by

$$F=2\frac{t_1t_2(t_1'+t_2')+t_1(r_2-1)(1-r_2')+t_2(r_1-1)(1-r_1')}{(t_1+t_2)(t_1'+t_2')-(2-r_1-r_2)(2-r_1'-r_2')}. \tag{3}$$

This can be rewritten as

$$F=2\frac{Ae^{i\theta}+Be^{-i\theta}}{De^{+2i\theta}+Ee^{-2i\theta}+C}, \tag{4}$$

where

$$A=t_1^2t_2+t_2(r_1-1)(1-r_1'),\quad B=t_1t_2^2+t_1(r_2-1)(1-r_2'),$$

$$D=E=t_1t_2,\quad C=t_1^2+t_2^2-(2-r_1-r_2)(2-r_1'-r_2').$$

The transmitted intensity ($\phi\equiv 2\theta$) is

$$T\equiv|F|^2=4\frac{\alpha+\beta\cos\phi+\beta'\sin\phi}{\gamma+\delta\cos\phi+\epsilon\cos2\phi}, \tag{5}$$

where $\alpha=|A|^2+|B|^2$, $\beta=2\,\mathrm{Re}(AB^*)$, $\beta'=-2\times\mathrm{Im}(AB^*)$, $\gamma=|D|^2+|E|^2+|C|^2$, $\delta=4\,\mathrm{Re}(DC^*)$, and $\epsilon=2|D|^2$. According to Landauer's formula the conductance is given by $G=(e^2/\pi\hbar)\,T/(1-T)$.

Let us first consider the case with no magnetic flux ($\varphi=0$). Even in this case T may exhibit oscillations as a function of the phases of the t's and r's[14] [which influence the coefficients α, β, γ, δ, and ϵ in Eq. (5)]. When $t_1=0$, an appropriate choice of the phases of r_1 and r_1' may result in $T=1$ or $T=0$.[15] That is, by tuning the nonconducting branch we can affect dramatically the transmission through the other channel. This effect is present also when $|t_1|$ and $|t_2|$ are finite. In particular, we can obtain $T=0$ even when $|t_1| \ll |t_2|$. On the other hand, one can improve the conductance of a scatterer by connecting to it in parallel a very poor (tuned) conductor. Notice that these resonances will disappear once the inelastic diffusion length $l_{\mathrm{in}}=(D\tau_{\mathrm{in}})^{1/2}$ (τ_{in} is the inelastic scattering time) becomes of the order of the size of the ring, L. Thus we may obtain dramatic changes (either increase or decrease) of G with the temperature. Another interesting effect may occur if we expose one of the channels (say channel 1 with $|t_1|\ll|t_2|$) to, e.g., electromagnetic fields, whose effect is similar to inelastic scattering. This may cause a dramatic change in the transmission of the weakly scattering channel. When $t_1=t_2=t$, Eq. (3) reduces to $F=t$. We expect this result to hold also for an n-branch system. Thus when the temperature increases the conductance should increase from

$G_t = (e^2/\pi\hbar)|t|^2/(1-|t|^2)$ to Ohm's law $G = nG_t$.

We now turn on a magnetic flux φ. In general T is periodic in φ with a period φ_0, although one can find specific conditions on the coefficients in Eq. (5) to enhance the effect of the second harmonic and make the effective period $\varphi_0/2$. Our effect is thus different from the one reported in Refs. 7 and 8. This effect may be very strong even in the limit of strong scattering ($l_{el} \ll L \ll l_{in}$). Assume, for example, that $|t_1| \sim |t_2| \sim t \ll 1$. In that case, unless very special phase relations hold, $\alpha \sim \beta \sim \beta' \sim \delta \sim \delta' \sim t^2$, $\epsilon \sim \epsilon' \sim t^4$, and $\gamma \sim 1$. These are then oscillations of a period φ_0 and an amplitude $\sim t^2$ appearing on a constant background of $\sim t^2$ (thus T may vanish for certain values of φ). In addition there are also (second-harmonic) oscillations of a period $\varphi_0/2$ and a smaller amplitude $\sim t^4$. If $|t_1| \ll t_2$ the relative size of the oscillations becomes smaller (and again, the harmonics with a period $\varphi_0/2$ are even smaller). A more detailed analysis of the structure of T as a function of the various phases is now in progress.

Finally we comment on the possibility of observing such effects experimentally. Values of l_{in} in the range of 10^3 Å and even more have been reported for the ~1 K temperature range. One may achieve $l_{in} \gg L$ by using temperatures, say, in the 10 mK range (with $L \sim 1$ μm) or by using modern nanometer-scale fabrication techniques[16-18] ($L \sim 10^3$ Å) at $T \sim 1$ K. These estimates are based on an inelastic time $\tau_{in} \sim \hbar/kT$, a rough phenomenological estimate which usually gives a correct order of magnitude and a temperature dependence which is not far from that observed experimentally. One may consider also spin-orbit scattering. However, the relevant length for this mechanism is for light metals as large as ~0.5 μm. As far as the contribution to, e.g., the magnetoresistance due to electron-electron interactions is concerned, it is believed to be additive[19] to the localization contribution. Thus, it is extremely improbable that the two contributions will exactly cancel the effect obtained here. Notice also that in some cases of two dimensions for the temperature range discussed here the interaction contribution to the magnetoresistance is expected to be much smaller than the localization one. Another requirement that should be satisfied is the one dimensionality of the ring. The periodicity in the magnetic field is satisfied even if there are a number of modes in the channels. However, in order to observe large amplitudes of relative oscillations and sharp resonance effects one should require that not too many modes, whose effects may cancel, be present. The question of many transverse modes requires further work. It is of interest whether or not modes in the radial direction and in the direction parallel to the flux could behave differently, which may mean that long thin cylinders may also exhibit these effects. The oscillation[7,8] with period $\varphi_0/2$ is due to backscattering interference—waves coming back to the initial point after traversing the whole ring in opposite directions[20] and thus accumulating a phase difference of 4θ. This is valid for strict Aharonov-Bohm flux or when the variation of the flux across the sample wall is negligible. For N parallel channels this effect adds coherently and yields a relative change of order unity in the resistance. For our effect (of period φ_0) the parallel channels may be thought to add randomly and therefore the relative effect to be of order $N^{-1/2}$. For $N \sim 10^3-10^4$, which is attainable, this difference may well be balanced by the requirement that the waves propagate *twice* the length without inelastic scattering for the effect of Refs. 7 and 8 as compared to ours. Also, one may argue that $O(N^2)$ terms will exist in our case as a result of all the interferences among the N channels in each of the two arms of the ring. If these add randomly, one will still get an $O(1)$ relative effect. Clearly, this needs a more definitive calculation.

We emphasize that our results are exact for uncorrelated electrons in the one-dimensional geometry studied. For example, multiple reflections and transmissions through the junctions and the scatterers are taken into account, as in series addition of Landauer-type obstacles.[11,12]

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