Effective Couplings of Grand Unified Theories in Curved Space-Time

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The curved–space-time generalizations of SU(5) grand unified theories are considered. The high-curvature limit of the effective gravitational and cosmological constants, as well as other coupling constants not present in flat space-time, is studied using renormalization-group methods. These effective coupling constants appear in the gravitational field equations and may have important cosmological implications.

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The role played by grand unified theories in cosmology is currently of great interest. By use of the renormalization group, it is possible to predict the high-curvature behavior of the gravitational and cosmological constants. As we shall see, these predictions may be important for the early universe.

Coupling constants not present in the flat–space-time Lagrangian appear both in the generalized Einstein-Hilbert action and in the terms linking the Higgs scalars to the scalar curvature R. These additional terms, which are required in curved space-time for renormalizability, appear in the following part of the total Lagrangian:

$$L_{\rm curv} = \Lambda + \kappa R + \alpha_1 R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} + \alpha_2 R^{\mu\nu} R_{\mu\nu} + \alpha_3 R^2 - \xi_\phi R\, \mathrm{tr}(\phi^2) - \xi_H R H^\dagger H. \qquad (1)$$

Here Λ and κ are related to the cosmological constant, Λ_c, and Newtonian gravitational constant, G, by $\Lambda = -(8\pi G)^{-1}\Lambda_c$ and $\kappa = (16\pi G)^{-1}$. The scalar field multiplets ϕ and H are the Higgs fields, and the trace is over the group indices of ϕ. In addition to $L_{\rm curv}$, the total Lagrangian will have contributions from the gauge, fermion, and scalar fields appropriate to the grand unified theory under consideration. The detailed expression, which is not essential for the present discussion, may be found in Parker and Toms.[1]

As is well known,[2] renormalization requires the introduction of a renormalization point μ with dimension of mass. The bare couplings must be independent of μ, leading to renormalization-group equations for the effective coupling constants. These equations show how the effective couplings change as μ is scaled by a dimensionless parameter s. It can be shown[3] that in curved space-time the parameter s corresponds to a scaling $g_{\mu\nu} \to s^{-2} g_{\mu\nu}$ of the metric.[4] By consideration of curvature invariants, such as R^2, it follows that the large-s limit gives the high-curvature behavior of the theory. The renormalization-group functions required to study this high-curvature behavior are found from a computation of the renormalization counterterms in $L_{\rm curv}$. We have computed these counterterms,[1] using the background-field method,[5] for the Georgi-Glashow theory[6] (in the unbroken-symmetric phase) as well as for an asymptotically free generalization of that theory.[7] Because we deal with the unbroken symmetry and with a background metric, our results are most relevant to the range between the grand unified theory (GUT) and Planck scales.

For the asymptotically free theory, the coupling constants which do not appear in $L_{\rm curv}$ are all expressed in terms of the gauge coupling constant g. All of these couplings are then small at high curvature as a result of asymptotic freedom.[8] Consequently, definite results may be obtained by perturbation theory for the high-curvature limits of Λ, κ, and the other effective coupling constants appearing in $L_{\rm curv}$. Results may be obtained for other GUT's provided that, as is generally assumed, the coupling constants do not grow too large.

We find that the value of the effective gravitational constant G at early times may be altered from its present value. The effective α_i coupling constants, which are coefficients of terms in the action that are quadratic in the curvature, are found to grow large in magnitude at high curvature. This behavior is valid even if these constants are extremely small or zero today. The cosmological constant Λ_c exhibits similar behavior. In the high-curvature limit, Λ_c becomes large and positive regardless of its present value. This may have implications for inflation[9] in the early universe.

The fact that quantum corrections can lead to terms in the effective action that are not present in the classical action is related to the idea of induced gravity.[10] One difference is that in the present work we are concerned with the high-curvature limit of the theory, whereas the work on induced gravity has been mainly concerned with obtaining unique predictions for Λ_c and G at low curvature.

Solutions to the renormalization-group equations in the asymptotically free theories of Chang *et al.*[7] for the effective coupling constants ξ_ϕ and ξ_H are found to grow increasingly large in magnitude at high curvature in the absence of further constraints. However, within the context of an asymptotically free theory it is natural to search for a solution in which the coupling constants are proportional to some power of the gauge coupling g. We find that, although this cannot be done for ξ_i ($i=\phi$ or H), it is possible to find a solution of the form

$$\bar{\xi}_i(s)=g^2(s)r_i, \tag{2}$$

where $\bar{\xi}_i=\xi_i-\frac{1}{6}$, the r_i are constants, and $g(s)$ is the running gauge coupling constant. The r_i are determined by the two-loop contributions to the renormalization-group functions which govern the behavior of $\xi_i(s)$. A full discussion is given in Ref. 1. In view of the asymptotic freedom of $g(s)$, it is evident from Eq. (2) that in the high-curvature limit $\xi_i(s)\to\frac{1}{6}$ as $s\to\infty$. In conjunction with the asymptotic freedom of the Higgs self-couplings, this behavior will suppress particle creation by isotropically expanding universes.[11]

The effective coupling constants $\alpha_i(s)$ are found to be

$$\alpha_1(s)=\alpha_1(1)-\frac{391}{(4\pi)^2 720}\ln s, \tag{3}$$

$$\alpha_2(s)=\alpha_2(1)-\frac{571}{(4\pi)^2 45}\ln s, \tag{4}$$

$$\alpha_3(s)=\alpha_3(1)+\frac{583}{(4\pi)^2 144}\ln s+\text{small terms}, \tag{5}$$

where the $\alpha_i(1)$ are constants of integration. The coefficients of $\ln s$ are determined by the numbers of fields present in the theory. The above values apply to the first paper in Ref. 7. Solutions of the same form, which are based on the one-loop approximation, are also valid for the Georgi-Glashow model[6] provided that the coupling constants which are not asymptotically free do not grow too large at large s. In the case of the Georgi-Glashow theory, the numerators 391, 571, and 583 of the fractions multiplying $\ln s$ become 797, 542, and 525 for Eqs. (3)–(5), respectively. In nonconformally flat space-time the contribution of the running coefficients $\alpha_i(s)$ to the effective Einstein equations is larger than the contribution of the trace anomaly[12] by a factor of $\ln s$ [if we set $\alpha_i(1)=0$]. Because the curvature during the GUT era is considerably larger than the mean curvature today, this contribution to the curvature-squared terms in the Einstein equations would be expected to be at least an order of magnitude larger than that of the trace anomaly. The effects of these terms will be of cosmological significance, and can be studied by established methods.[13]

In the high-curvature limit, we find that the effective gravitational Lagrangian takes the form $L_{\text{curv}}=AC^{\alpha\beta\gamma\delta}C_{\alpha\beta\gamma\delta}+BG$, where $C_{\alpha\beta\gamma\delta}$ is the Weyl curvature tensor, and G is the integrand of the Euler characteristic. A and B depend on the number of fields and are proportioned to $\ln s$. Regardless of the particle content of the GUT, A is negative and B is positive. This form of the Lagrangian leads to field equations with interesting conformal properties at high curvature. In conformally flat space-times, the coefficients of the quadratic curvature terms in the field equations approach constant values. (For a more complete discussion, see Ref. 1.)

The renormalization-group function which governs the effective behavior of κ depends upon ξ_ϕ and ξ_H. If we use solutions for $\xi(s)$ of the form given in Eqs. (2), then the result for the running Newtonian constant $G(s)$ is given by

$$[16\pi G(s)]^{-1}=K\mu_\phi^2(1)[1+3(2\pi)^{-2}g_0^2\ln s]^{2.106}+\text{const}, \tag{6}$$

where $\mu_\phi^2(1)$ and g_0 are the values of the Higgs mass and gauge coupling constant at a given renormalization point $\mu=\mu_0$. We have taken three generations of fermions. The constant K is

$$K=1.90r_1-0.37r_2, \tag{7}$$

where r_1 and r_2 were defined in Eq. (2).

The effective cosmological constant $\Lambda_c(s)$ is given by

$$[8\pi G(s)]^{-1}\Lambda_c(s)=0.35g_0^{-2}\mu_\phi^4(1)\{[1+3(2\pi)^{-2}g_0^2\ln s]^{3.106}-1\}+\text{const}, \tag{8}$$

where $G(s)$ appears in Eq. (6).

The constants of integration appearing in Eqs. (6) and (8) may be determined in terms of the present values of the Newtonian and cosmological constants, although this would require a more detailed analysis, taking account of symmetry breaking. If we ignore these details in order to obtain crude estimates of the behavior of $G(s)$ and $\Lambda_c(s)$, and assume $|K| \sim 1$, $\mu_\phi(1) \sim 10^{15}$ GeV, and $g_0 \sim 1$, then because the constant of integration in Eq. (6) is extremely large, $G(s)$ is essentially unchanged from today's value through the GUT era. On the other hand, the constant in Eq. (8) will be very small or zero because of the present value of Λ_c, so that at the GUT time, $\Lambda_c(s)$ is at least of order $m_{\rm P}^{-2}\mu_\phi^4(1)$ where $m_{\rm P}$ is the Planck mass. (The presence of the $\ln s$ term could increase this estimate by several orders of magnitude.) This contribution is of the same order and sign as that obtained from the vacuum energy in the inflationary scenario.[9]

In GUT models which are not asymptotically free (such as the Georgi-Glashow model[6]), if, as is usually assumed, the coupling constants remain small at the GUT time, then the above order-of-magnitude estimates for $\Lambda_c(s)$ and $G(s)$ would also be valid. For $G(s)$ this is because the constant appearing in Eq. (6) will still be of order $m_{\rm P}^2$ and dominate the first term which remains of order $\mu_\phi^2(1)$. Because $G(s)$ remains essentially constant, $\Lambda_c(s)$ in these theories will still be of order $m_{\rm P}^{-2}\mu_\phi^4(1)$.

We have considered how renormalization-group effects may be of importance for grand unified theories in the early universe. We were particularly interested in the behavior of the coupling constants occurring in $L_{\rm curv}$ given in Eq.(1). Further investigation, including the effects of symmetry breaking, is underway.

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